Computer experiments in harmonic analysis

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Abstract

It is conventionally understood that computers play a rather limited role in theoretical mathematics. While computation is indispensable in applied mathematics and the theory of computing and algorithms is rich and thriving, one does not, even today, expect to find computers in theoretical mathematics settings beyond the theory of computing. Where computers are used, by those studying combinatorics , algebra, number theory, or dynamical systems, the computer most often assumes the role of an automated and speedy theoretician, performing manipulations and checking cases in a way assumed to be possible for human theoreticians, if only they had the time, the memory, and the precision. Automated proofs have become standard tools in mathematical logic, and it is often expected that proofs be published in a computer-checkable format.

It is not surprising, then, that most philosophical work on computers in theoretical mathematics has been on computers' roles as supplementary mathematicians. Donald MacKenzie's 2001 book Mechanizing Proof demonstrates the rich potential for social and historical studies to complement the substantial analytic debate in this area of philosophy. But what of computers in theoretical mathematics behaving as computers, and not as mere mechanized mathematicians? Very little role is commonly assumed for computers working as supplements to mathematicians, rather than as supplementary mathematicians themselves. Accordingly, very little philosophy has attempted to grapple with theoretical mathematics in which computers play an essential but essentially non-theoretical role.

My presentation will draw on work I conducted as a researcher in harmonic analysis on fractals at Cornell University. I will analyze the explicit and implicit conceptual apparatus employed in my and my fellow researchers' use of computers in the theoretical study of second order differential equations, such as those for sound and heat flow, on various fractal analogues of the Sierpinski gasket. Such gaskets are easy to visualize in very crude approximation in a low number of dimensions. As one increases the complexity of the gasket or the refinement of one's analysis, visualization and precise computation become impossible, and soon computers are unable to produce even approximate data to model differential equations in these situations. We thus had to carefully choose analytic approaches and methods to make our theoretical mathematics amenable to computer simulation.

In my case, studying the transformation of the gaskets as they are expanded into increasingly high dimensions, computer simulation eventually required that the problem be reimagined entirely in terms of interlinked systems of parameters. This computer-approximation-driven theoretical orientation shaped my mathematical intuitions toward the problem and guided my fellow researchers and me in both theoretical and computational directions. We discovered both that computer approximation could be incredibly powerful as an aid to intuition, and that it can be incredibly difficult to transfer computer-oriented mathematics back into the purely theoretical standards of our area of specialty.

I will address the philosophical implications of computer-driven theoretical mathematics, asking how computer experiments can shape both the content and standards of theoretical sciences.

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