PHILOSOPHY OF MATHEMATICS EDUCATION: THE CONSTRUCTION OF GEOMETRICAL KNOWLEDGE, LIVED EXPERIENCE, AND LIVED TIME

BICUDO, Maria Aparecida Viggiani – Universidade do Sagrado Coração (USC) and Universidade Estadual Paulista (UNESP), Rio Claro campus, São Paulo, Brazil.

Verilda Speridião - Doctoral student, Universidade Estadual Paulista (UNESP); Professor, Universidade Metodista de São Paulo, São Paulo, Brazil.

Rosa Monteiro – Doctoral student, Universidade Estadual Paulista (UNESP); Professor, Universidade Cruzeiro do Sul, São Paulo, Brazil.

Objective:

To present studies carried out by the Phenomenology and Mathematics Education Study Group, based in the Graduate Program in Mathematics Education of the State University of São Paulo (UNESP), in Rio Claro, São Paulo, Brazil.

The ideas presented concern the conception of the philosophy of mathematics education, the construction of geometrical knowledge, lived experience, and lived time. The conceptions that shape the homeland of the research comprise a phenomenological focus, in terms of a philosophy and method of inquiry.

CONCEPTIONS REGARDING THE PHILOSOPHY OF MATHEMATICS EDUCATION

As described in BICUDO[1] , BICUDO & GARNICA[2] , and BICUDO[3], philosophy of mathematics education deals with themes from philosophy of mathematics as well as philosophy of education, employing methods of inquiry from the field of philosophy. Being imbedded in this field of inquiry, questions regarding *reality, the production of knowledge, epistemology, and values* are placed in evidence, viewed from the perspective of mathematics, with a focus on education.

Philosophy of mathematics education assumes the position of thinking and reflecting on themes that are broad enough to cover the entire field of mathematics education. This does not imply that mathematics education can be reduced to the philosophy of mathematics education, but rather that the latter reflects on and thinks reflectively about the former, seeking to know and interpret what has been or is being carried out. This is a meditative task that leads one to self-knowledge, self-criticism, and the delineation of one's identity. It is in this way that mathematics education is fortified, while also providing glimpses of the future and informing choices. Regarding activities within the field of mathematics education, it is our understanding that the following themes represent convergencies that should be the focus of reflective and critical analysis by philosophers of mathematics education:

- the conception of education and of mathematics education;
- the conception of reality and of knowledge;
- the conception of reality of mathematical objects;
- didactic-pedagogical positions and guidelines regarding the work of the mathematics teacher[4]

THE CONSTRUCTION OF GEOMETRICAL KNOWLEDGE

As an example of studies in the field of philosophy of mathematics education, considered as a field of inquiry constituted at the interface of the fields of philosophy, philosophy of education, and philosophy of mathematics, within a phenomenological perspective, we present a brief description of some research being carried out that revolves around the theme of the construction of geometrical knowledge.

One of the studies, *What happens in the subject-mathematics encounter?*[5], focuses on the construction of geometrical knowledge in the pre-reflective, based on the intentional man-world relation described by Edmund Husserl as the noesis-noema relation. The other, which deals with the exploration of the square as a flat geometric figure with four congruent sides, seeks the lived experience of the *square* geometric form, discussing spatial perception and spatiality.

These studies work with the description of perception that takes place at the man-world encounter, according to the conception presented in Merleau-Ponty's book, which follows the path of Edmund Husserl regarding the phenomenological dimension of philosphy, and of Martin Heidegger, concerning the existential dimension.

The first study analyzes data obtained through descriptions conducted from the perspective of the *Web of Meaning* [6]. According to Bicudo[7],

...the Web of Meaning is not composed of points, constituted of concepts, which interconnect forming a web. Rather, each knot in the web expresses the lived experience that bears the

circles or vortexes within them, of which each element is representative of all the others and is linked to the vortexes as if by vectors.

This makes it possible to comprehend the perceived meanings regarding the object of inquiry, since in the very process of constructing the web, we open up the layers of understanding that take place in the pre-reflective, also known as the pre-logical and pre-propositional, to the degree to which the different meanings lead to the same idea, forming it as if it were the nucleus of meaning. The interpretation of the web that is constructed makes it possible to comprehend the Lebenswelt (world-life), where meanings are made, as it is constituted on the historical homeland where the person exists, and is thus with others.

In the *man-world encounter*, mathematics reveals itself as a *cultural object* that has its origins "[...] in the coexistence of an infinite number of consciousnesses and inter-sensorial units that compose the intersubjective meaning of the experience of the mathematical phenomenon"[8]. The construction of geometry, understood here as a field of inquiry which is inserted in that of mathematics, shows itself in this encounter as being knowledge whose primacy is in perception and is constituted, at its origin, by a notion of form, the *form perceived* by the subject in the unit of his/her *Own Body*. This makes sense when we come to understand Merleau-Ponty:

Form is a configuration which is visual, sonorous, or even precedes distinction by the senses, where the sensorial value of each element is determined by its function in the complex whole, and varies with it. /.../ This notion of form makes it possible to describe the mode of existence of primitive objects of perception. They are, as we said, before being known as true objects, experienced as reality.[9]

One's own body is understood as a field of perception, the point from which each of us beholds the other; it is the place from which we see the world and where the world makes itself present for us. *One's own body*, as a subject of perception, makes perception the first layer of meaning, making it possible for the *perceived form* to become the homeland of another stage of

knowledge construction, referred to by this author as *intersensorial exploration*, which will constitute the *sensed form*, the visual and sonorous configuration that, when elaborated within the sphere of its structures, gives origin to the *produced form*, which are the geometric forms instituted and that we work with in the field of geometrical knowledge.

The construction of knowledge, when comprehended as a movement that effects syntheses of transition, taken here as *perceived form, sensed form, and produced form*, allows the tension that exists between the subject and the world to be coherent as much with the experiences of the subject as with the percept in the world, without imprinting on one or the other anything that is alien to it.

This coherence can be achieved when we assume that sensible objects have a physiognomy which can be comprehended in the manifestations of *One's own Body*, since the qualities of the sensible object can be recognized by the type of behavior that aims for that which is characteristic of the object, for example, in its structure or mode of use.

The study of *modes of sensing-experiencing the perception of geometry itself* revealed to us that geometric objects are sensible objects that can recognized by a type of behavior, and that at the moment *One's own Body* assumes this attitude, they obtain a *quasi-presence* of the qualities of their physiognomy as: *perceived form, sensed form, and produced form*.

This study opens up didactic-pedagogical possibilities for the construction of knowledge of the geometric object as well as to help that object reveal itself to the student. The geometrical object is to be comprehended in its operational and organizational aspects, but must also be perceived in its physiognomy and its possible qualities and applications. This is what a phenomenological approach seeks in the teaching of geometry in the classroom.

The pedagogical perspective that emerged from the investigation of the *subject-mathematics encounter* informed the elaboration of mathematics activities for the classroom, addressing themes related to the teaching of geometry such as: the condition of the existence of the triangle[10], symmetry[11], the Golden Ratio in the relation of the pentagon with the pentagram[12], etc. These activities delineate educational actions in mathematics that, when carried out in the elementary classroom, and in teacher education programs, reveal possibilities for comprehending the construction of geometrical knowledge itself and these educational activities.

We will now discuss some examples of possibilities for working in the classroom.

1. THE LIVED EXPERIENCE OF THE PERCEPT AND THE DEALT-WITH (*O TRABALHADO)* IN THE CONSTRUCTION OF GEOMETRICAL KNOWLEDGE

The objective of the study entitled *An Phanomenen orientiertes Erleben im Mathematikunterricht*[13], *A vivência sob o foco da visão fenomenológica na aula de Matemática*, is to explicate the term *lived experience* in the construction of geometrical knowledge. It sought to respond to the question: *what is revealed of the lived experience at the level of sensing-experiencing?* when dealing with mathematical subjects regarding the condition of the existence of the triangle, that is, when the sum of the two sides of a triangle is greater than the third side. This activity was developed in a high school classroom, with students aged 15 to 17, drawing on didactic-pedagogical methods based on the comprehension of the phenomenological attitude.

In this activity, three students positioned themselves inside a large elastic band and worked with the idea of a segment of a straight line of a triangle, experiencing the rules that emerged from these geometrical objects and later verifying them by comparing measurements of straight line segments formed by the distance between the students.

After experiencing the activity, the students wrote depositions on the question: What did you feel in this activity? The depositions were analyzed from the perspective of the *Web of Meaning* described earlier.

This research, with its focus on the lived experience, points to the way in which its *transitoriness* occurs at the level of sensing-experiencing. Some states are revealed in which the contents of the *lived experience* can be found: at times as verification, at times as something constituting itself, and at other times presaging syntheses of comprehension and transition. These states were grouped into three categories: *category of verification, category of meaning-making*[14], and *category of syntheses of transition.*

In the category of verification, the assertion *The student feels like a whole revolving around an accomplishment,* indicates the possibility of living-experiencing the whole. This does not refer, however, to a whole closed in itself, but to a whole that returns to something that is being realized. In this, we see focused intentionality.

In the *category of meaning-making*, when we analyze the assertion *The student feels as though he/she is constituting a triangle*, we understand that the *triangle* is enunciated in the construction. It is not placed like a display, or a representation in the form of a *triangle*. Its construction is not an act of cloning. The triangle undergoes construction, in the certainty of that which constructs it. It is a mode in which certainty presents itself as if incarnated, in such a way that the student feels like a triangle. The triangle is *foreseen*, presenting itself as a whole.

In the *category of syntheses of transition*, the assertion *The relation between the vertex and the side reveals itself* shows that the student perceives that there is a relation between the number of vertices and sides. When feeling that he/she is occupying the position of the vertex, and perceiving the others who are occupying the other two vertex positions, he/she perceives the *side* that unites these vertexes, forming a triangle.

The categories overlap one other. We can perceive an intimate connection between units of the subject and the intersensorial unit of the triangle. Let's consider the assertion *The student feels bounded when feeling like a triangle*. This assertion, when analyzed from the perspective of the *unit of the subject*, indicates a verification. Feeling like a triangle gives him/her a sensation of being bounded, although this assertion, when interpreted from the perspective of the *intersensorial unit of the triangle* shows itself as a leap taken by the subject in the process of comprehending the triangle, once the triangle reveals itself, in its geometric form, as a bounded figure. This is a field in which the pre-predicative comprehension installs itself, and where the roots of the construction of knowledge of the definition of a triangle occur, showing a bounded region, which reveals an area.

2. THE LIVED EXPERIENCE OF THE SQUARE AND SPATIALITY

This study was conducted with students enrolled in a specialization course in psycho-pedagogy at a private university in the state of São Paulo. The students were encouraged to discuss the square geometric figure, and were divided into small groups of four to do so. They were given a long piece of string that was tied together at the ends, forming a closed line. We asked them to place the string at their waists and to move themselves around in such as way that the string stretched between them would form a square.

The participants organized themselves to form the figure suggested, positioning themselves in relation to the others. The initial concern, revealed in the movement of the group members as they gave instructions to each other, was with the *measurement of the sides*. They sought to distance themselves from, or get closer to, each other in such a way that the piece of string separating them would be the same size. However, as they oriented themselves regarding the position each should occupy in order to from the square, their talking and gestures revealed a preoccupation with the "alignment".

Instructions like, “closer” or “further away” were given, accompanied with hand gestures indicating the need for one or the other to move more to the left or to the right. In addition to the distance observed between participants to form the square, the alignment revealed a concern about the internal angle of the square and the parallelism of the sides. In other words, although this concern was not verbalized explicitly by the participants, we perceive it in the action of the movement that orients the occupation of the space. The position each group member is to assume is revealed using the others as reference, considered in pairs - *who should I be aligned with and keep myself equidistant from?* This was the preoccupation revealed in the movement of the group. There is knowledge that is not expressed in words but which makes itself present in the formation of the square figure. The participants know *what* and *how* they should do to construct it. They orient themselves spatially for this, until the figure pleases everyone in the group and they announce that the square is formed.

An analysis of the group’s perception regarding the members’ involvement in the activity, presented here in succinct form, reveals that the lived-experience of the square favored the discussions about the spatial perception and spatiality. Upon carrying out the proposed task, the students sought a place, a position that would allow them to achieve their objective, which was: to position themselves in relation to the others in such a way as to form a square with the string that surrounded them.

This search for the appropriate place demands modes of organization and disposition from the students that will be comprehended beginning with the existential spatiality. That is, assuming an existing world that is familiar to them, this world forms the foundation for the first sense-experience, and possibilities for the subject to distance and direct him/herself spatially are revealed.

There is a configuration perceived by the students in the process of organizing the group. Together they establish an environment, a scenario, to be explored and define the place to be occupied. The place is perceived globally, based on this scenario that installs itself around the group and makes it determine the distance that should separate its members. This perceived distance is not measured objectively. It is lived-experienced. It is defined by the power of the body to discover a region and occupy it, *evaluating* “how far” a group member should position him/herself from the other, aiming to acheive the proposed objective.

This takes place, since the distance between them separates two beings who have the power to comprehend the shared experience. The students do not show the need to measure the string stretched between them objectively. The distance is covered by “*my body as a system of possible actions”* (Merleau-Ponty, 1994, p.336). The body defines its “phenomenal place” based on its task, on that which it must carry out. *My body as power to perform certain gestures, as demanding of certain levels /.../ perceives the spectacle that invites it to perform the same gestures, theater of the same actions (id. p. 337)* and permits the establishment of a pact, a co-existence that “*gives us the right to enjoy the use and advantages of the space*” (id. ib.)

Measuring objectively, using a measuring tape, for example, is a necessary activity of we want to draw the quadrilaterals formed in the lived experience, identify them, name them, and explore them from the perspective of geometrical construction. However, this is a secondary task, in relation to the primacy experienced in the perception of the position of *One’s own Body* as it occupies the position to form the square with the other members of the group.

What prevails in the lived experience is the *evaluation of the distance, the determination* of the position to be occupied. Measurability has a different significance here. It is based as much on the extension of the line that is to be measured as on the possibility of the action of measuring employed by the subject in relation to that which he/she is to measure. The significance of measuring reveals itself in the involvement that exists in the action of measuring when the subject seeks to determine and evaluate the distance lived-experienced.

We understand this to mean that, in the effort to form the square, the subject of the research, the student, explores the space of the world in which he/she lives, co-present with the others. He/she orients him/herself according to primitive spatiality. He/she is guided by circumvision, that is, by the sight that discovers one's surroundings, a region in which all the "*where's*" occur as possibilities to be occupied and emerge in the *being together*, in the co-habitation, making it possible to experience the form of the square differently from any other. He/she chooses a "where", a place to occupy, not because it is a specific place that bears in itself all the directions of space, or the meaning of all its determinations, but because it is, in the primordial world that he/she inhabits, the place that gives basis to a first sense-feeling that is experienced, making him/her *see* the square form being sought.

This "doing geometry", understood phenomenologically, makes it possible to carry out activities that make evident the action of *One's own Body* in the act of knowing, in the search for the structure of what is revealed in the way the objects appear. They are actions in which the gestures are seen as an expression of the intentionality of One's own Body in the act of knowing. When we propose a phenomenological attitude towards teaching and learning geometry, we understand that we privilege the involvement of the subject in the path to be followed, i.e., in the very act of doing geometry, focussing on the comprehension of definitions at another time. This path reveals itself in the process of walking it and is not followed by the subject alone. It is constructed in the action of walking that shows itself to be open. It is made known during the process being carried out, because nothing demands that the way one should know be established beforehand.

Ultimately, proposing an approach in the classroom that prioritizes the primacy of perception means involving oneself in a pedagogical practice that contributes to a world that makes sense to teachers and students. It signifies making an effort so that activities that are carried out individually and collectively contribute to the comprehension of scientific rigor, *placing the genesis of knowledge in suspension, reconstructing the evidence of clarifying thought, of perceptions, of the intuitions that reveal the phenomenon, in the activity of a being that is One's own Body, live intentionality, in its movements of seeking, doubting, questioning, together with the other and in the temporal horizon that brings us the certainty of the world lived through experiences, of the other, of feeling/sensing, of theories, of the meaning of praxis* (Bicudo, 1999, p.49). It is seeking the expression of meaning that geometry has for every subject involved in the act of knowing.

3. LIVED-TIME IN THE CONSTRUCTION OF GEOMETRICAL KNOWLEDGE

The study entitled *O tempo vivido na construção do conhecimento de simetria matemática (Lived time in the construction of knowledge of mathematical symmetry)* seeks to explicate *lived experience* in the dimension of time. In this inquiry, *time* is not being approached as a sequence of facts, the sum of presents that become the past and that await a future. We understand that time is born of *our* intentional relation with things. It never becomes constituted, since in temporality, i.e., in the way of being time, being is synonymous with passing. The lived present, upon becoming the immediate past, does not cease to exist. We are not spectators of this passing. It is not thought by us; we bring it to pass. According to Bicudo:

To focus on the phenomenon of lived time is to fix our gaze on life, on the way life flows; in other words, on the way we live the moments that, on a continuum, interconnect themselves in the flow of the movement of being itself.[15]

The research reported here investigates the possibilities visualized by mathematics teachers when they carry out an activity related to symmetry within a didactic approach that assumes a phenomenological attitude.

This activity was carried out with: *corporal* exercises that reveal the symmetry of reflection as *perceived form*; exercises of *filling the pre-determined space* with certain figures, that announce the symmetry of translation; and exercises involving *drawings* done with a ruler and compass that denote the symmetries of rotation and reflection.

At the end of the activity, the teachers who participated in the course responded to the question: How did this activity about symmetry contribute to your education as a teacher?

Their depositions were analyzed phenomenologically, constructing a *Web of Meaning*. This web revealed that the structure of the horizons visualized by the subjects fell into in three categories: *lived time, the manifestation of symmetry, and the contextualization of symmetry.*

In the category *lived time*, when we analyze the deposition – *the return of things that were abandoned or neglected, that did not show they existed long ago,* we perceive that the past penetrates the present with all its anguishing power and completely occupies it, placing in epoqué a slice of all the past, revealing the positive manifestations of memory that emerge from the mass of forgetfulness. Thus, what is revealed is a new understanding of memory interwoven with forgetfulness, and its importance for the construction of mathematical knowledge.

In the category *the manifestation of symmetry*, we perceived in the depositions *the presence of symmetry* and *the distinction between rotation, reflection, and translation* that the phenomenon of symmetry reveals itself as it shows itself in its variations, namely: rotation, reflection, and translation.

In the category *the contextualization of symmetry*, the deposition *the simplicity of the activity of symmetry linked to esthetics provides an immediate application or intent,* speaks of the lived experience of symmetry as something intrinsic to lived time, hence as a profound man-world union, contextualizing it in the sphere of esthetics. The context here cannot be understood as something isolated from the object of study, but rather as an integral component of the lived experience.

FINAL CONSIDERATIONS

These studies lead us to consider that, in human actions concerning mathematics in an educational setting, a vision of the world is implicit, a conception of human being and, consequently, a philosophical conception. Often, because these visions are not the object of analysis and reflection, the meaning of the actions remains obscure, and in this way, the activities do not find ways to be comprehended reflectively and analyzed critically and contextually.

According to our understanding, it is incumbent upon philosophers of mathematics education to study these actions, seeking to understand them and explicate them as regards their bases, procedures, ends, conceptions, intervening actions, and historical-social meanings.

BIBLIOGRAPHY

BICUDO, Maria A Viggiani & GARNICA, Antonio Vicente M Garnica. *Filosofia da Educação Matemática*. 3ª ed.,Coleção tendências em Educação Matemática. Belo Horizonte: Autêntica, 2003.

_____ *Filosofia da Educação Matemática: constituição de suas múltiplas faces*. *In Filosofia da Educação Matemática — Concepções & Movimentos*. Org. Maria A V. Bicudo. Brasilia: Plano, 2003.

_____”Qualitative Forschung in phänomenologischer Perspektive am Beispiel des Mathematikverständnisses von Kindern und Jugendlichen” **in** Fichtner, Bernd, Freitas, M.T, Monteiro, R. (Hgg) *Kinder und Jugendliche im Blick qualitativer*

Forschung. Bamberg: ATHENA-Verlag, 2003,p.349-356..

_____& Monteiro, Rosa Paulo. Eine phänomenologische Untersuchung zum Verstehen von Geometrie **in** Fichtner, Bernd, Freitas, M.T., Monteiro, Roberto. *Kinder und Jugendliche im Blick qualitativer Forschung.* Bamberg: ATHENA-Verlag, 2003, p.357-367.

_____ . F*ilosofia da Educação Matemática: um enfoque fenomenológico*. In *Pesquisa em Educação Matemática: concepções e perspectivas*. Org. Maria A V Bicudo. São Paulo: UNESP, 1999.

_____ . *Tempo, tempo vivido e história.* Bauru: EDUSC, 2003

TONI, Adlai Ralph. *Filosofia & Matemática: construção de um olhar*. *In Filosofia da Educação Matemática – Concepções & Movimentos*. Org. Maria A V Bicudo. Brasilia: Plano, 2003.

GADAMER, Hans-Georg. *Verdade e Método.* Trad. Flávio Paulo Meurer. Petrópolis: vozes, 1997

DEGGER, Martin. *The Concept of Time*. Trad. Willian MacNeill. Oxford: Blackwell, 1992.

_____. *Introdução* à Metafísica. Rio de Janeiro: Tempo Brasileiro, 1987.

_____. *Seminários de Zollikon*. São Paulo: EDUC; Petrópolis, RJ: Vozes, 2001.

JTH, Verilda Speridião. An phanomenen orientiertes Erleben im Mathematikunterricht. In Kinder und jugendliche im Blick qualitativer Forchung. Kultur historishe Schule, Phanomenologie und Ethnografie. Bernd fichtner, Maria Teresa Freitas, Roberto Monteiro(Hgg.) Bamberg, 2003.

_____ . *O tempo vivido na construção do conhecimento de simetria matemática*. Anais do II SIPEM- II Seminário Internacional de Pesquisa em Educação Matemática. SBEM, Santos, 2003.

_____. *Conhecimento Geométrico – uma rede corpórea. In Filosofia da Educação Matemática – Concepções & Movimentos*. Org. Maria A V Bicudo. Brasilia: Plano, 2003.

_____. *O conhecimento geométrico: trama de vivências corpórea-sócio-cultural*. *In: Formação Humana e Educação*. Org. Bicudo & Belluzzo. Bauru: EDUSC, 2002.

vência sob o Foco Fenomenológico da Educação Matemática*.* Documento base do 1º Painel Interinstitucional de Investigação Qualitativa. UFJF – Juiz de Fora – MG, 2001.

_____. *O triângulo e a sala de aula.. Uma visão Fenomenológica da Geometria. Educação Matemática em Revista*. Ano 8, nº 11, December 2001.

_____. *A rede de Significados: imanência e Transcendência: a Rede de Significação. In:* Maria A V. Bicudo. *Fenomenologia – confrontos e avanços*. São Paulo: Cortez, 2000.

_____. *O Ensino da matemática na visão construída no estudo da interrogação: o que acontece no encontro Sujeito-Matemática?* Anais II Encontro Brasileiro de Estudantes de Pós-graduaçào em Educação Matemática. Rio Claro, 1998.

_____. *O que acontece no encontro Sujeito-Matemática?*. EPSILON. *Revista de La S.A E.M*. 'THALES", nº 42, volume 14(3), España, 1998.

_____. *O que acontece no encontro Sujeito-matemática?* Masters thesis. UNESP – Rio Claro, 1997.

RLEAU-PONTY, M. Fenomenologia da percepção. Trad. Carlos Alberto de Moura. São Paulo: Martins Fontes, 1994.

_____. A estrutura do comportamento. Trad. José Alencar Corrêa. Belo Horizonte: Interlivros, 1975.

_____ Merleau-Ponty na Sorbonne: resumo de cursos filosofia e linguagem. São Paulo.Papirus,1990.

NKOWSKI, Eugène. *Lived Time – Phenomenological and Psychopathologial Studies*. Trad. Nancy Metzel. Northwestern University Press. Evanston, 1970.

ULO, Rosa Monteiro. A compreensão geométrica da criança: um estudo fenomenológico. Dissertação de Mestrado. UNESP/Rio Claro: São Paulo, 2001.

YL, Hermann. *Simetria*. Trad. Victor Baranauskas. São Paulo: EDUSP, 1997.

ICUDO, Maria Aparecida Viggiani. Filosofia da Educação Matemática: um enfoque fenomenológico. *In* Pesquisa em Educação Matemática: concepções e perspectivas. Org. Maria A V Bicudo. São Paulo: UNESP, 1999, p. 27.

[2] BICUDO, Maria A Viggiani & GARNICA, Antonio Vicente M Garnica. *Filosofia da Educação Matemática*. 3ª ed.,Coleção Tendências em Educação Matemática. Belo Horizonte: Autêntica, 2003.

BICUDO, Maria Aparecida Viggiani. *Filosofia da Educação Matemática: constituição de suas múltiplas faces*. *In Filosofia da Educação Matemática — Concepções & Movimentos*. Org. Maria A V. Bicudo. Brasília: Plano, 2003, p.15.

[4] BICUDO, Maria Aparecida Viggiani. Filosofia da Educação Matemática: um enfoque fenomenológico. *In* Pesquisa em Educação Matemática: concepções e perspectivas. Org. Maria A V Bicudo, *op.cit*., pg.27/28.

[5] KLUTH, Verilda Speridião. *O que acontece no encontro Sujeito-matemática?* . Epsilon - Revista de la S.A E.M. "Thales" . nº 42. vol. 14(3), 1998. This study was a finalist for the Premios Internacionales de Investigação y de Renovación Pedagógica em Educación Matemática. "Thales-San Fernando" por La Originalidad Del Tema Planteado, Por la Seriedad De Su Desarrollo Y las Perpectivas Abiertas Hacia Nuevas Vias De Investigación, in the area of Research in Mathematics Education.

[6] Detalhes em: KLUTH, Verilda Speridião. *A rede de Significados: imanência e Transcendência: a Rede de Significação. In*: Maria A V. Bicudo. *Fenomenologia – confrontos e avanços*. São Paulo: Cortez, 2000.

[7] BICUDO, Maria Ap.Viggiani. *Fenomenologia; Confrontos e Avanços.op.cit.p.97/98*

[8] KLUTH, Verilda Speridião. *O que acontece no encontro Sujeito-matemática?*. Dissertação de Mestrado. UNESP – Rio Claro,1997, p. 176.

MERLEAU-Ponty, M. *A estrutura do comportamento*. Trad. José Alencar Corrêa. Belo Horizonte: Interlivros, 1975.

[10] KLUTH, Verilda Speridião. *O triângulo e a sala de aula: Uma visão Fenomenológica da Geometria*. Educação Matemática em Revista. Ano 8, nº 11, dezembro de 2001.

[11] KLUTH, Verilda Speridião. *O tempo vivido na construção do conhecimento de simetria matemática*. Anais do II SIPEM- II Seminário Internacional de Pesquisa em Educação Matemática. SBEM, Santos, Brasil. 2003.

[12] KLUTH, Verilda Speridião. *A proporção áurea: uma visada fenomenológica.* Anais VI EPEM, Catanduva, Brasil. 2001.

KLUTH, Verilda Speridião. An phanomenen orientiertes Erleben im Mathematikunterricht. In Kinder und jugendliche im Blick qualitativer Forchung. Kultur historishe Schule, Phanomenologie und Ethnografie. Bernd fichtner, Maria Teresa Freitas, Roberto monteiro(Hgg.) Bamberg, Deutschland, 2003.

[14] By *meaning-making* we want to say that mathematics shows itself, becoming a phenomenon for the subject, bringing with it the entire history of its communion with humans, who constructed it and used it in their social activities.

[15] Bicudo, Maria Aparecida Viggiani. Tempo, tempo vivido e história. Bauru: Edusc, 2003, p. 33.