

An Analysis of Errors Made in the Solution of Simple Linear Equations

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ABSTRACT

This is an investigation into the errors made by pupils when solving simple linear equations. Data was collected from a final examination and analyzed with reference to recent literature. After finding three error types identified in the literature, six new error types not discussed in the literature were identified by the researcher. This constituted the pilot study. An expanded large-scale study employing the same methodology was carried out on a sample of 246 pupils' answers to between three and six linear equation questions, both to test the robustness of these six new error types and to find examples of the Transposing error mentioned in the literature. During this process the new data, and the 166 errors identified, was analyzed and, because of the nature of the new examination questions in this large-scale study, examples of the Transposing error were found and the unidentified errors were reduced to five. Thus nine error types appear in the analysis of the large-scale study. Three of these errors, Transposing, Switching Addends, and Division, are found to account for approximately three-quarters of the total number of errors. Transposing and Switching Addends errors are classified as structural errors, and mechanisms are suggested for their commission, both from the literature and from the experiences of the researcher. An interesting finding of this study, and one that may deserve further study, is that Transposing errors occur due to what may be oversimplification of the transposing process. Also, the Switching Addends error appear more frequently in 'algebraic' than in 'arithmetical' equations, and the resulting differing success rates confirm the findings of the literature on the difference in difficulty between these two types of equation. The findings of the large-scale study also highlight the importance of subordinate skills such as division, especially among younger pupils.

Chapter 1 - Introduction

The purpose of this inquiry is to identify, and classify by relative frequency, the most common errors made by a sample of pupils in their attempts to solve simple linear equations. The further analysis of the possible mechanisms responsible for these errors may facilitate the design and development of improved teaching strategies in the classroom.

Linear equations are a central part of any Mathematics course, especially at Lower Secondary level. Not only do they appear in their own right, but they are also an integral part of a wide variety of Algebraic, Geometric, and Trigonometric problems. In Physics especially, as well as in other sciences and social sciences, linear equations are used in the solution of problems. For example, Density is defined as Mass divided by Volume, which leads to the simple linear equation $D = M / V$. It is this equation, and other homogeneous equations from Physics such as $R = V / I$ (Ohm's Law), $S = D / T$ (speed, distance and time), and $I = W / V$ (power equation for determination of fuses in an electrical circuit), to name but a very few, where pupils have a rich opportunity to apply their mathematical knowledge in a physical situation in the real world, in other words, to use mathematics as a tool. It follows that any attempt to analyse and improve the teaching and learning of this topic is important.

In referring to a study by Livingstone & Borko (1990) into differences in actual teaching between novice and expert teachers, Koehler & Grouws observe that novice teachers "had little knowledge of student misconceptions. Their schemata was adequate for their own understanding, but was insufficiently developed....to enable them to be responsive, flexible teachers" (1992, page 121). Armed with knowledge of errors likely to be made by pupils, novice teachers may be able to improve the quality of their teaching of mathematics not only by being aware of errors or misconceptions, but also by alerting pupils to such errors while the initial teaching is taking place. Further, this potential for improvement in teaching may not be limited to novice teachers because, as Koehler & Grouws (1992, page 121) point out, "experts become in some ways like novices when teaching new content" i.e. it is hoped that the findings of this dissertation may be helpful to novice and expert teachers alike, in improving the initial teaching of linear equations in four ways.

First, the analysis of the range of errors made by a large sample of pupils might provide information to enable the development of teaching strategies, which anticipate probable, documented difficulties. Second, this analysis could also give guidance to the teacher as to the development of the pupil's initial thought processes during the learning of the material, although the drawing of such

inferences would have to be done with caution. Writers of textbooks could use third, such information in that instead of presenting only correct material, common misconceptions and errors could be discussed at relevant points in model solutions (although this would have to be done carefully to avoid confusion). Fourth, the teacher could match a certain pupil's error with one from the list in order to better understand what the pupil did wrong.

To make it more useful, an informative error analysis could include the relative frequency by type of each classified error. It may then be possible to establish a frequency hierarchy of common errors. This would be of special help to the teacher of a large class, allowing her to both anticipate the more frequent errors and to enable her to identify the most important intervention points. For instance, if error X is the most frequently-made in the study, the teacher could demonstrate to the class during the initial presentation of simple linear equations exactly what many other pupils believe (erroneously) to be the next step in the solution. Conversely, class time might not be wasted in attempting to forestall what appears to be, according to the studies, relatively infrequent errors.

This dissertation provides a brief overview of the current literature pertaining to error analysis in Algebra, and presents and discusses the findings of two analyses of errors found in linear equation questions, based on a pilot study and a large-scale study. There are already examples in the literature of large-scale studies in this particular field. It is hoped that this study will add to the results of researchers such as Kieran (1984, 1989, 1992) and Matz (1981) who identify and classify errors, by introducing relative frequency of errors made. However it will not go into the exhaustive detail as did Carry et. al. (1980), who analysed in such detail that it is not in a concise enough form to be used by the classroom teacher. The aim is that the study will build on the work and ideas of these and other researchers in such a way that the practicing teacher may quickly form an opinion on what to look out for in the teaching of this topic, while being careful not to assume unwarranted generalisability, for reasons including the fact that the study was conducted using samples of students only from Bermuda.

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