


MATHEMATICAL DISCUSSION IN THE CLASSROOM

Elizabeth Lovell
elizabethall@yahoo.com

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Exploring the role of discussion in the Mathematics classroom, particularly with reference to the aims and use of the plenary within the three part lesson of the National Numeracy Strategy.

INTRODUCTION

The context in which my work began

I am a secondary Mathematics teacher, with two years teaching experience. My aim as a teacher is to develop my pupils' understanding of Mathematics, rather than just their ability to answer test questions. The National Numeracy Strategy (NNS) in secondary schools has been introduced, in pilot schools, for the year 2000 cohort of Year Seven pupils. I teach at a Local Education Authority comprehensive school in Hertfordshire, teaching mathematics to pupils aged from eleven to eighteen. This school is not part of the official pilot. Nevertheless the Mathematics department is using a new scheme of work for these Year Seven pupils, based on the NNS. All staff teaching Year Seven are intending to work within the NNS framework, using the three part lesson structure which consists of 'starter', 'main body' and 'plenary'. Some members of the department took part in training sessions prior to the beginning of the year intended for schools piloting the NNS.

During the first half-term two members of the department, including myself, monitored the introduction of the NNS by observing lessons, looking at exercise books and asking teachers about their general experience of using the framework. With regard to the first of the three parts of the lessons our findings were very positive. Learning objectives for the starters are outlined in the scheme of work. We found that both staff and pupils appeared to enjoy and participate enthusiastically in the starter activities. Staff were making use of a wide variety of resources and ideas for these activities and sharing these effectively within the department. The main body of the lesson resembled very closely the practices generally used in teaching all year groups and the new scheme of work appeared to be working reasonably well with a few reservations expressed.

With regard to the final part of the lesson our findings were less positive. All staff who taught Year Seven expressed confusion about the use of the plenary section of the lessons. This tended in practice to consist of a few minutes in which the teacher set homework, collected in books or went through the answers to the class work. Staff felt that this was typically the same as the end of many lessons not based around the NNS.

The NNS seems to suggest that the plenary could be used to contribute more significantly to the learning process than this. My belief was that its aim was related to the idea of promoting understanding. I began to wonder whether my colleagues and I could either use the final part of the lesson differently or change something else in our teaching in order to ensure that the aims of the plenary (and the NNS as a whole) could be achieved within our lessons. I was concerned that the NNS could not be effective in improving teaching and learning if teachers changed only the physical structure of lessons in order to meet requirements rather than implementing changes in teaching and learning methods.

My research project

I decided to learn more about the NNS, particularly the intended aim and uses of the plenary section of each lesson. I went on to learn about the importance of language and the role of discussion in the Mathematics classroom. As a result I decided to take as my research basis that discussion about mathematics can help pupils to develop understanding. Hence there is a need for pupils to be taught how to, and to be given opportunities to, discuss mathematics. This is detailed in the Research section of this report.

I undertook a practical investigation based on the outcome of the above research. The investigation focussed on different ways of involving pupils in worthwhile mathematical discussion. I experimented by using various teaching and learning activities with the Key

Stage Three classes that I teach. I describe and analyse three of these activities, used with a high-ability Year Seven class, in the Investigation section of this report.

INITIAL RESEARCH

'In mathematics you don't understand things. You just get used to them.' (von Neumann, quoted by Gaither, p.249).

First I shall consider sources relating to the role of the plenary in the NNS. Then I shall consider the place of discussion in the Mathematics classroom where the aim is to dispel the attitude described by von Neumann.

Plenary

On its website the DFEE (2001) describes the role of the plenary, particularly for able pupils, as follows:

- celebrate achievement and raise expectations by showing good work and explaining what is good about it
- make learning explicit, by requiring able pupils to explain their thinking and use the appropriate terminology
- ask able pupils to explain the criteria for success in their work and to reflect on how well they have met them
- invite able pupils to make generalisations and to provide evidence to support their conclusions and opinions
- allow able pupils to lead the session occasionally, asking them to prepare questions or points to put to the rest of the class
 - whet the appetite for the next day's work, negotiating challenging targets for future lessons.

It is clear from this that at least part of the intended use of the plenary is to provide opportunities for pupils to discuss Mathematics. This is in order for pupils to develop understanding, going beyond the stage of being able to do specific calculations that have been modelled for them. This understanding should enable them to apply what they have learnt to a variety of problems and feel confident that they can think about and use what they have learnt without confusion.

The DFEE (2001) outline of uses for the plenary also suggests that one of its roles is to generate interest in the subject and motivation for study in the pupils. It is to achieve this by providing opportunities for reflection on the standard of work being produced, for challenging work going beyond textbook/examination style questions and for looking forward to the next topic of study so that pupils can see their study in context.

In its Evaluation of the NNS (2001), OFSTED states:

" The best plenaries are used to draw together the key ideas of the lesson, reinforce teaching points made earlier, assess what has been understood, and correct errors and misconceptions ... The least successful element of the daily mathematics lesson is the plenary. Only four in ten plenaries were good and one-quarter were weak. The impact of many of the weakest plenaries was reduced by lack of time." (page 10)

This fits in with the experiences of Mathematics teachers at this school in highlighting weaknesses in the use of the plenary and lack of time for it to fit in to a three-part lesson. In Secondary schools this problem is likely compounded by the fact that in many cases the lesson slots are less than the one hour length recommended in Primary Schools (at this school, for example, lesson periods are 50 minutes long).

Repeating, summarising and assessing seem to be emphasised as being the expected uses of the plenary here, rather than challenging, placing in context, stimulating interest and providing opportunities for discussion, as recommended by the DFEE.

Koestler (1964) states that,

The conventional test of understanding is verbal explanation - the subject is invited to name the general rule of which the event to be explained is a particular instance. But the availability of such neat and ready explanations is the exception rather than the rule - unless the explanation was learned by rote. (page 619)

He explains this by outlining 'A whole series of graduations in understanding and explanation,' and goes on to claim, 'Thus instead of talking of insight and understanding as all-or-nothing processes, and making verbal explanation a test for passing school exams, we should proceed by more cautious statements. (page 620)

This is in order to recognise true levels of understanding of concepts and relations, rather than making superficial judgements.

Koestler's theories outline the relationship between understanding and the ability to explain. They support the idea that it is more beneficial to the learning process to provide opportunities for pupils to increase their understanding by discussing and explaining mathematics than to provide opportunities for teachers to assess the understanding of their pupils by requiring them to explain concepts. This implies that the purposes for the plenary suggested by the DFEE are of more educational value than those implied by OFSTED (see above).

Schools are assured that they do not need to follow the recommended NNS structure of a lesson prescriptively (DFEE framework, 2001, page 7). The importance of talking and listening are emphasised by their inclusion as items to be considered in the audit document which must be completed by schools before beginning to introduce the NNS (DFEE audit, 2001).

They [pupils] listen attentively to their teachers and to each other.

They answer questions willingly, explaining and demonstrating their ideas clearly using subject-specific vocabulary. (page 7).

If the plenary is an essential part of the NNS because it is expected to provide opportunities for pupils to learn through discussion, then the flexibility within the implementation of the framework implies that these opportunities need not always be at the end of a lesson.

Discussion

It has been assumed above that discussion does assist the development of understanding. According to Cockcroft (1982)

Mathematics teaching at all levels should include opportunities for ... discussion between teacher and pupils and between pupils themselves. (paragraph 243)

The reasoning behind this is articulated well by Vygotsky (1986)

The relation of thought to word is not a thing but a process, a continual movement back and forth from thought to word and from word to thought... Thought is not merely expressed in words; it comes into existence through them. (page 218)

From personal experience it is clear to me that human beings are often able to develop clarity of thought through the process of talking. Intuitively it follows that this should apply to mathematical understanding. Mathematical language is important to allow access to mathematical understanding, and as Vygotsky (1986) discusses, pupils develop their understanding of the meaning of words by observing how they are used by others and practising their use.

Not all talk, even about mathematics, directly contributes to the growth of understanding. Clarkson (1973) states that

It was found that verbal interaction between children may be extremely productive of ideas and development in certain settings. Preparation, even priming, for such a task can be crucial to the success of the effort, and the role of sensitive adult intervention may be crucial. (page 4)

Experience in the mathematics classroom and within the wider school environment bears out the idea that pupils need to be guided in how to discuss, i.e. engage in 'purposeful talk,' (Simmons, 1993). Therefore it is necessary for the teacher to not just provide time

for pupils to discuss but also to structure their experiences to build up their ability to use such time productively.

Developing understanding

Using the terminology of Skemp (1976), Brissenden (1988) asserts the importance of discussion in learning mathematics for the following reason:

We should aim for 'relational understanding' (knowing why rules work), and 'logical understanding' (being able to explain them to others) rather than the 'instrumental understanding' (using rules without knowing why they work) which results from learning mainly by imitation, as at present. (page 9)

This is exactly what I am aiming to do in the classroom. Whilst there is a place for imitation in learning I believe, with Brissenden, that without the development of understanding imitation is ultimately fruitless. In order for pupils to be able to explain to others they need appropriate language (see above) and opportunities to practise. Hence the use of a plenary section within each lesson, or other regular time within lessons, to develop the skills needed to discuss mathematics is a potentially valuable measure.

Wiles (1985) sets out a list of criteria for competency in communication. In looking at the worth of discussion in the mathematics classroom, Brissenden (1988) relates this to Mathematics:

What then can mathematics contribute to Wiles' communicative competence?...

- a. articulating and presenting an idea publicly, in a clear and intelligible way
- b. explaining a method
- c. arguing logically in support of an idea
- d. criticising an argument logically, including one's own
- e. evaluating the correctness of an idea, or its potential in attacking a problem
- f. speculating, conjecturing, entertaining an idea provisionally
- g. accepting an idea provisionally and examining the consequences
- h. keeping track of a discussion, reviewing
- i. coping with being stuck, supporting others in difficulty
- j. drawing others out, using 'show me' or 'how did you get that?'
- k. acting as spokesperson for a group's ideas. (page 203) (letters in italics added).

I shall take as the basis for my investigation, that attempts to develop these skills in the mathematical classroom have the potential to improve pupils understanding of mathematics, their motivation in studying the subject and their general social skills. Therefore I shall experiment with activities in the classroom designed to help develop these skills. In analysing these activities I shall refer back to the letters that I have inserted above (in italics) to indicate the individual skills.

INVESTIGATION

The class

The class consists of thirty-two high ability Year Seven pupils, in the top set of three in one half of their year group. Most pupils achieved level five (one achieved level four and one level six) in their Year Six Key Stage Two Mathematics tests. In the first weeks of term they exhibited enthusiasm and ability in written and oral answering of numerical questions, but appeared confused when asked to explain things. For example, three quarters of the class would typically volunteer an answer to a sum but only one quarter would offer an explanation as to how the answer was reached, and these pupils would provide a description of the method used rather than a reason why the method was chosen or why it worked.

The first activity

The first activity I used with the class deliberately to cause them to think about and discuss how they answer questions was in

relation to the distributive law. As part of a session of the Numeracy course, we were reminded that children are encouraged as part of the NNS to use strategies such as the following

To do the sum 56×7 :

$$\text{Do } 50 \times 7 + 6 \times 7 = 350 + 42 = 392.$$

We were then challenged to consider whether pupils had a clear picture of why this works and whether they might consequently believe that this strategy was suitable for use with division sums.

The pupils in the class did indeed use this method and state it as the way they had worked out questions, as a result of having been taught it at Primary School. I thought that probably a few of the pupils did have a clear picture of how the strategy worked but that most did not. I decided to test my opinion whilst providing the pupils with an opportunity to discover how it works and to discuss mathematics. I presented the task on the whiteboard at the beginning of the lesson as follows:

Which of these are correct?

Discuss in pairs.

$$\begin{aligned} 18 \times 60 &= (10 + 8) \times 60 \\ &= 10 \times 60 + 8 \times 60 \\ &= 600 + 480 \\ &= 1080 \end{aligned}$$

$$\begin{aligned} 100 \div 25 &= 100 \div (20 + 5) \\ &= 100 \div 20 + 100 \div 5 \\ &= 5 + 20 \\ &= 25 \end{aligned}$$

$$\begin{aligned} 8 \times 57 &= 8 \times (50 + 7) \\ &= 8 \times 50 + 8 \times 7 \\ &= 400 + 56 \\ &= 456 \end{aligned}$$

$$\begin{aligned} 45 \div 15 &= 45 \div 10 - 45 \div 5 \\ &= 4.5 - 9 \\ &= -4.5 \end{aligned}$$

I asked the pupils to work in pairs to read through the four sums with their suggested solutions, to decide which solutions were correct and to explain the reasons for their decisions. I had planned to give pupils ten minutes to work on the task and then as a plenary to spend five minutes discussing it as a whole class.

Most of the pupils were slow to discuss the solutions to the sums, and appeared not to notice that some of the answers were in fact wrong. As I moved around the room asking pupils what they had found, I asked questions such as "But what is $45 \div 5$?" After about ten minutes of pupils writing the sums, the "solutions" and the actual answers in their exercise books, and talking quietly and intermittently, more enlivened discussion between pupils began, and so I left the discussion going for ten more minutes. Now, as I talked with pupils, they told me which were correct and I asked them why some worked and some did not. Still pupils struggled to explain, coming up with ideas about adding or subtracting for multiplying or dividing. Gradually consensus appeared that the "splitting method" worked for multiplication but not for division.

I brought the class together and asked the pupils to describe what they had found and to try to explain it. They were united in the fact that the method they had been taught for multiplication worked for it but not for division. No one was able to explain why independently. I prompted pupils to describe what multiplication means and how this was equivalent to the addition sum they did in the method. I then prompted them to consider that the same breakdown could not be made for division. Finally I wrote notes with regard to the distributive law on the board and the pupils copied these into their exercise books. In total, this activity took thirty five minutes.

I felt that the pupils were unsure and nervous when starting this activity, and because of their fear of being wrong were unwilling to really engage with the problem. Gradually this changed, as I prompted pupils to talk about their thoughts and encouraged them, right or wrong. As the activity went on, pupils did begin to talk to one another about the mathematics involved and to attempt to explain things. With a high level of support the pupils did make progress in understanding. This experience fitted in closely with those of Clarkson (1973).

Skills *d*, *e* and *g* were worked on by most pupils, and some also worked on skills *a*, *f* and *i* (1). The focus was on skill *d* - logically

criticising an argument. At the beginning, many did not appear to be familiar with the use of this skill, but eventually all were able to make use of it to some extent. I felt that it was a positive early step, but that I had not really achieved my aim of getting the pupils to develop understanding through their own discussion, because I had so strongly led and influenced the content and structure of almost all of the discussion.

Following on from this lesson, I made a conscious effort to encourage discussion in general in lessons, and to listen to pupils' thoughts and contributions, whether or not they were immediately relevant to the point I was trying to make in a particular lesson or activity. I hoped that this would help the pupils to develop more confidence as well as providing a positive model of listening skills.

The second activity

In the summer term there was a gap in the Year Seven scheme of work and I used this to spend six lessons giving pupils the chance to lead the class. This was in the approach to the summer examinations, so I gave the pupils, in groups of two or three, the task of leading the class for ten to fifteen minutes in work on a topic of their choice from those covered during the year. They were given two objectives: to revise the topic with the class; and to promote discussion of mathematics. These objectives were frequently reiterated so that all the pupils were aware of them. They spent two lessons preparing for the presentations and then four lessons presenting their work and participating in other presentations.

Amongst the groups, many styles of presentation were used. The amount of time allocated to talking was different in each case. How much of the talking was by members of the presenting group, and how much was required from members of the rest of the class, varied widely. Some of the talking was descriptive, some explanatory. Some talking was organisational (telling other pupils what to do for an activity) and out of all the presentations, one negative comment about mathematics was made. Along with talking, some groups explained concepts by acting them out, such as division according to a ratio. Other groups concentrated on describing how to tackle different types of questions and then practising questions of that type.

Subsequently the pupils wrote reflections on the presentations, commenting on their own group's level of success in fulfilling the two objectives and on the one other presentation that they thought had been the best in meeting the objectives. In these pieces of writing, many of the pupils (particularly those who analysed their own work in depth) wrote about their difficulties in talking about mathematics as opposed to doing exercises. Most pupils selected either the 'Angles' or the 'Ratio' presentations as having been the most successful. These two presentations had each mixed explanatory talking and demonstrating by the group with activities for the rest of the class to attempt.

I felt that the activity had been successful in making pupils aware of the importance of talking about mathematics and what could be gained from doing so. I felt that in some cases it was successful in helping pupils to develop understanding by articulating their thoughts. The main skill being practised by the pupils was a - presenting an idea clearly in public. Pupils also worked on skills *b*, *j*, and *k* as they prepared and presented, and skill *h* as they took part in other presentations ([1](#))

I now wanted to see if this, along with the other work we had been doing, would enable the pupils to discuss new mathematical ideas without a high level of intervention from myself.

The third activity

I decided to use an extension activity by Gardiner (2000) entitled 'True, False and 'Iffy' number statements.' I drew an adapted table, with the following column headings, on the board:

Always True	Sometimes True	Never True	Not Sure
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I had the pupils copy this table into their books and gave them photocopied sheets with eighteen number statements on (see [Appendix](#)), to be categorised.

I spent fifteen minutes setting up the activity and modelling how to discuss the statements by leading the pupils in whole-class discussion about three statements. 'The product of two numbers is a whole number', 'The product of two odd numbers is odd' and 'The product of three whole numbers is never the same as their sum' were the statements that I used as examples.

For each, I first asked the pupils to think silently for a moment and then to raise their hands to vote whether they thought the statement was always, sometimes or never true. Then I picked individuals who had voted for each and asked them to explain their choice. As they did so the correct choice became apparent and the pupils themselves noticed and pointed out mistakes or extra things to consider (for example the effect of using negative numbers or fractions). Then I asked the pupils to vote again and they were unanimous in choosing the full correct answer.

Having gone through this process with three statements, I gave the pupils thirty minutes to place as many of the rest of the statements as they could into the correct category. I was very impressed by the high standard of mathematical discussion that I heard as I moved around the room. The pupils participated in enthusiastic mathematical debate, considering all possibilities rigorously and confidently. As they did so they used correct mathematical language and spoke clearly, often demonstrating a process of thinking out loud.

I listened with interest to the discussions and did not feel the need to intervene with any of the pupils - they all appeared to be able and willing to engage with the task. Some pupils asked me to arbitrate between differing views - when this happened I asked them to explain what they each thought and each argument was resolved by the pupils themselves.

The final five minutes of the lesson were spent on the plenary activity. For this I asked the pupils to volunteer their thoughts about any of the statements they found particularly interesting or surprising. Several pupils volunteered and I chose two to speak, which they did with clarity and obvious understanding, before it was time to end the lesson. The first of these pupils described how he had realised that a fraction multiplied by a fraction gave a smaller fraction, and so the square root of a fraction would be larger than the original fraction. The second described how she had worked out that it was true that any square number has an odd number of factors by describing how to find factors of a number to one of her peers.

I felt that the pupils had achieved the objective that I had set of developing understanding through mathematical discussion in this lesson. The learning that took place in the lesson happened through the pupils themselves thinking and talking to each other. Unlike at other times, for example in the first activity, it was not necessary for me to lead them point-by-point through the thinking process. I feel that this was valuable, as they are more likely to retain this learning since they have worked it out for themselves, and as they have had the opportunity to develop their thinking skills. In this activity all the pupils worked on skills *c*, *d*, *e*, *f* and *g*. Some also worked on skills *a*, *h*, *i*, *j* and *k* (1).

In lessons in general, I have noticed in these pupils more willingness to talk about mathematics, and to volunteer explanations without having to be coaxed. Often they display a high level of awareness of their own understanding. When the pupils are stuck they tend to ask a fellow pupil or myself to explain how and why something works rather than how to do a particular question.

CONCLUSION

The outcome of my research

In the course of my research and investigation I have discovered the importance of talking about mathematics as a part of developing understanding. I have seen that this is part of the intended use of the plenary within the NNS, and that plenary activities do not necessarily have to take part at the end of lessons. I have experienced in my classroom the fact that pupils are able to discuss mathematics and do learn through doing so, but that they need to be given the skills and confidence necessary before this can happen effectively.

For the future

In NNS training the main emphasis with regard to the plenary appears to be assessing understanding, with some mention of clearing up misunderstandings and challenging pupils. I believe that it would be valuable to build regular space into lessons for pupils to learn through talking rather than be assessed on how they are able to talk (although of course informal assessment of understanding may still take place, despite not being the objective of the activity). This could be in plenary sessions or in the main part of some lessons.

The investigative work described here was carried out by one teacher with one class. Obviously the same activities may have had different outcomes if led by a different teacher and involving different pupils. Nevertheless, I was encouraged in my own classroom practice by the results of the investigation. In continuing to investigate I shall consider further ways in which one might measure the effect of the activities on the pupils' understanding and their thinking skills.

I shall to continue to try to encourage the pupils in my classes to discuss mathematics in such a way as to be able to develop their understanding through their talking. I intend to investigate two further activities. Firstly, short activities involving solving a problem and then convincing a partner that the solution is correct. Secondly, having 'hot seat' tests at the end of studying a topic, where pupils are divided into groups: one group studies in order to be successful in answering questions when tested; the other groups prepare the test questions.

I shall also continue to research to think about different ways of developing understanding through discussion and of developing thinking skills.

Final thoughts

If you ask mathematicians what they do, you always get the same answer. They think. They think about difficult and unusual problems. They do not think about ordinary problems: they just write down the answers.

Egrafov, quoted by Gaither, page 156.

It is not the job of mathematicians ... to do correct arithmetic operations. It is the job of bank accountants.

Shchatunovski, quoted by Gaither, page 237.

Accessed

In general the NNS in Primary schools appears to have been successful in improving the perception of mathematics by pupils and their numerical skills. At Secondary level it should therefore be possible to concentrate on developing their analytical skills and helping them to become mathematicians.

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APPENDIX

A The product of two numbers is a whole number.	B Adding a zero to the end of a number multiplies it by 10.
C The square root of any number is smaller than the original number.	D The product of two odd numbers is odd.
E Every square number has an odd number of factors.	F When you square a number the answer is positive.
G Suppose you divide a number by 2 and then by 10. If instead you divide the number by 10 and then by 2, the answer could be different.	H Calculating two fifths of a number is the same as dividing it by 5, then multiplying the answer by 2.
I Prime numbers are odd.	J The sum of the digits of any multiple of 3 is divisible by 3.
K The sum of two numbers is greater than their difference.	L The only factors of a perfect square are the square itself, a square root, and 1.
M The product of three whole numbers is ever the same as their sum.	N The product of a negative number and a positive number is negative.
O Dividing by a number less than 1 gives a larger number.	P When you multiply two numbers the answer is bigger than either of them.
Q The sum of two odd numbers can sometimes be odd.	R The cube of any number is bigger than its square.

