ANALYSIS ON INCLINATION ANGLE OF SHEAR BAND UNDER LOW CONFINING PRESSURE BASED ON GRADIENT-DEPENDENT PLASTICITY*

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Abstract The inclination angle of shear band is analyzed considering heterogeneity of rock material when a single shear band is formed in the center of specimen under triaxial compression. The analytical solution of post-peak axial stress-axial strain curve is deduced using the assumption that the total post-peak deformation is composed of entire uniform elastic deformation and localized shear plastic deformation dependent on the thickness of shear band. The obtained solution shows that the post-peak stiffness is related to the inclination angle of shear band，confining pressure，thickness of shear band and elastic modulus，etc. Using the solution，the expression for the inclination angle of shear band can be presented easily and it is dependent on constitutive parameters of rock material and geometry parameters of rock specimen. Larger dilation angle or loading rate leads to increment of the inclination angle. In addition，the inclination angle increases with the thickness of the shear band，which cannot be explained or forecasted by other existing solutions，such as Coulomb inclination，Roscoe inclination and Arthur inclination，etc. The presented analytical results are compared with earlier experimental investigations and the agreement is good.
Key words gradient-dependent plasticity，localization，inclination angle，shear band，confining pressure
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1 INTRODUCTION

Shear band analysis is a classical issue with great importance for engineering applications. The inclination angle of shear band renders us an insight into instability of rock materials and phenomenon of strain localization. The angle of shear band has been theoretically and experimentally investigated by a number of researchers，with the main thrust provided by Coulomb(1776)，Roscoe(1970)，Arthur(1977)，Vardoulakis(1980)，Arthur and Dunstan(1982)，Vermeer(1990) and Lade (2001)et al[1~3].

The main disadvantage of earlier theoretical solutions is that the thickness of shear band is regarded as zero and the distributed plastic shear strain in shear band is not considered.

Based on gradient-dependent plasticity，Pan et al obtained the solution of inclination angle of shear band dependent on dilatant coefficient[4]. Based on the same theory and resorting to conservation of energy，Wang and Pan got the expression of shear band dependent on constitutive and geometry parameters of rock specimen[5]. Suppose that the failure pattern of rock specimen subjected to axial compressive stress is shear band，a formula for size effect was derived by Wang and Pan[6]. Using the formula proposed，the size effect of inclination angle of shear band which can not be forecasted by other formulas was discussed.

In the present contribution we shall utilize gradient-dependent plasticity to investigate how the inclination angle of shear band can vary with confining pressure.

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Analytical results are compared with earlier investigations.

2 ANALYSIS

As shown in Fig.1，a rock specimen with length L is subjected to uniform compressive stress σ in the two ends and confining pressure p in the four lateral surfaces. Strain localization is initiated at the peak stress in the form of single shear band. The stable orientation of shear band is $\alpha = \pi/2 - \theta$ of being parallel to the axial direction and the width of the band is w. The failure pattern of rock in shear band is caused by shear stress τ parallel to the tangential direction of the band. After the localization is initiated，the rock outside the shear band remains intact and is elastically unloaded according to elastic modulus. According to the knowledge of mechanics of materials，shear stress and its maximum τ_c can be expressed as

$$\tau = 0.5(\sigma - p)\sin 2\alpha \tag{1}$$

$$\tau_c = 0.5(\sigma_c - p)\sin 2\alpha \tag{2}$$

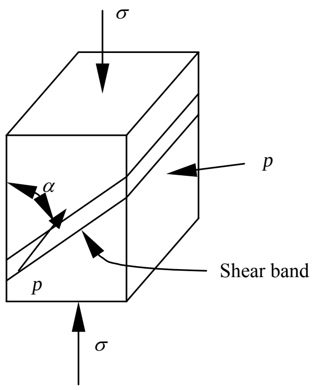

Fig.1　The rock specimen under triaxial compression

The constitutive relation between shear stress and shear strain is bilinear. In elastic region，shear modulus governs the relation：$\tau = G\gamma$. After τ_c is attained，strain softening occurs and the slope of curve τ - γ is $-\lambda$. λ is called shear softening modulus and its value is always positive. For simplicity，the shear band is treated as a one-dimensional problem in its normal direction (y-direction). In the boundary between elastic and plastic zone，$y = \pm w/2$. Supposing that the plastic relative shear displacement across the shear band is $2s$，the displacement s can be calculated by using

$$s = \int_0^{\frac{w}{2}} \gamma^{\mathrm{p}} \mathrm{d}y = \frac{w}{2} \cdot \frac{\tau_c - \tau}{c} \tag{3}$$

where $c = G\lambda/(G+\lambda)$，γ^{p} is local plastic shear strain in shear band[5，7]. The axial compressive strain of specimen can be given by

$$\varepsilon_a = \frac{\sigma}{E} - 2\upsilon\frac{p}{E} + 2s\frac{\cos\alpha}{L} \tag{4}$$

where E is elastic modulus，and υ is Poisson's ratio. Differentiation of Eq.(4) yields

$$\mathrm{d}\varepsilon_a = \left(\frac{1}{E} - \frac{w\sin\alpha\cos^2\alpha}{cL}\right)\mathrm{d}\sigma \tag{5}$$

Supposing that the slope of the post peak portion of complete axial stress-axial strain curve is $-\lambda'$，$\lambda' > 0$，we can get

$$\lambda' = \left|\frac{\mathrm{d}\sigma}{\mathrm{d}\varepsilon_a}\right| = \left(\frac{w\sin\alpha\cos^2\alpha}{cL} - \frac{1}{E}\right)^{-1} \tag{6}$$

The post-failure portion of axial stress-axial strain curve becomes less steep as the confining pressure is increased，as is depicted in Fig.2. In general，for rock specimen having moderate length to diameter ratio，many experimental values of the inclination angle of shear band is $\theta \in [55°，85°]$ and $\cos\theta(1-\cos^2\theta)$ decreases with θ. Therefore，if θ lies between 55° and 85°，then θ increases with λ. That is to say，increasing confining pressure leads to decreasing inclination angle of shear band. From Eq.(6)，we can get

$$\cos\theta = \sqrt[3]{t+s} + \sqrt[3]{t-s} \tag{7}$$

where $t = -0.5Q$，$Q = \dfrac{L}{w}\dfrac{1/\lambda' + 1/E}{1/\lambda + 1/G}$ and $s = \sqrt{(Q/2)^2 - 1/9}$.

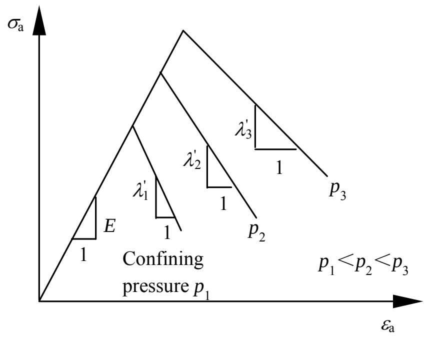

Fig.2　The influence of confining pressure on post-peak srain softening behavior

Many experimental results show that dilation angle decreases with increasing confining pressure. From Eq.(6), we can conclude that increasing dilation angle, i. e. decreasing confining pressure or increasing λ', leads to increasing inclination angle of shear band. The results are consistent with Roscoe and intermediate inclination qualitatively[1~3].

Some experimental observations show that larger loading rate can cause more brittle axial response in strain softening region, i.e. λ' increases with loading rate. Therefore, according to Eq.(6), the inclination angle of shear band increases with loading rate. Numerical results[8] also show similar phenomenon.

In order to study the influence of λ' and w on α, we take parameters as follows: $M = 5\lambda'/E$, $E = 20$ GPa, $G = 0.25\,E$, $\lambda = 0.003\,2\,E$, $L = 0.1$ m. The relation between α and M is shown in Fig.3.

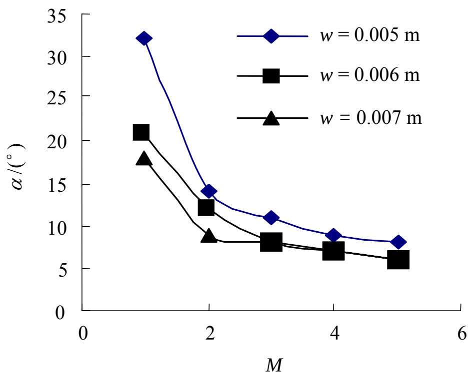

Fig.3 The influence of thickness of shear band on inclination angle of shear band

For the lower λ', i.e. the larger confining pressure, inclination angle of shear band θ has the lower value. The results consist with experimental results obtained by Besuelle et al[9]. The orientation of the shear band with respect to the minor principal stress increases with the width of the shear band, which cannot be forecasted by other expressions for the inclination of shear band.

Herein, for simplicity, it is assumed that the relation between p and $1/\lambda'$ is linear:

$$1/\lambda' = kp + b \tag{8}$$

where k and b are two parameters. If λ' is not sensitive to p, then $k = 0$. Upon substitution of Eq.(8) in Eq.(6), a relation is obtained between confining pressure and inclination angle of shear band.

Ord et al conducted an extensive investigation on Gosford sandstone and some experimental results are listed in Table 1. The reason for scatter in data on inclination angle of shear band is that the result is very sensitive to boundary conditions[3, 10].

Table 1 The some experimental results obtained by Ord [10]

Experiments	Confining pressure/MPa	Shear modulus /GPa	Elastic Modulus/GPa	Inclination angle /(°)
RA0624	5	5.5	13.7	63/65
RA0627	10	6.4	18.0	58/59
RA0629	7.5	7.1	18.3	57/66
RA0636	20	6.3	16.3	54/55/56
RA0640	15	4.6	12.1	62/63

We take other parameters as follows: $L = 0.08$ m, $w = 0.004\,63$ m, $\lambda = 0.003\,2\,E$, $bE = 5$, $kE = 1/19.7$. The comparison between theoretical and experimental results is shown in Fig.4. Straight line is the results of linear regression on experimental results and shows that increasing leads to decreasing inclination angle of shear band. Compared with results of linear regression, the maximum error is 3° and the agreement is good. The agreement between experimental and presented theoretical results is good except for the data in confining pressure of 10 MPa.

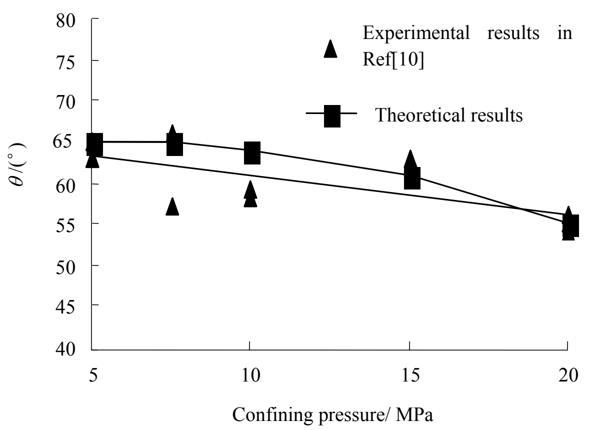

Fig.4 Comparison between theoretical and experimental results for inclination angle

3 DISCUSSION

The presented analysis of inclination angle of shear band is applicable for rock specimen under low confining pressure. Firstly，if the confining pressure is high，the slope of axial stress-axial strain curve can be positive in the post-failure region，which disagrees with the presented bilinear constitutive relation of rock material. Secondly，for higher confining pressure，multiple shear bands are universal and some shear bands even traverse lateral surface from upper or lower end，as the case is not in agreement with the assumption that only a single formed shearing band goes through the two lateral surfaces. Thirdly，the inclination angle of shear band can lower than 55° due to high confining pressure. Fourthly，for higher pressure，all the bands can be concentrated in half of the specimen[9]. In the end，the condition modeled is triaxial compression，i.e. $\sigma_2 = \sigma_3 = p$. Present results show that σ_2 has no influence on shear stress in boundary between elastic and plastic zone. Bt in fact，σ_2 affects not only failure mode，but also the inclination angle of shear band. It should be，noted that the inclination angle of shear plane is analyzed based on twin shear theory[11]. The main advantage of gradient-dependent plasticity is that the thickness of shear band can be determined according to internal length parameter reflecting heteropeneity[12～14].

It is apparent that further theoretical investigations are necessary to reveal the regularities of strain localization of rock under high confining pressure and under other experimental conditions.

4 CONCLUSIONS

The inclination angle of shear band is dependent on constitutive relation of rock material and geometry parameters of rock specimen. The inclination angle of shear band decreases with increasing confining pressure. Larger dilation angle or loading rate leads to increase of the inclination angle. The inclination angle increases with the width of the shear band. Analytical results are compared with earlier investigations and the agreement is good.

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