WHEN AFFECTIVE ISSUES BECOMES INEFFECTIVE: A CALL FOR PREVENTING TERMINOLOGY POLLUTION

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ABSTRACT. In the mathematics education literature, the term *belief* has been subject to a variety of interpretations; moreover, most research studies on belief avoid giving an explicit definition and settle for an operational definition. This paper argues for developing a sharp definition of belief explaining why scholars in mathematics education have suggested different definitions and discussing the ensuing difficulties that have been encountered.

In the nearly seven decades since Brownell's "Psychological Consideration in the Learning and Teaching of Arithmetic" was first published in the 10th yearbook of the National Council of Mathematics (NCTM), we have come to see that questions concerning affective educational outcomes are not in all respects as simple as once thought. Studying students' and teachers' affective educational outcomes -- beliefs, perceptions, norms, attitudes, emotions and values -- remains popular. Furthermore, most mathematics educators feel conflicted by debates over how to define affective educational outcomes, which might not be definable. Is it possible to develop sharp definitions for *belief, perception, norm, attitude, emotion, and value* that could be used in all empirical research? More important, are such definitions necessary or appropriate? Answering these questions requires understanding why scholars in mathematics education have suggested different definitions for these terms and the ensuing difficulties they have encountered. This article focuses on problems arising from the need to develop a sharp definition for *belief*.

THE GENESIS OF THE PROBLEM

The desire to clarify the ambiguous relationship among beliefs, perceptions, norms, attitudes, emotions, and values in mathematics education has been expressed by several researchers (Furinghetti & Pehkonen, 1999; Pajares, 1992). These terms, however, are used in studies with different meanings, and mathematics educators have not reached any agreement on their meanings. Aside from not being explicit about meaning, mathematics educators often use these terms interchangeably and, even when a definition is given, may not be satisfied with it (Furinghetti & Pehkonen, 1999). Some mathematics educators might argue that to analyze both theoretically and empirically the constructs denoted by these terms, it is not necessary to define them explicitly. They might say that the purpose of research on belief, emotion, and attitude is less about trying to formalize terms used and more about trying to understand individuals. They might also argue that such understanding also constitutes science. Accordingly, in the effort to understand the human condition, it may be productive not to attempt to provide sharp definitions of the constructs being investigated since humans are not always totally rational: Their emotions involve some superstitions or, as Green (1971) would say, some nonevidentially held beliefs.

These ideas and assertions are partly rooted in the early development of mathematics education as a scientific field, when there was a lot of discussion about sharpening the distinction between what constitutes concepts and what constitutes generalizations. Historically, this debate subsided when it became apparent that discussion was not really moving the field forward. Therefore, one might argue that the debate over what constitutes beliefs vs. attitudes vs. values and so forth is not moving the field forward either and that progress is not dependent on having sharper definitions. It goes without saying that mathematics education has made considerable progress over the past 30 years by attending to what leads teachers and students to do what they do --whether those driving forces are beliefs, perceptions, norms, attitudes, emotions, or values. More is now understood about teachers' and students' behaviors than decades ago. It is natural therefore, to think that the focus of research should be on understanding human behavior, and what leads to particular behavior -- however ultimately defined -- is called an operating force. The argument, however, exists for sharper definitions to analyze these relationships both theoretically and empirically.

The advancement of mathematics education as a academic field requires the identification of affective terms by both constructing and adhering to a vocabulary that makes the necessary distinctions in a consistent way (Hart, 1989). The need for coherent and sharp definitions of educational outcomes to prevent *terminology pollution* has been expressed by various researchers. Suchting (1992) described the problem by bringing up the danger of unexamined rhetoric: " Certain words and combinations of words are repeated like mantras" (p. 247). Pajares (1992) reiterated the importance of using terms "consistently, accurately, and appropriately once their definitions have been agreed on" (p. 315). In addition, the need to resolve this problem is expressed by Steffe (1995), who says that "conflicts [in terminology] can paralyze action" (p. 489). Thus, in mathematics education, there is a need to analyze terms referring to belief more deeply and to develop a more coherent framework for research on issues of affect, thereby clarifying the relationships among the terms. This clarification will also help mathematics educators facilitate and investigate the interaction of affective factors with cognitive factors in mathematics learning and teaching.

Although the problem partly stems from the complexities of affect itself, the lack of consensus on definitions among mathematics educators is also a major factor that contributes to and increases the complexity of the problem. Despite the acknowledgement among mathematics educators of the need for explicit definitions to distinguish or separate one affective educational outcome from another (Thompson, 1992), not only do mathematics educators use theoretical definitions of *belief, knowledge,* and *attitude* inconsistency, but also most research studies on these affective educational outcomes avoid giving an explicit definition and settle for an operational definition (Cooney, Shealy, & Arvold, 1998). Considering the number of studies that have been conducted on affective educational outcomes without having a strong theoretical basis and an explicit definition, it is unlikely that these studies will contribute much to the development of mathematics education as a scientific field (or some might argue as an academic discipline) failing to produce important new knowledge, ideas, and methods. To wit, these research findings will not be based on scientific arguments and will not have good evidence to support them (Kilpatrick, 2001).

I definitely agree with my colleagues who claim that mathematics education has made considerable progress over the past 30 years by attending to what leads teachers and students to do what they do. It is now time to analyze belief both theoretically and empirically by defining it explicitly. A glance at the historical development of the ideas of calculus provides an example. Dismissing the metaphysical view of mathematical objects helped Newton and Leibniz[1] develop the ideas of calculus. Only after mathematicians felt comfortable with these ideas and they had proven useful did Cauchy give a rigorous foundation to these ideas. As is well known, the modern theories of analysis -- and one might even say today's mathematics and our civilization -- owe their existence to this historical development. The development of Fourier series and complex numbers is another example of how a successful combination of practice and theory helped the overall development of mathematics as a scientific field. Without a consensus on what a belief, perception, norm, attitude, emotion, or value are, therefore, how could one be sure that humanity was not experiencing a kind of schizophrenia. Furthermore, research findings and the legitimacy of mathematics education as a scientific field (or an academic discipline) would be questionable (Kilpatrick, 2001).

Without doubt, the problem of not having explicit definitions for constructs is rooted in major differences among mathematics educators. Some colleagues might be critical of any attempt to define the boundaries of mathematics education research. Researchers in mathematics education need to reach a common understanding regarding their views of research on educational outcomes. There is a need to resolve main discrepancies over what it means to be a mathematics educator and what kind of knowledge mathematics education should aim to disseminate (Sierpinska, Kilpatrick, Balacheff, Howson, Sfard, & Steinbring, 1993). Those opposed to defining the research methodologies and methods in mathematics education can be asked why the findings of mathematics education research are being questioned by various people and organizations. If society questions the validity of research findings because mathematics education is an academic discipline and not a scientific field, why is it not questioning the findings of psychology, philosophy, or sociology? I believe that understanding the interdisciplinary and eclectic nature of mathematics education will help the overall growth of mathematics education as an academic discipline. It is time to discuss and to search for a common ground in how mathematics education research differs from research in philosophy, sociology, and psychology by acknowledging these fields contributions to the field. The investigation of problems associated with affective educational outcomes may help the effort of determining and identifying research methodologies and methods in mathematics education.

THE INTERDISCIPLINARY AND ECLECTIC NATURE OF MATHEMATICS EDUCATION AND ITS IMPACT IN RESEARCH ON AFFECTIVE EDUCATIONAL OUTCOMES

In the early development of mathematics education, mathematics educators imported various terms from different academic disciplines (e.g., sociology, psychology, and philosophy). They also used and adopted different research methods and theoretical frameworks (e.g. grounded theory, phenomenology, and constructivism) that have been used by these academic disciplines. Although this initiative of using research methods from other disciplines initially helped mathematics education research to flourish, problems eventually arose because these research methods, terms, and theoretical frameworks were not specifically developed for mathematics education. These problems are especially revealed in the investigation, documentation and evaluation of affective educational outcomes (Kulm, 1980; Leder, 1992).

Any coherent account of a particular research methodology in mathematics education requires the incorporation of ideas from philosophy, sociology, and psychology. This requirement is partly a result of the fact that mathematics education deals with human beings and the metaphysical views of human nature, which are indispensable to all empirical studies in mathematics education. One cannot sharply separate theoretical and empirical statements; moreover, gathering empirical evidence and making inferences from research findings provides (or requires) both a general introduction to mathematics education inquiry and the use of a variety of methodological approaches. The values that constitute the different forms of mathematics education knowledge also require a collaboration among different philosophical, psychological, and sociological views about the nature of mathematics education itself. Hence, any attempts to confine values (why, to whom, and how mathematics should be taught) to separate realms of philosophical, sociological, and psychological assumptions and to exclude these assumptions from empirical concern would be counterproductive to furthering an understanding of affective educational outcomes. In the same vein, the scientific status (or legitimacy) of mathematics education research will always depend on particular philosophical, sociological, and psychological views about mathematics itself and its role in society.

THE PRECARIOUS NATURE OF BELIEF RESEARCH IN MATHEMATICS EDUACTION

In the investigation of one's beliefs, mathematics educators have been tackling the following questions: What is a belief? How can a person's beliefs be documented? Is it possible to distinguish between beliefs and knowledge? What is the relationship between beliefs and attitudes? Distinguishing a belief from knowledge has also been a subject of contention in philosophy, sociology, and psychology (Baker, 2001; Bloor, 1991; Dretske, 2000). Not only has it been difficult for philosophers, sociologists, and psychologists to agree on what in general distinguishes a belief from knowledge, but they have also had much difficulty even determining what belief and knowledge are. They all attempt to find pragmatic and communal definitions of knowledge by stressing the fact that knowledge depends not just on the evidential status of an individual claim but also on such factors as concepts that are commonly agreed upon by others in the community. They also cannot dismiss the fact that a particular knowledge is not independent and self-reliant, and they also acknowledge that formulating conditions of justification of knowledge in such terms as *adequate evidence* (Corsini, 1999), *sufficient warrant* (Marshall, 1998), *beyond reasonable doubt* (Runes, 1983), and so on, might require the collaboration of one academic field with another.

It is widely accepted that what distinguishes knowledge from belief is an epistemological criterion that knowledge requires some form of justification, evidence, or supporting reasons. This statement does not necessarily imply, however, that a justified true belief is knowledge, because the justification might be incomplete in certain crucial respects. Moreover, the justification is different for different items of belief. Although pure mathematical knowledge (such as theorems) is considered justified if derived from axioms and definitions, what about pedagogical knowledge and pedagogical content knowledge? Consequently, the path from belief to knowledge is vague and convoluted in mathematics education, and there is also no clear distinction between beliefs and knowledge (Pajares, 1992; Thompson, 1992). By asking what counts as knowledge, researchers are inquiring about the kinds of knowledge valued by mathematics educators.

Current practice in mathematics education research exhibits a tendency to distinguish knowledge from a belief on the basis of the possibility of objective evaluation of its validity (Thompson, 1992). This tendency begs the question of what is going to count as an objective evaluation. Moreover, is it possible to evaluate something objectively? Thompson was not the first to introduce the terminology of *objective evaluation* to distinguish beliefs from knowledge. Abelson (1979) portrayed features of distinguishing a belief from knowledge as "existential presumption", "alternativity", "affective and evaluative loading", and "episodic structure". The existential presumption challenges the existence or non-existence of one's beliefs about an entity and alternativity deals with how significantly one's ability to conceptualize ideal situations differs from present realities. According to Abelson, "the affective and evaluative loading" principle attempts to distinguish knowledge and beliefs by linking them to effective and evaluative components (e.g., feelings, moods, and personal evaluations) and "the episodic structure" principle tries to make a distinction based on personal and/or cultural events. Later, Nespor (1987) added two features, "non-consensuality" and "unboundedness" to Abelson's original formulation to show that knowledge is more dynamic than belief and is also bounded with logical reasoning. Although Abelson's and Nespor's proposals are enticing, it is usual to find these characteristics in the knowledge systems, as Abelson himself admitted. In addition, what beliefs one is capable of holding are restricted to the sorts of things that one is capable of knowing, and one's knowledge consists of those beliefs that one might confidently hold.

So far, it is well established that belief and knowledge are closely connected. Therefore, it is useless to seek new answers to the question of whether it is possible to distinguish between beliefs and knowledge in mathematics education. First, mathematical reasoning itself includes various psychological, philosophical, sociological, and even theological components like beliefs and expectations, analogies, and paradigms. Historically, even mathematicians' religious affiliations influenced their practices, their view of mathematics, and therefore the production of pure mathematical knowledge (Kline, 1980). A recent study demonstrates that mathematicians' religious convictions still continue to influence their practices, independently of the religious group to which they belong (Norton, 2002).

Second, all mathematics educators conduct their research with a background of theory that comprises a unitary package of beliefs about mathematics and mathematical knowledge. In mathematics education, neither epistemological stances nor pedagogical theories are strongly prescriptive of one another, and every theoretical perspective embodies a certain way of understanding ontology as well as a certain way of understanding epistemology. Mathematics educators' beliefs and knowledge may intrude upon the conduct of their work, especially in their choice of research problems and in their views about the practical uses of their research results. The problem in the identification of knowledge and belief is a failure among mathematics educators to reach a consensus regarding what should count for justification of knowledge in mathematics education. Obviously, it is not fruitful to seek an absolute justification for knowledge in mathematics education, because of Gödel's work that showed pure mathematical knowledge itself might not be justified since most mathematical systems contain truths that cannot be proven within the system. Thus, my proposed solution to this problem is to give up the idea of seeking a full or complete justification by agreeing on the basic qualifications for a commonly accepted justification for what constitutes knowledge in mathematics education.

A brief summary, critique, and comparison of previous belief studies and definitions

In the mathematics education literature, studies can be divided into three categories based on one's beliefs about: (1) the nature of mathematics as a discipline, (2) mathematics learning, and (3) mathematics teaching. In the first category, Ernest (1988) categorized one's beliefs about the nature of mathematics as Platonist, problem-solving, and instrumental. Other researchers (Borasi, 1992; Lester, 1994; Maurer, 1987), however, defined beliefs as students' or teachers' subjective knowledge about mathematics such as seeing mathematics as a finished product and knowing mathematics as remembering and applying the correct formulas.

In the second category, one's beliefs about oneself are investigated along with their individual relationship to mathematics including beliefs related to self-confidence, self-concept, casual attributions of success, their authority concept, and so on. Lester (1988) found that students' belief systems, as well as other affective characteristics like motivation and anxiety, guided and regulated their mathematical problem-solving processes. Schoenfeld (1989) found students' beliefs about what mathematics is had a very strong effect on the their mathematics learning. Although both Lester and Schoenfeld investigated the impact of students' belief on their mathematical learning, their belief definitions are slightly different. Schoenfeld's (1989) definition of belief, "an individual's understandings and feelings that shape the ways that the individual conceptualizes and engages in mathematical behavior" (p. 358), weighs the psychological dimension of belief. However, Lester, Garofalo, and Kroll's (1989) definition of belief -- "the individual’s subjective knowledge about self, mathematics, problem solving, and the topics dealt with in problem statements"(p. 77) -- tries to cover both philosophical and psychological dimensions of belief.

There is a need to use an interdisciplinary definition of a belief (as mentioned above) in the investigation of one's beliefs on mathematics learning since one's beliefs about what counts as a mathematical knowledge and what one finds interesting and important have influence upon one's mathematical behavior. Moreover, in mathematics education, it is not easy to separate beliefs into context and capability categories since the mathematics elicits the emotion itself (Schoenfeld, 1989).

In the final category, research studies focused on the role played by teachers' beliefs about mathematics and how to teach mathematics on their classroom behavior. Research findings display a discrepancy between intended and actual teaching behaviors that were caused by teachers being unaware of the impact of their values on classroom dynamics and students' learning (Clarkson & Bishop, 1999, Mura, 1995; Thompson, 1984). I propose that the investigation of teachers' beliefs requires a successful integration of the sociological dimension of belief with philosophical and psychological dimensions since teaching mathematics itself is a product of a social interaction. This necessity is reflected in Thompson's (1992) definition of belief, "a teacher’s conceptions of the nature of mathematics may be viewed as that teacher's conscious or subconscious beliefs, concepts, meanings, rules, mental images, and preferences concerning the discipline of mathematics" (p. 132).

So, why do mathematics educators use different belief definitions in their research? Although various researchers' definitions of belief might appear contradictory, their definitions are consistent with their interest areas and their perceptions of what kind of knowledge mathematics education should produce. Whereas Hart's (1989) definition -- belief is a construct that reflects certain types of judgments about a set of objects -- tries to include psychological and sociological dimensions of a belief, Ponte's (1994) definition stresses the psychological dimension of a belief, "the incontrovertible personal truths held by everyone, deriving from experience or from fantasy, with a strong affective and evaluative component" (p. 169). These belief definitions are compatible with Hart's and Ponte's research interests. Hart focuses mainly on mathematics education research dealing with gender issues and equity in mathematics classes, whereas Ponte focuses on learning and problem solving. Aside from the researchers' interest areas, their academic disciplines also influenced how they defined the term *belief* (e.g., Pajares, 1992 & Nespor, 1987). As a result of being trained or having an extensive knowledge in psychology, Pajares' definition of belief is aligned with a psychological definition.

The need for developing an explicit definition of belief

The main reason for developing an explicit definition of belief is to prevent linguistic ambiguity that makes it difficult to separate research on beliefs from research on attitudes, values, emotions, and knowledge. Partly as a result of differing views of what the words beliefs, knowledge, values, emotions, and attitudes refer to and partly as a result of using them synonymously, it often becomes difficult to interpret the meanings of these terms in mathematics education. Most researchers agree that having an explicit definition of belief would clarify the ambiguity between beliefs and attitudes. Although Silver (1985) underscored this need and the problem by saying, "Are all attitudes also beliefs; if not, then how do we distinguish those that are from those that are not?" (p. 256), McLeod (1992) restated same ideas in the following words "in the literature it is difficult to separate research on attitudes from research on beliefs" (p. 58).

Without doubt, having an explicit definition of belief will contribute to the efforts of having an explicit definition of attitude, which is especially important in efforts to help students develop positive attitudes toward mathematics. The lack of an explicit definition of belief also creates an incompatibility and ambiguity between its definition and the instruments used to assess or measure them. Moreover, without an explicit definition of belief, the assertions such that investigation of beliefs acquisition or modification requires an exploration of the nature of sets of beliefs or belief systems rather than an examination of the nature of one belief alone (Green, 1971) would not move beyond solipsism. Therefore, having an explicit definition of belief is crucial to understanding not only what one's beliefs are but also how these beliefs are structured and held. This effort will also help to clarify different conflicting theoretical interpretations of this construct and its relation to other constructs such as the difference between conceiving emotions as belief systems (Averill, 1986) versus a part of belief systems (Furinghetti & Pehkonen, 1999).

The need for developing an interdisciplinary definition of belief

The inclusion of a philosophical dimension of belief is necessary in the investigation of students' or teachers' beliefs about the nature of mathematics since the investigation of what mathematics is requires a reference to one of the following philosophical stances of mathematics: realism, conceptualism, formalism, and intuitionism (Ernest, 1991). Furthermore, the investigation of one's beliefs related to mathematics learning requires the inclusion of philosophical, sociological, and psychological dimensions because of linkages between students' beliefs and their achievement, students' characteristics (e.g., race and socioeconomic status) and the impact of social organization of schools and society on to students' beliefs on mathematics and mathematics. Moreover, the investigation of teachers' beliefs also requires the inclusion of philosophical, sociological, and psychological dimensions because the investigation of teachers' beliefs requires a consideration of the teachers' prior school experiences (as a student, in a teacher preparation program) and the ways in which these experiences influence their social teaching norms (school setting and curriculum), and their beliefs about mathematics (Ernest, 1991).

CONCLUSIONS AND EDUCATIONAL IMPLICATIONS

This paper has raised even more questions than it began with. I hope to channel research on beliefs in the direction of coming up with an explicit and interdisciplinary definition of belief. In summary, there are two main reasons for the use of an interdisciplinary definition of belief: First, mathematics teaching practice in schools is intimately tied to political and social realities of society, and second, the research problems in mathematics education reflect the discrepancies between the mathematics educational philosophical stance it seeks to foster and the actual mathematics education practices. Moreover, every research program in mathematics education assumes a prior commitment to philosophical, psychological, and sociological views of the nature of mathematics education. The researcher's explanation of beliefs (one's beliefs about the nature of mathematics, learning mathematics, and teaching mathematics) may have social, psychological, and philosophical components, which the beliefs themselves might lack. Hersh (1986) emphasizes the effect of these three dimensions on mathematics teaching and learning practice as follows: " Ideas have consequences. One's conception of what mathematics is affects one's conception of how it should be presented. One's manner of presenting it is an indication of what one believes to be most essential in it" (p. 13).

Therefore, studying beliefs requires examining the context and research goals in which they are considered. The time has already passed to understand and to decide the ways in which the views of psychologists, philosophers, and sociologists differ from or resemble those of mathematics educator's in their investigation of affective educational outcomes. Moreover, mathematics educators must also reach a consensus among themselves regarding what a belief is. Having an explicit definition of belief will also contribute to efforts of finding a balance between two different paradigms that try to explain mathematics education practices, and at the same time, attempt to articulate a vision of the ideal of mathematics education regardless of what the mathematics educator actually does in practice.

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[1] In here, the author is not asserting Newton and Leibniz did not think about neither ontological nor epistemological status of calculus ideas. The point that he is trying to make is they did not allow the debates on the metaphysical status of mathematical objects to prevent them from developing their theories.