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Knowledge in mathematics: Here one has to keep on reminding oneself of the unimportance of the ‘inner process’ or ‘state’ and ask “Why should it be important?” What does it matter to me? What is interesting is how we *use* mathematical propositions.

Ludwig Wittgenstein, *On Certainty*, # 38, p. 7e.

Introduction

The “new Wittgenstein” coalesces around a series of common interpretive protocols: Wittgenstein is not advancing theories in metaphysics but employing a therapeutic method; he is helping us to work free of the confusions that become evident when we begin to philosophize; at the same time, Wittgenstein is disabusing us of the notion that we can stand outside language and command an external view, and; that such an external view is both necessary and possible for grasping the essence of thought and language. By contrast, Wittgenstein, on the new reading, encourages us to see that our intuitions about meaning and thought are best accommodated “by attention to our everyday forms of expression and to the world those forms of expression serve to reveal” (Crary & Read, 2000: 1). This new schema for reading Wittgenstein also puts less emphasis on the decisive break in his thought, represented by the *Tractatus* and the posthumous *Investigations*, to emphasise, by comparison, significant continuities of his thought centring around his therapeutic conception of philosophy. The standard narrative, attributed to Michael Dummett (1991) and taken up by scholars such as Kripke and Pears, runs:

in the *Tractatus* Wittgenstein advocates a *truth-conditional* theory of meaning which has the characteristic features of *realism*, and later on he rejects and embraces a theory of meaning as consisting in *assertability-conditions* which has the characteristic features of *anti-realism* (Crary & Read, 2000: 2).

This account is often accompanied by a description of a shift from the metaphysics of the *Tractatus*, dedicated to offering an account of the general form of the proposition and how language pictures the world, to “the non-metaphysical descriptions of our linguistic practices” (p. 3) that characterise the *Investigations*. In contrast to the standard view, the new reading insists on the therapeutic character of Wittgenstein’s philosophy such that the abandonment of the idea of an external standpoint carries no consequences for our entitlement to basic epistemic ideals.

What is more, the new reading emphasises that his modes of philosophical criticism and the methods he employs serves to highlight and elucidate his therapeutic aim: “the dialectical structure of Wittgenstein’s writing makes an internal contribution to the philosophical instruction it contains” (p. 7). This is what we have called the “pedagogical” aspects of Wittgenstein’s later philosophy (see Peters & Marshall, 1999b). It is this aspect, as Crary & Read (2000: 7) point out, that Stanley Cavell’s early work was first to reveal and it is a theme that he has returned to again and again. [\[2\]](#)

The new reading that emphasises the therapeutic character of Wittgenstein’s philosophical aims and method, I would argue, is sympathetic to and consistent with the “postmodern” view of Wittgenstein we present in our recent book *Wittgenstein: Philosophy, Postmodernism, Pedagogy* (Peters & Marshall, 1999). Our reading explicitly provides

an emphasis on a literary, cultural and (auto)biographical reading of Wittgenstein's works, their intertextuality, the expression of the spirit of European (Viennese) modernism in the *Tractatus*, and the anticipation of certain ‘postmodern’ themes in his later works which, on the one hand, cast him in close philosophical proximity to Schopenhauer, Nietzsche and Heidegger and, on the other, project his writings into an interesting engagement with poststructuralist thought (Peters & Marshall, 1999: 19-20).

This cultural reading, in part, was inspired by Cavell’s work, which we took as an exemplar both in reading Wittgenstein in relation to the movement of *modernism* and against Wittgenstein’s Viennese cultural background. Cavell’s writings also draw widely upon the philosophical tradition and emphasise the parallels between Wittgenstein and many contemporary thinkers, including both Derrida and Foucault.

In our reading, while we do not maintain that there are two Wittgensteins as in the standard narrative, we do argue that the *Tractatus* is *modernist in its formalism*, while the *Investigations* anticipates certain “postmodernist” themes (see also Peters & Marshall, 2001). The distinction is principally a matter of the style of *doing* philosophy and it is reflective of the impact of larger cultural forces upon Wittgenstein and, significantly, also the six years break from philosophy (in an institutional sense) which Wittgenstein spent as a schoolteacher. It does not deny that there are significant continuities in his thought, say, for instance in his view of philosophy. In this reading it is possible to argue that the therapeutic aim became more manifest in Wittgenstein’s “pedagogical” style and in a view we called “philosophy as pedagogy” (Peters and Marshall, 1999a, Chapter 10). Yet nowhere in the presentation of this view did we use the terms “social constructivism”, nor do we think that “postmodernism” (whatever that elusive term means) necessarily entails social constructivism in any of its versions. [\[3\]](#)

The cultural and postmodern reading of Wittgenstein, like much of postmodernism, considered as a whole, tends to emphasise a number of cluster concepts, including the following:

- anti-foundationalism;
- anti-essentialism;
- anti- or post-epistemological standpoint;
- anti-realism about meaning and reference;
- suspicion of transcendental arguments and viewpoints;
- rejection of the picture of knowledge as accurate representation;
- rejection of truth as correspondence to reality;
- rejection of canonical descriptions and final vocabularies;
- suspicion of metanarratives. [\[4\]](#)

To this list we might add an emphasis on linguistic use and *therapeutic view of philosophy*; that is, an embodiment of many of the features of the list above and an ethos, above all, concerning philosophy as a critique of language summed up best in the famous quotation from the *Investigations*: “Philosophy is a battle against the bewitchment of our intelligence by means of language” (#109). [\[5\]](#) It is a view that underlies the development of social sciences and cultural studies in the latter half of the twentieth-century; perhaps, sloganised in the twin methodological imperatives: the linguistic turn, the significance of representation, and the so-called “social construction of reality”, on the one hand; and, the attempt to overcome the dualisms, the search for certainty and essences, and the subjectivism that is the legacy of the Cartesian thought. [\[6\]](#)

Wittgenstein has been portrayed as a foundational thinker of the twentieth century who articulated a version of social construction in relation to language, to our epistemic ideals, and even to mathematics. The emphasis has fallen unfortunately on “social construction” (Wittgenstein never used this terminology) instead of a therapeutic view of philosophy. This paper, examines the work of Wittgensteinians who embrace a social construction thesis in the philosophy of mathematics, notably Dummett’s “conventionalism” and, in the philosophy of mathematics education, namely Paul Ernest, before arguing, following Gerrard (1996) and others, that Wittgenstein -- particularly, in his later period, dominated by the “language-game conception” – was neither an objectivist nor a social constructionist, i.e., one who denies objectivity in mathematics.

Social Constructivism: Dummett’s Conventionalism and Ernest’s Constructivism

Michael Dummett's conventionalism is laid out in his influential 1959 paper "Wittgenstein's Philosophy of Mathematics" (Dummett, 1959).^[7] In his interpretation, Dummett argues that Wittgenstein proposes a "knock-down" argument of the objectivity of mathematical truth and he labels Wittgenstein's view as "full-blooded conventionalism" (p. 425, page references to Pitcher). Gerrard (1996: 182-3) characterises Dummett's position as one that picks up on Wittgenstein's rejection of the metaphysical picture inherent in Hardy's objectivism but "missed the Fregean anti-subjectivism in Wittgenstein's anti-psychologism, and thus ... concluded that he [Wittgenstein] must be recoiling into the anarchist [read "constructivist"] camp in another of its forms". Thus, Dummett credits Wittgenstein with the view that "the logical necessity of any statement is always the *direct* expression of a linguistic convention" (p. 425, cited in Gerrard, p. 183). Gerard (1996: 183), I think correctly, remarks: "It is not that our calculations and proofs follow from an antecedent set of agreed-upon conventions, but that at *every step* a new decision is required".

Dummett's attribution of "full-blooded conventionalism" contributes to a picture of Wittgenstein-as-anarchist, supported further by David Bloor's (1983) characterisation of Wittgenstein's "anthropological method". Our mathematical language is a part of our language-games, which themselves are contingent and based upon our linguistic conventions that have developed as forms of community agreement. On this reading, "Wittgenstein reduced any mathematical necessity to the level of the contingent and the conventional" (Gerrard, 1996: 183).^[8]

In the field of the philosophy of mathematics education, Paul Ernest from University of Exeter, has been the influential in putting forward a social constructivist philosophy of mathematics.^[9] Ernest (1999) argues that the traditional absolutist (read "objectivist") account of mathematics should be replaced by a "conceptual change" philosophy of mathematics built upon principles of radical constructivism that, nevertheless, does not deny the existence of the physical and social worlds. He hopes to overcome the criticism that radical constructivism is necessarily solipsistic is overcome.

In addressing the questions: "What is the basis of mathematical knowledge? What gives it its seeming certainty, and is this certainty justified?"^[10] Ernest identifies two main currents in the philosophy of mathematics: what he terms "absolutist" and "conceptual change" philosophies of mathematics. The former, exemplified by Platonism but also including Logicism, Formalism and Intuitionism "assert that mathematics is a body of absolute and certain knowledge"; the latter, by contrast, "assert that mathematics is corrigible, fallible and a changing social product."

As he explains, the second position is the radical one for most of us still look to mathematics as a primary source of certainty. While in science, absolutist view have fallen away under the combined attack of philosophers such as Hanson, Kuhn, Lakatos and Feyerabend, in the philosophy of mathematics the absolutist view remains the dominant conception. "Absolutists believe that mathematical truths are universal, independent of humankind (mathematics is discovered, not invented), and culture- and value-free."

Ernest briefly considers two objections to absolutism, which he thinks can be overcome by embracing a thesis of social constructivism. First, he suggests, following Lakatos (1978), that deductive logic, as the means of proof, does not establish mathematical certainty for such a quest inevitably leads to an infinite regress. There is no escaping the set of assumptions, however minimal, mathematical systems require. (Ernest does not really examine this argument or state it with much precision; he simply relies on Lakatos). Second, even within an axiomatic system, mathematical theorems can not be considered to be certain for Gödel's second incompleteness theorem, demonstrates that consistency requires a larger set of assumptions than contained within any mathematical system (but for the very simplest).^[11]

He states the social constructivist thesis as "mathematics is a social construction, a cultural product, fallible like any other branch of knowledge" – a view entailing two further claims: the allegedly uncontroversial idea that "the origins of mathematics are social or cultural" and the notion that "the justification of mathematical knowledge rests on its quasi-empirical basis." The second claim he attributes to Wittgenstein (1956), among others.^[12]

Equipped with two realist assumptions – the existence of both an independent physical world and a social reality – Ernest suggests that a social constructivist epistemology can be developed from the principles of radical constructivism, which he elaborates as follows:

a. "knowledge is not passively received but actively built up by the cognizing subject"

@. b. “the function of cognition is adaptive and serves the organization of the experiential world, not the discovery of ontological reality” (von Glasersfeld, 1989, p. 182).

c. the personal theories which result from the organization of the experiential world must 'fit' the constraints imposed by physical and social reality;

d. they achieve this by a cycle of theory-prediction-test-failure-accommodation-new theory;

e. this gives rise to socially agreed theories of the world and social patterns and rules of language use;

f. mathematics is the theory of form and structure that arises within language.

This social constructivist epistemology, he asserts, “draws on Wittgenstein's (1956) account of mathematical certainty as based on linguistic rules of use and ‘forms of life’, and Lakatos' (1976) account of the social negotiation of mathematical concepts, results and theories.” In this paper I am principally concerned with Ernest’ attempt to base a social constructivist epistemology on Wittgenstein to which I do not believe Wittgenstein would consent. It is not enough, indeed it is misleading, to say that Wittgenstein bases his account of mathematical certainty on linguistic rules of use and ‘forms of life’, as it is to claim (referred to earlier) that Wittgenstein argued that “the justification of mathematical knowledge rests on its quasi-empirical basis.” I shall deal with these criticisms below.

Ernest claims that this account of a social constructivist epistemology for mathematics overcomes the two problems he identifies with absolutism. First, (if I can condense his account) he argues “the concepts of mathematics are derived by abstraction from direct experience of the physical world” through negotiated meanings forged with others and thus, “mathematics is a branch of knowledge which is indissolubly connected with other knowledge, through the web of language.” He concludes “the unreasonable effectiveness of mathematics” is a direct result of the fact that it is built into language and the description of the world for “It derives from the empirical and linguistic origins and functions of mathematics.” Second, the apparent certainty and objectivity of mathematical knowledge, he claims:

rests on natural language, and that mathematical symbolism is a refinement and extension of written language. The rules of logic and consistency which permeate the use of natural language provide the bedrock upon which the objectivity of mathematics rests. Mathematical truths arise from the definitional truths of natural language, acquired by social interaction ... The truths of mathematics are defined by implicit social agreement - shared patterns of behaviour - on what constitute acceptable mathematical concepts, relationships between them, and methods of deriving new truths from old. Mathematical certainty rests on socially accepted rules of discourse embedded in our ‘forms of life’ (Wittgenstein, 1956).

Finally, Ernest suggests that since “the principles of radical constructivism are consistent with, and can be supplemented by assumptions of the existence of physical and social reality” the social constructivist account he offers can avoid criticisms of solipsism and subjectivism, levelled by the likes of Goldin (1989) and Kilpatrick (1987).

Wittgenstein’s Philosophy of Mathematics

In readings of the new Wittgenstein it is maintained that his philosophy of mathematics was not separate from his philosophy of language and that, against Dummett, there is a unity of Wittgenstein’s later philosophy of mathematics with the *Philosophical Investigations* (see Gerrard, 1996: 171). Also in keeping with this new view the emphasis is placed not on any substantive position or doctrine that Wittgenstein held, but rather -- in accord with the therapeutic notion of philosophy to which he subscribed -- on the fact that Wittgenstein constantly warned against the attempt to construct theories in philosophy (see e.g., Floyd, 1991, 1995). Wittgenstein might thus be interpreting with criticizing a range of views in philosophy of mathematics – for example, psychologism, logicism, Platonism, intuitionism, formalism, conventionalism – but not positively associated with any doctrine or position. [\[13\]](#)

Others, like Glock (1996), have suggested that Wittgenstein’s major contribution to the philosophy of mathematics was his *normative* conception of the role of mathematical propositions in empirical reasoning:

Mathematical propositions describe neither abstract entities nor empirical reality, nor do they reflect the transcendental workings of the mind. Their a priori status is due to the fact that, in spite of their descriptive appearance, their role is a normative one; nothing which

contravenes them counts as an intelligible description of reality.

It is this normative conception, Glock (1996) argues, that

separates his *position* from the established schools of twentieth-century philosophy of mathematics, which are united by the idea that mathematical propositions refer to some kind of reality, whether physical signs (formalism), mental processes (intuitionism) or abstract entities (logicism) (my emphasis).

The difficulty with Ernest's position is that it attributes to Wittgenstein a view he didn't hold. Certainly, Wittgenstein would not have held anything like the social constructivist thesis Ernest marries him to and his descriptions of Wittgenstein's philosophy of mathematics, *viz.*, "Mathematical certainty rests on socially accepted rules of discourse embedded in our 'forms of life'", are either too simplistic in not taking account of the complexity and change in Wittgenstein's views, or, plainly wrong, at least on accounts springing from readings of the new Wittgenstein. [\[14\]](#)

To take the first charge, it is clear, for instance, that Wittgenstein changed his views on the philosophy of mathematics over the years. By this I do not mean, as Floyd (1998: 233) ironically puts it, that "in his youth he was an 'ideal language philosopher', in his early middle age he was a 'verificationist', and in his old age an 'ordinary language philosopher'" but rather that it is possible to distinguish a number of philosophies or stages in the development of his philosophy. [\[15\]](#) First, we can identify an early philosophy, given in the *Notebooks, 1914-16* and the *Tractatus*, which while informed by Frege's and Russell's view on the foundations of mathematics, nevertheless, was critical of their attempt to reduce mathematics to logic. Wittgenstein never rejected Frege's anti-subjectivism or anti-psychologism and he retained the idea that there is a difference between tautologies and mathematical propositions. [\[16\]](#)

Second, after 1929 and through the early 1930s, Wittgenstein developed a transitional "calculus" view, where he held that mathematical propositions had meaning only within the calculus which was considered an autonomous, self-enclosed, arbitrary system. [\[17\]](#) On this view, the rules alone determine meaning and grammar is not accountable to any reality. Third, during the mid-1930s Wittgenstein shifted from the "calculus" to the "language-game" view, where he considered mathematics as a *motley* of language-games, thus not only rejecting the self-contained nature of the calculus conception but also explicitly tying the language of mathematics together with the use of its signs ("in *mufiti*") outside mathematics, as part of an activity in "the stream of life".

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[1] A version of this paper was published in *Theory and Science*, 3 (3):

(<http://theoryandscience.icaap.org/content/vol003.002/peters.html>).

[2] I have sought to demonstrate the significance of Cavell's work for philosophy of education. In particular, I have drawn attention to the way in which he emphasises the importance of the "voice" of the child in the opening sections of the *Investigations* and how the structure of the figure of the child is intimately related to the theme of learning language-games (see Peters, 2001).

[3] It is difficult to state the thesis of social constructivism with any precision, although, perhaps, one version might be called "conventionalism" after Dummet's reading of Wittgenstein. The term first came to prominence with Berger and Luckmann's (1967) *The Social Construction of Reality* which was an account of the social phenomenology of knowledge. It was quickly picked up by social psychologists in the 1970s focusing on the cultural acquisition of language, appealing to the work of Piaget and/or Vygotsky (e.g., Brunner, 1990). Piaget and Vygotsky still remain the central figures in social constructivist pedagogy. Others, like Harré (1983, 1986) explicitly followed the pragmatist G.H. Mead and Wittgenstein, as a means of criticising the biological determinism of Piagetian "stages", offering, by contrast, a socio-cultural analysis of cognitive development. Notice here that social constructivism is not directly applied to knowledge but rather to the psychology of human development. Similar views, utilising Wittgenstein (among other theorists), were advanced by Shotter (e.g., 1993) and Gergen (1985, 1991), both of whom were instrumental in linking social constructivist psychology with the postmodern turn. Yet the "postmodernism" of Shotter and Gergen, at least in its early stages, while critical of the universalist explanations of modernist psychology, tended to embrace a form of (liberal) humanism; whereas the "poststructuralist" orientation of the critical psychologists (e.g., Walkerdine, 1984), was critical of the alleged humanism, individualism and liberalism underlying social constructivism (see Morss, 1996).

[4] The list is taken from Bernd Magnus' (1989) discussion of Nietzsche in relation to postmodern criticism. In Peters & Marshall (1999a) we devote considerable space to exploring the nature of the historical and intellectual connections between Nietzsche and Wittgenstein (see Chapters 2 and 4), seen as thinkers sharing a significant range of concerns. To Magnus' list we might also add what Rorty calls "antirepresentationalism" and also add, alongside "suspicion of metanarratives", the turn to narrative and narratology, more generally -- the "*petite recits*" pitted against metanarratives by

Lyotard (1984). Significantly, Lyotard (1984) makes central use of Wittgenstein (see Peters, 1995), in a creative misappropriation to emphasise the conflictual or *dissensual*, nature of language-games.

[5] While Wittgenstein says at one point that he is a disciple of Freud, there is no sense in which we should treat his notion of therapeutic philosophy in Freudian discourse as a “depth hermeneutic”.

[6] See Ian Hacking’s (1999) excellent discussion of social construction as a code embroiled in “the culture wars” and “the science wars” that dominate intellectual life.

[7] As become obvious in this section I follow Gerrard’s (1996) characterisation of Dummett’s view. All I need for my purposes here is a broad characterisation of the social constructivist Wittgenstein.

[8] Here Gerrard (1996: 183) notes, quoting a remarks from Dummett in 1978, that Wittgenstein is regarded as “being truly radical”.

[9] I shall draw on Ernest (1999) “Social Constructivism As A Philosophy Of Mathematics: Radical Constructivism Rehabilitated?” available at:

[10] In this regard see Hilary Putnam’s (1998) “Rethinking Mathematical Necessity” where, rethinking his earlier attachment to Quine’s view that mathematics is best seen as an integral part of science, he attempts to detranscendentalize mathematics while still holding on to the Kantian (Fregean and Wittgensteinian) insight that the laws of logic are *sinnlos* (without content) but the ground of their truth resides in their being the formal presuppositions of judgement. The effect of his line of thinking is, in agreement with Quine, still to jettison the analytic-synthetic distinction (and the notion of “analyticity”) but, in a very Wittgensteinian way, to cast doubt on the notion of ontological commitment and talk of the “existence” of mathematical entities. He suggests that the notion of “naturalized epistemology” is obscure in the realm of the philosophy of logic and mathematics because it is not “clear what it means to naturalize a normative ... notion” (p. 229). He concludes: “And the trouble with talk of epistemology in the case of mathematics is that this talk depends on the idea that there is a problem of justification in this area. But perhaps mathematics does not require *justification*; it only requires theorems” (p. 229).

[11] Ernest, prematurely in my view, takes these objections as “disposing of absolutism”. I think the arguments require greater care and examination.

[12] In addition, he lists, Lakatos (1976, 1978), Davis and Hersh (1980), Kitcher (1983), Tymoczko (1986).

[13] Some, like Gerrard, seemingly accepting Floyd’s point, follow Burton Dreben, to talk of offering a “tame” or “domesticated” Wittgenstein.

[14] This is not to say that I am unsympathetic to the argument Ernest puts forward, only that I entertain doubts concerning its attribution to Wittgenstein and that Ernest’s argument insofar as it is buttressed by appeals to Wittgenstein, fails. Yet see his more recent work which spells out the case for a social constructivist philosophy of mathematics (Ernest, 1999) and provides a provocative and interesting critical account of mathematics education in relation to the justification problem “Why teach mathematics?” (Ernest, 2000). To the traditional aims -- to reproduce mathematical skill and knowledge based capability, and to develop creative capabilities in mathematics – he suggests adding: to develop empowering mathematical capabilities and a critical appreciation of the social applications and uses of mathematics, and to develop an inner appreciation of mathematics: its big ideas and nature.

[15] I based this account on Gerrard (1996: 173 pp).

[16] For a brief account of Wittgenstein Tractarian, early anti-logicist views see Glock (1996:). Floyd (1998: 234) notes that while Wittgenstein criticised Frege’s and Russell’s logicism as “presenting diverse mathematical techniques in the guise of just one sort of technique”, nevertheless, “he never retracted his early remark ‘that the great work of the modern mathematical logicians ... has brought about an advance in Logic comparable only to that which made Astronomy out of Astrology, and chemistry out of Alchemy’”.

[17] Gerrard (1996: 193, Fn 11) suggests that recognition of a distinct “middle” period is a recent discovery and suggest a possible fourth post-*Investigations* stage, based on the philosophical psychology. Significantly, he argues that the failure to distinguish the different stages, in part, contributed to the unsympathetic ear the early secondary literature gave Wittgenstein’s philosophy of mathematics as against the more recently positive evaluations by Crispin Wright (1980), S. G. Shanker (1987) and Cora Diamond (1991).