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MATHEMATICAL SOPHISTICATION AND EDUCATIONAL PHILOSOPHIES AMONG NOVICE MATHEMATICS TEACHERS

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This study investigated the mathematical sophistication and educational beliefs of novice secondary mathematics teachers. Five high school mathematics teachers answered structured interview questions relating to their level of mathematical sophistication and their underlying philosophy of mathematics education. This study used Weinstein's (1998) Ways of Knowing Mathematics, a hierarchy based on intellectual development theory, to determine levels of mathematical sophistication. Ernest's (1991) classification of philosophies of mathematics education provided the basis for interpreting the teachers' educational philosophy.
This investigation was fueled by a desire to document practices of novice teachers with reference to the ideal teacher who possesses relativistic means of knowing and implements mathematical learning based on the constructivist model. Weinstein and Ernest are used to support this goal, since each framework holds a constructivist approach as the ideal. Furthermore, these interpretive tools can provide insight to the educators and mentors of mathematics teachers. Both frameworks provide a means to illuminate novice teachers' frames of mind as well as promote self-reflection.

Background

Intellectual Development Theory

The process for determining a teacher's level of mathematical sophistication derived from the work of three theories of intellectual development: Perry (1970), Belenky et al. (1986), and Baxter Magolda (1992). Perry documented the stages of intellectual development of Ivy-League undergraduate men as moving from dualism, through multiplicity and relativism, and to commitment. The basic growth transformed an adult from believing in the absolutism of right and wrong to understanding the infusion of contextual judgment and uncertainty into all aspects of life. Belenky et al. examined the applicability of Perry's stages to women. Their research used the metaphor of voice to describe stages of intellectual development in women. In this metaphor, the least empowered women are voiceless and silent. The following stages include listening to the voices of others (received knowing), the inner voice (subjective knowing), the voice of reason (procedural knowledge), and the integration of voices (constructed knowing). Baxter Magolda provided the most recent comprehensive theory on the development of mathematics sophistication. This framework consists of four stages: absolute knowing, transitional knowing, independent knowing, and contextual knowing. Each stage contains five categories: the natures of knowledge, the role of the learner, the role of the instructor, the role of peers, and evaluation.

Much of the current research on teacher development uses hierarchical developmental schemes descended from and relating to the work of Perry and the other intellectual development theories. Research reviewed by Brown and Borko (1992) reported that most preservice and in-service teachers were classified in the dualistic stage of Perry's development theory. Teachers in this stage may have difficulty embracing the constructivist teaching methods and reform movements. The authors concluded that because of differences in development, not all novice teachers are prepared for the actions required by a good mathematics teacher. Even when a teacher embraces the talk of constructivist teaching, Burton (1995) claims that underneath most social constructivists lies an absolutist by training. Therefore, changing philosophies will occur through slowly integrating constructivist activities and language into daily practices. Continued research is needed on how teacher education programs and experiences can help teachers attain higher levels of intellectual development.

Mathematical Sophistication

In response to the challenges inherent in applying general theories of intellectual development to the specific context of learning mathematics, Weinstein (1998) developed a hierarchical framework for mathematical sophistication. The theory, based on Baxter Magolda's epistemological reflection model, focuses on students' ways of learning and verifying mathematics. This framework describes five levels of sophistication in students' methods of learning mathematics and five more in verifying mathematics. The titles of these stages are given in Table 1.

Leaerning Mathematics	Verifying Mathematics
Mimicking the Procedure	Receiving Absolute Truth, Alone
Choosing among Procedures	Verifying Answers Alone
Understanding many Procedures	Verifying Answers Together
Understanding the Structure	Verifying the Structure Together
Constructing the Concepts	Agreeing on Social Structure

Table 1. Weinstein's Ways of Knowing Mathematics, arranged from least to most sophisticated.

Learning Mathematics, at the lowest level of development, corresponds to students mimicking the mathematical procedure given by an instructor. Choosing Among Procedures, stage two, describes students who understand the existence of multiple methods to solve a problem but depend solely on the one that works for them. Students who value the multiplicity of problem solving fall into the third level of the hierarchy, Understanding Many Procedures. Though the students may have preferences for solving problems, they understand that other procedures provide additional learning opportunities. Students capable of Understanding the Structure of mathematics, stage four, comprehend that there is more to learning mathematics than knowing all the procedures. Beyond understanding structure, the final level consists of Constructing Concepts. Learning mathematics at this level means understanding the concepts that connect the procedures and structure.

The framework for verifying mathematics develops from Receiving Absolute Truth, Alone to Agreeing on Social Structure. The initial stage for verifying mathematics embraces absolute authorities such as teachers. Listening to authorities is the only method of verification. The next phase, Verifying Answers Alone, continues with the absolute belief in the ability of authorities, such as teachers and textbooks. A student understands the importance of seeking verification, which they interpret solely as consulting with authorities. Stage three, Verifying Answers Together, is the first stage in which learners believe in their ability to check problems with others. This verification includes repeating the original procedure or checking with another person who completed the same problem. As in the case of learning mathematics, the final two stages shift from an emphasis on problem solving to an emphasis on structure. The fourth stage, Verifying the Structure Together, extends beyond checking answers to agreeing on the role of new knowledge within the existing structure of mathematics. The final stage of the framework consists of learners who believe in the relativistic nature of mathematics: the structure and organization of mathematics derives from both logical and social structures. At this stage, verification of mathematics requires a community of people whose authority is derived from technical skills and social position.

Weinstein notes that students do not uniformly progress through these two dimensions of knowing mathematics. For example, it is possible for a student to be at a low level in the framework for learning mathematics but possess an appreciation for the socially structured nature of mathematical knowledge. Weinstein's results are based on the qualitative analysis of interviews of students at different levels of mathematical sophistication. The interviews were designed to probe their ways of studying, learning, and verifying mathematics as well as their personal definitions of mathematics and beliefs about it.

Philosophies of Mathematics Education

Research in mathematics education has changed dramatically over the last three decades, with interest in the role of teacher belief systems developing relatively recently. In the 1960's, little research on mathematics education existed. Research on mathematics education increased in the 1970's with a focus on identifying teacher characteristics related to student success. In the 1980's, the epistemological framework of constructivism emerged. This movement sparked new research in the field of mathematics education, much of it focused on understanding teacher beliefs and teachers' construction of mathematical concepts (Cooney, 1994)

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Cooney and Wilson (1995) claim that an understanding of the underlying structure of teacher beliefs may help remove the randomness associated with teacher reform movements. According to these researchers, teachers possessing rigid belief systems are less likely to engage in reflective thinking. These teachers may disregard educational research and reform movements that require adjustments in their teaching styles. Research on teacher beliefs has categorized the majority of preservice teachers as possessing absolutist or dualist beliefs about mathematics when Perry's scheme of intellectual development is used as a means to categorize teachers' mathematical conceptions.

Cooney et al. (1998) discusses the belief systems of teachers. They are convinced that "an indoctrinated view of mathematics minimizes the impact of rationality in favor of memorization. This view is the antithesis of mathematics as a human endeavor"(p. 312). Researchers in teacher preparation advocate the development of programs that encourage relativistic beliefs and reflection. It is "crucial that we develop a way of thinking about how teachers orient themselves to their students, to the mathematics they are teaching, and to the way that they see themselves teaching mathematics"(Cooney, 1994, p. 42).

Different researchers have provided classification systems describing the different philosophies of teaching mathematics and their implications. Ernest (1991) classified teacher beliefs into five categories, which will be explored in the next section. Lerman (1989; 1990) identified two opposing conceptions of mathematics as absolutist and relativist and explored the consequences of these conceptions on teaching. Other studies identify connections between teachers' mathematical conception and their instructional techniques. Kuhs and Ball (1986) classified instructional philosophies into four categories: learner-focused, content-focused with an emphasis on conceptual understanding, content-focused with an emphasis on performance, and classroom-focused. Teachers using learner-focused instructional strategies often possess beliefs about mathematics closer to the constructivist viewpoint. Content-focused instructional beliefs align with the Platonist viewpoint of mathematics. Instruction using the content-focused view with emphasis on performance allies itself with the view of mathematics as an instrument. Finally, a classroom-focused instructional view has less grounding in mathematical beliefs but rather in the belief that classroom structure strongly influences student learning (Thompson, 1992).

These researchers have proposed theories that answer the call of Cooney et al. to develop a means of interpreting the effects of teacher belief systems. One classification system by Ernest (1991) provides a detailed description of five mathematical philosophies and implications for the classroom. Ernest's framework for analysis and reflection is similar in structure to the one proposed by Weinstein. However, instead of looking at the impact of beliefs on the behavior of the teacher as a learner, Ernest explores the impact of beliefs on teaching behavior.

Ernest's Philosophies of Mathematics Education

Ernest (1991) describes five mathematical philosophies that provide adequate depth and breadth in examining educator's philosophies: Industrial Trainer, Technological Pragmatist, Old Humanist, Progressive Educator, and Public Educator. A brief summary of these five classifications is included in Table 2. The five classifications move from the conservative to the constructivist. The Industrial Trainer, a conservative, views mathematics as a set of absolute truths stemming from authority and favors a back to basics movement with emphasis on hard work, drill and practice, and an absence of technology. Ernest relates this philosophy to the dualistic stages of Perry's theory.

Social Group	Description
Industrial Trainer	View of Mathematics: Set of Truths Theory of Society: Rigid Hierarchy Theory of Ability: Fixed and inherited Mathematics Aims: Back to Basics Theory of Learning: Hard work, practice rote Theory of Teaching Mathematics: Authoritarian Theory of Resources: Anti-calculator Theory of Social Diversity: Differentiated schooling by class
Technological pragmatist	View of Mathematics: Unquestioned body of useful knowledge Theory of Society: Meritocratic

	Theory of Ability: Inherited Mathematics Aims: Useful math to appropriate level Theory of Learning: Skills acquisition, practical experience Theory of Teaching: Skilled instructor, motivate through relevance Theory of Resources: Hands-on, computers Theory of Social Diversity: Vary curriculum by future observations.
Old Humanist	View of Mathematics: Body of structured, pure knowledge Theory of Society: Elitist Theory of Ability: Inherited Mathematics Aims: Transmit body of knowledge Theory of Learning: Understanding and application Theory of Teaching: Explain, motivate, pass on structure Theory of Resources: Visual aids to motivate Theory of Social Diversity: Vary curriculum by ability
Progressive educator	View of Mathematics: Process view Theory of Society: Soft hierarchy, welfare state Theory of Ability: Varies but need cherishing Mathematics Aims: Creativity, self-realization through math Theory of Learning: activity, play, exploration Theory of Teaching: facilitate personal exploration Theory of Resources: rich environment to explore Theory of Social Diversity: humanize neutral math for all
Public educator	View of Mathematics: social constructivism Theory of Society: inequitable, hierarchy needing reform Theory of Ability: cultural product Mathematics Aims: critical awareness and democratic citizenship Theory of Learning: Questioning, Decision making, Negotiation Theory of Teaching: Discussion, Conflict Questioning Theory of Resources: Socially relevant, authentic Theory of Social Diversity: Accommodation for social and cultural diversity

Table 2. Summary of Ernest's Philosophies of Mathematics Education

Like the Industrial Trainer, the Technological Pragmatist views mathematics as an unquestioned body of useful knowledge. This educator values the utilitarian nature of mathematics, the acquisition of skills, and the use of technology to solve practical problems. These educators seek to make mathematics a pleasurable pursuit for the greatest number of students. "The main features of this ideology are an unquestioning acceptance of existing structures and models coupled with an action-oriented world-view, treating intellectual and ethical matters in terms of practical outcomes "(p. 153). Ernest likens this ideology to the multiplistic viewpoint in Perry's theory. A Technological Pragmatist maintains an absolutist viewpoint of mathematics but the pragmatic nature of this philosophy values the plurality of views in society. Though consciously considering the needs of society and industry, this view is too limited in its valuing of mathematics. The philosophy disregards the growth and development of mathematics as well as the means for personal understanding. The strong focus on application minimizes any appreciation for the richness of theoretical pursuits.

The Old Humanist educator describes mathematics as a body of structured knowledge and emphasizes the learning of structure and the beauty of mathematics. This philosophical outlook favors the use of tracking in mathematics so as not to hold back the best students. The intent is to provide the highest level of instruction to the best students while providing an acceptable level for others. The Old Humanist, according to Ernest, serves the need of a small minority of students while disregarding the needs of the majority. This viewpoint has grave consequences for education. It allows the needs of the elite minority to determine the education of all members of society. This disregard of the needs and interests of all members of society would cause Old Humanist teachers to look askance at "social justice" academic agendas such as the "Algebra for All" movement; they would resist changes to the curriculum for non-college-bound or vocational students, denigrating them as "watering down the curriculum." A consequence of this somewhat inflexible viewpoint is to present mathematics as a remote ideal, which is unappealing to many students.

Progressive Educators maintain a child-centered philosophy of education. They value the process view of mathematics and the role of the teacher to facilitate self-realization through creative exploration and discovery. Those with this philosophy attempt to minimize failure and maximize the self-worth of an individual. A Progressive Educator would be quite likely to embrace the "Algebra for All" movement, seeing it as a way for each child to realize their full mathematical potential, whether the "Algebra" they take is a highly abstract preparation for a scientific major in college or a pragmatic exploration of patterns and functions for a vocational student. However, three important factors in the teaching of mathematics are not sufficiently emphasized within this philosophy: the teacher's role in the transmission of knowledge, the necessity of teacher intervention in a child's learning for corrections and challenges, and the importance of the teacher as a role model.

Lastly, the Public Educator possesses a social constructivist view of mathematics. The belief in the fallibility of mathematics underscores this philosophy. Since math exists only in the minds of people, it must be recreated in each student's mind. This viewpoint values the use of discussion, cooperative group work, and projects. Socially relevant materials and the politicization of the classroom are fundamental aspects of teaching and learning mathematics - this type of "liberatory education" is described well by Freire (1970) and Frankenstein (1987). This philosophy also leads to the generation of fair assessment of abilities regardless of gender, race, or other social categorizations. However, students' unfamiliarity with this teaching style hampers the implementation of teaching based on this philosophy. Ernest believes that the Public Educator philosophy is ideal for successful mathematics teaching. These teachers likely support many of the recent constructivist reform movements. A Public Educator, like a Progressive Educator, would embrace the "Algebra for All" movement, but would be additionally motivated by broad sociological issues such as the desire to address historical class- and ethnicity-based injustices in the availability of advanced mathematics, moving well beyond the Progressives comparatively simplistic desire to help each student reach his or her potential.

Method

This study aimed to determine the level of mathematical sophistication and the philosophy of mathematics education of novice teachers. Five novice teachers, recruited from the Washington, D.C. area school systems, participated in the study. Length of teaching experience was the only criteria for selection of teachers. All participants had completed less than one year of independent teaching with one participant completing a year of practicum and student teaching. The teachers participated in a 45 minute structured interview as shown in the appendix. The interview included questions on their personal experiences with learning and teaching mathematics as well as questions pertaining to their definition of mathematics and mathematics education.

The structured interview provided an efficient means to gather evidence about math sophistication and philosophy. Although the interviews diverged due to each participant's experiences, the structured questions guaranteed that all teachers would be given the same opportunity to respond to similar questions. Additionally, the use of standard questions provided a link between the theoretical frameworks and the teachers' responses.

The interviews were analyzed to find information on the teachers' level of mathematics sophistication and their philosophy of mathematical education. The transcripts were coded by key words from the two theoretical frameworks. For the analysis on learning mathematics, key words included references to problem solving, processes, procedures, concepts, structure, and proofs. The difference between teachers who valued the utilitarian nature of mathematics and those that valued the theoretical structure helped to distinguish levels of sophistication. Also, comments about the development of mathematics tended to favor either the Platonic viewpoint or the constructivist viewpoint, and thus provided divergent categories among the responses. For determining the mathematical sophistication for verifying mathematics, key words included the terminology describing the process of judging the correctness of an answer. Discussion about the social nature of mathematics was also coded in this analysis. The comments were consolidated and compared to Weinstein's framework and to each other.

The analysis of the teachers' philosophy of mathematics education proved a greater challenge. In this area, comments related to mathematics and classroom practices were noted, especially comments on technology, teaching styles, goals of teaching, student roles in teaching, and motivational strategies. The interviewer did not expect teachers to provide detailed information in all areas. Teachers were encouraged to share their classroom practices, not comment on general educational possibilities. Comments about student opinions and abilities provided insight into interpreting their philosophy of mathematics education.

Results

Although the five participants were all white middle-socioeconomic-status teachers, they possessed diverse educational backgrounds and experiences. The five teachers taught in three jurisdictions and five different schools. A summary of the five participants in included in Table 3. In spite of these differences, many similarities existed among their responses.

Name	Degree	Years of Teaching	Course Load	Age
Irene	BA Math MAT	one	Consumer Math, Geometry, Algebra	Late 20's
Mary	BA Math MAT	one	Algebra I, Geometry, Trigonometry	Early 20's
George	BA Education and Math	zero	Geometry	Early 20's
MaryJo	BA Math	one	Consumer Math, Algebra I, Algebra II	Early 20's
Gabrielle	BA Accounting MS Accounting	one	Basic Math, ESL	Late 20's

Table 3: Summary of characteristics of the five participants.

MaryJo completed her teaching experience at a local, private university. She switched from a business major in her junior year and completed a mathematics degree with teaching certification in five years. She was in her first year of teaching at the same school in which she completed her student teaching. Her course load consisted of five classes with three preparations: Consumer Math, Algebra I, and Algebra II.

Irene received her undergraduate mathematics degree at a private university in New York State. She completed an MAT program at a local public university and was in her first year of teaching in the same school system in which she student taught. She was teaching five classes with three preparations: Consumer Math, Algebra I, and Geometry.

Mary finished both her undergraduate degree and MAT at a public university in Virginia. Her undergraduate degree included a double major in mathematics and religious studies. This was her first year of teaching and she taught five classes with three preparations: Algebra I, Geometry, and Trigonometry.

Gabrielle was the only uncertified teacher in the study. She had been hired in late September of that year to fill an unexpected vacancy when a teacher took maternity leave. Gabrielle's background included both an undergraduate and a master's degree in accounting. She had not started the process to attain certification because she continued to doubt whether or not she would remain in education. In fact, Gabrielle did not expect to return to her teaching position the following year.

George was the only participant in the study who was not a regular classroom teacher. George was attending a private university and had completed a degree in mathematics with a double major in education. George was interviewed at the conclusion of a yearlong process of practicum and student teaching experience. This included teaching five sections of Geometry. George had been offered a job in the same school in which he completed his student teaching.

A few similarities among the participants should be noted. All of the participants came from similar cultural and economic backgrounds, describing themselves as middle class and Caucasian. Three of the participants had grown up in the Washington area and four of the participants had completed at least one degree in the Washington area. Four of the participants had bachelor's degree in mathematics. Only one of the participants had completed an undergraduate degree in education. Of these participants, two acquired their teaching certificate with their undergraduate degree, two completed MAT programs, and one was uncertified.

Level of Learning Mathematics

In the Learning Mathematics component of Weinstein's Ways of Knowing Mathematics, three of the participants can be classified in the fourth stage, Understanding Structure. These teachers indicated that there were multiple procedures in mathematics and that these procedures are all valuable and indicative of the underlying structure of mathematics. These teachers indicated that understanding in mathematics can only exist if there is an understanding of the underlying structure. Mary, Irene, and George had all completed undergraduate degrees in mathematics. They expressed a love of mathematics since elementary school but reservations about their abilities to understand and participate in graduate level mathematics. A summary of these classifications is included in Table 4.

	Ways of Knowing Mathematics		Philosophy of Math Ed
	Learning	Verifying	
Irene	Level Four: Understanding Structure	Level Three: Verifying Answers Together	Level Two: Technological Pragmatist
Mary	Level Four: Understanding Structure	Level Three: Verifying Answers Together	Level Two: Technological Pragmatist
George	Level Four: Understanding the Structure	Level Three: Verifying Answers Together	Level Undetermined: Not Yet Formed
MaryJo	Level Three: Understanding Many Procedures	Level Two: Verifying Answers Alone	Level Two: Technological Pragmatist
Gabrielle	Level Two: Choosing among Procedures	Level One: Receiving Absolute Truth, Alone	Level One: Industrial Trainer

Table 4: Summary of Classification of Participants

The following quotes illustrate their belief that understanding mathematics includes understanding the underlying structure.

> Mary: I think that [understanding mathematics] means to understand the laws upon which math is founded. Like the laws where Euclidean Geometry works. Laws can't do certain things. You have to understand that before you can understand Euclidean Geometry. I think that it is understanding the laws and rules by which the math is governed.
>
> Irene: Mathematics is a process ... to get you to solve a problem. It is a logical background that makes everything work. On the first level I think the first thing that you are able to understand... is how to solve the problem. And then . . . you can get into well why does this work.
>
> George: I am not going to say that mimicking procedure is understanding. Just because they have been given the quadratic formula just because they can find the roots. Like if I say give me the roots and they say -1 and 4 big deal! They have to have understand more. They have to be comfortable that they are doing it right.

The other two participants were at a lower level of sophistication of learning mathematics. Both of these teachers had some training in business and accounting. MaryJo switched her major from business to mathematics education in her junior year and graduated with a minor in business. She expressed frustration over her proof oriented coursework and did not see its usefulness in her classroom teaching. MaryJo limited her definition of understanding mathematics to the correct application of different procedures. This is consistent with the third stage, Understanding Many Procedures, of Weinstein's framework:

> MaryJo: I guess you can understand a concept by doing well and being able to regurgitate the information that you've

been given. But I guess really understanding means taking it and applying it to other situations. Other situations that you see like when you do work in algebra and you might not see how it relates but it does relate to things that you can use later on. You are learning facts that will help you with anything that you do. They [students] can be very creative sometimes at how they find their answers. They usually come up with the agreed upon answer and that is how I can tell if they understand math.

Gabrielle's education included two business degrees -- a bachelors and a masters in accounting. Her comments represent level two of Weinstein's framework: Choosing among Procedures. She focused the application of an appropriate process without any discussion of understanding the mathematics behind the process:

> Gabrielle: I like solving problems. I am a problem solver. It doesn't even have to be a math problem. I'm analytical, like step by step. I try to bring in my problems from my accounting background like interest problems. You know money problems and word problems.

Regardless of level of learning mathematics, all the participants emphasized the logical processes of mathematics. They wanted students to value the procedures in mathematics that can be used to solve problems. All five participants expressed a belief that problem-solving skills within mathematics can lead to better problem solving strategies in other areas.

Level of Verifying Mathematics

The participants answered two types of questions about the verification of mathematics: the process used by the mathematical community to verify ideas or theorems and the process used by their students to seek verification. The responses regarding their views on verification within the mathematical community contributed to the analysis of their levels of verifying mathematics. A summary of this analysis is included in Table 4. The four respondents with degrees in mathematics possessed a much more developed sense of verification than the one participant with the business background. All four of the math majors expressed the need to verify mathematics in a social structure. They felt that math could not be true without the approval of a specified social group. The following quotes include responses to the interview question "How do you know something is right in mathematics?

> Mary: They [theorems] have gone through a variety of mathematicians - not one mathematician. You have to have at least two or three to check. If you are using a proof and you are doing it logically then that proof is govern by certain steps. Those steps have to be valid. If the steps are valid then the proof is valid.

> Irene: It should be verified. Especially if you are talking about a new discovery. There should be other people reviewing it, just as if someone would edit a book. Someone has to go through and make sure that ideas are addressed and an outside reviewer is better than you yourself going back through.

> MaryJo: [Math is verified] through people, a lot of people agreeing on what was true and not true in some way -- that is the way that we judge what is true.

> George: You can't really come up with anything new without... You can't just say it works. You gotta to be able to show [people that] it works and you do that with a proof. This takes us back to what we were talking about in the beginning when we were talking about is there more than one correct answer or not. And we were talking about train of thought. In this case a proof is a train of thought using correct words in the right order.

Gabrielle, on the other hand, identified the teacher and the textbook as the absolute authority. This represents the lowest level of Weinstein's framework, Receiving Absolute Truth, Alone.

> Gabrielle: You mean, is there another way [to verify answers] other than ask? I mean hopefully you can go back to the book see how it is done again and check. Usually there is some way that you can check it.

Mathematics Education Philosophy

In the process of analyzing the teachers' philosophical beliefs, key words relating to their theory of learning, teaching, assessing, and the use of technology were used as indicators of philosophical beliefs. A summary of this analysis is also included in Table 4. Three participants, Mary, Irene, MaryJo, can be classified as technological pragmatists. These people emphasized the practical applications of mathematics. They were interested in the acquisition of problem solving strategies. Their role in the classroom included the need to motivate and involve students in the learning process. Although they each emphasized technology differently in their classrooms, calculators and computers played an inherent role in the classroom environment. Under Virginia's Standards of Learning, these three teachers were responsible for preparing students for the end-of-course examination. This goal remained paramount in their minds, reinforcing the technological pragmatist value on external testing. Motivation comes from the utilitarian nature of mathematics. Its usefulness is emphasized more than its structure:

> Mary: [I want them to understand that math] is not a scary thing. Right now they are going in with a fear of math. It may be a difficult subject but it is worth the effort. The skills and logic that you learn in math will carry over in every other subject.
>
> MaryJo: I try to explain why we have to learn this. Why do we have to do this and try to tell them it is not necessarily for that concept but we need to learn the process to go through to be able to reason logically through things. There are certain things that you need to be able to do logically and try to figure out. I want them to like it and to be able to look at it in a different way. To be able to see to try to determine how thing that they have done relate to things that they see everyday. And that is one of the hardest things to do. Why are you learning that? I am still trying to figure out why we learn all the stuff that we do.
>
> Irene: I want them to at least appreciate that there is some value to it. That it is not a course to make them suffer. Maybe they are not good at it but at least understand that it is an important tool to have in their life. It is worth trying to understand. I try to relate it back, to try to relate problems in the real world. How they might use it in a career or in there everyday life. How they might use it to build a deck in their backyard. The utilitarian nature. . . .

These teachers valued an active classroom and encouraged interaction between the students and teachers.

> Mary: When I am introducing concepts, I try to do a little activity. What we were talking about and go over terms. Try and relate something back to something else that we have done before. For example when we are learning square roots, I talk about the perfect squares and write them. Using the calculator to check. Also, using drawings. In geometry I make the students write out the formal definitions and theorems. We go back and look at what does it mean?
>
> MaryJo: There is a lot of interaction. I don't go through and just do the whole problem for them. I have them go through and interact with each other and try to pick out the steps that we need to go through in order to get the answer.
>
> Irene: Sometimes I do a discovery thing. Where we might investigate, especially [in] Geometry. Take different shapes and try to find the area. Or I start out with the theorem and let's see why it works. Let's see if it does work. If we try an example will it work? Is it something that we can use a formula for? Is it something that we can do by hand?

All three teachers acknowledged that they remained the students' main source of verification. However, they mentioned that social verification is a means for students to ascertain truth.

> Mary: [To check answers they can] bounce ideas off of each other. I encourage them to work in groups.
>
> Irene: [They check answers] either with me or by comparing problems and answers with other students.
>
> MaryJo: That is the one thing I liked about math. There is some way you can judge and see for yourself. For the students it is the teacher but I guess that it is the way things have been done. And you want to form them the way that things have always been done. And that is why it is correct or not correct.

The technological pragmatists described an appreciation for technology. However, the use of technology depended on the teacher's level of comfort with the technology.

> Mary: We use technology all the time. We graph x2, x3, x4, x5.... When I was in school we had to graph it by hand, now I can manipulate it. It is a teaching tool, a learning tool, a presentation tool. You can ask questions like what happened? I use it in my trig class a lot because I do a graphing lab where they have to use a calculator and graph things. They look at it to find the asymptotes, horizontal and vertical asymptotes. It doesn't draw them in. But if there

looks like there is a line there, what is happening? It is connecting the dots, if it is in dot mode then that line is not there anymore. So really teaching them how to use it and using it as a teaching tool.

> MaryJo: I use technology but not as much as some people. I think that it is important for the students to understand the concepts before they can see it and become too dependent on the calculator. But I do try to bring it in at the end after they have learned material. That way they can do a quicker, easier check.

> Irene: Graphing calculators are very big for both doing some discovery things and simplifying tasks. I did some work with computers this year. I used the tesselmania program, which is very good. They are very good for doing translations, reflections, rotations. Plus, the students really enjoyed it. I think that it definitely helps them. In that tessellations unit, I did the work with that program, not the chapter. At the end of the quarter, I gave them a test that included questions from that chapter even though we had never done definitions or theorems or any of that stuff. I was really pleased that they were able to pick up the information without it being spoon-fed for them.

One teacher, Gabrielle, fit Ernest's description of an Industrial Trainer. Her description of mathematics and teaching remained rigid and authoritarian: the teacher is the dispenser of knowledge and students must master skills. She believed in the necessity of mastering the basics before technology and exploration become valued. Included here are her responses to some questions on mathematical educational philosophy and teaching styles.

> Gabrielle: [Does technology play a role in your classroom?] No, I mean it is just the basics. Unless you could get maybe some program to show them how to divide. I don't know. It's pretty basic.

> [How do you usually present new topics?] I like to try to come up with a visual. Like in my math class, we are doing division. So, I have cards and they can count out how many and divide into piles

> [How can students verify their answers?] You mean is there another way than asking me? Oh boy, I mean, hopefully, they can go back to the book see how it is done again and check themselves.

The fifth participant, because of his relative inexperience, proved difficult to classify into any of Ernest's categories. Unlike the others, George identified and valued the constructivist philosophy. He spoke at length about his desire to include discovery learning and projects into his classroom. However, he questioned his ability to implement such ideals. Therefore, it is too early to categorize George as a Public Educator. He needs more experience in the classroom to create a firmer philosophy of mathematics education. The following quotes illustrate the decisions not yet made about his educational philosophy.

> George: I still think that I am fairly traditional in that when the information is first introduced I would like to be the guy at the board and then when they have it then you guys can go back to your calculator and mess around with it for a while. Stuff like that. I would definitely like to use the technology while I am at the board lecturing or at that time. But I am still pretty traditional in that way. I would lecture and then let the kids fool around with it. That might change.
> It would be nice for them to do something to show me that you have understood it. Give them some kind of thing that has to be worked out. How would they display it? I think in that way they could display whether or not they value math.

Conclusions

The difference in mathematical sophistication among the teachers fell into a predicable pattern. The teachers with degrees in mathematics showed a notably higher level of sophistication than the business major. The mathematics majors identified the social nature of verification and talked about its importance in the classroom. Further studies targeting a wider population of mathematics teachers, including education majors and alternatively certified teachers, would complement this study.

Even though four participants had completed teacher certification programs, the teachers did not talk about the larger mathematical education community. Only two teachers spoke of NCTM standards in mathematics. Of these, only one mentioned them without being prompted by the interviewer to discuss reforms. Questions about this larger community were not explicitly stated in the structured interview. More research focusing on the novice teachers' perceptions of the mathematics education profession could provide further insight into their philosophical outlooks.

These two frameworks for understanding mathematical sophistication and philosophies of mathematics education provide valuable information for teacher educators and professionals involved in mentoring novice mathematics teachers. Mentors can use these frameworks to help understand the perspectives of new teachers and to develop techniques that push novice teachers towards more advanced perspectives.

In addition to helping teacher mentors, these frameworks can be introduced to teachers as a way to generate personal reflection on learning and teaching. In fact, the five research participants remarked on how valuable being interviewed was for them. They were delighted and intrigued by the questions that provoked them to think about and explain their views of mathematics, mathematical aims, and views on teaching mathematics. They had reflected very little on these issues, but enjoyed the chance to grapple with them and as a consequence, grow as teachers.

Appendix 1: Interview Protocol

1. Describe your past experiences in mathematics in college, high school, and earlier.

Family math background
Childhood experiences
Contests and standardized tests
Degrees
Memorable teachers

2. What attracted you to become a mathematics teacher?

3. Give a brief definition of mathematics and share your notions about what it is.

How did it come to be?
How does it continually evolve?
How would you share this with your students?

4. What does it mean to understand mathematics?

How do you know something is right in mathematics?
Who has the authority to decide what is right?
Does every mathematics problem have a correct answer?
How is mathematics verified and supported?
What is mathematical knowledge?

5. What to you want your students to understand about mathematics?

How do students display their understanding?
How can students verify their knowledge?
What do students do with their knowledge?

6. Describe the way you present mathematics?

How do you introduce concepts?
What activities to you like to use in a classroom?
What roles to teachers and students play in the classroom?
What role does technology play in the classroom?
Has your presentation of mathematics been influenced by the recent
Reform movements?
Group Work?
Discovery Learning?

7. What experiences have you had with mathematical proofs?

What is the role of proof in mathematics?
How do you do proofs?
Why do you do proofs?

Who do you do them for?

8. Have you taught proofs to students?
How did you introduce them?
What were your students' reactions?

9. Do you plan to study (formally or informally) any additional mathematics?

Content?
Education Related?

10. Do you have anything else to add to our conversation?

11. Would you care to share any demographic information with me?
Age?
Ethnicity?
Grades?
Politics?
Other?

12. May I contact you by phone or email to clarify any questions I might have?

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