THE NEED FOR DIVERSITY TO IMPROVE UNDERGRADUATE EDUCATION IN DEPARTMENTS OF MATHEMATICS: THE PROBLEM OF STATISTICS AND APPLICATIONS

Patricia B. Cerrito

Introduction

Over the years, there has been much debate about the nature of statistics. Its primary impetus in this century was at the London School of Economics under the direction of Karl Pearson (Porter, 1986). The magnificent work of R. A. Fisher was entirely motivated by applications to agriculture (Geisser, 1993). Neyman and Pearson gave statistical inference its theoretical mathematical foundation (Lehmann,1993). From there, statistical research was virtually divided into two parts. Mathematicians insisted on mathematical rigor; applied statisticians demanded practical application. Tukey (1962) clearly outlined the nature of the debate. He declared that statistics is an empirical science and not a deductive system like mathematics. He wrote (p 3):

> To the extent that pieces of mathematical statistics fail to contribute, or are not intended to contribute, even by a long and tortuous chain, to the practice of data analysis, they must be judged as pieces of pure mathematics, and criticized according to its purest standards. Individual parts of mathematical statistics must look for their justification toward either data analysis or pure mathematics. Work which obeys neither master, and there are those who deny the rule of both for their own work, cannot fail to be transient, to be doomed to sink out of sight.

Chandler Davis (1994, p131) examines the conflict between theory and applications:

> What we would expect to find, on this basis, is a division of mathematical sciences into several specialities, some of them motivated by intellectual or aesthetic questions, some of them by the needs of the economy and public policy. We would expect to find the second sort, the more applied scientists, securing funds more readily, communicating more easily with the rulers and policy-makers of society; we would expect to find the first sort, the theoretical scientists, convinced of the greater importance of their problems (which after all were chosen freely and not imposed by any customer or client) and perhaps even of their own individual superiority as scientists.
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> So we are seeing more than a mere rejection of applications. The contemporary "pure" mathematician does not say only, "I am too noble to get my hands dirty on mechanical problems like you mere engineers," but something even more hostile in its defensiveness: something like, "You are destroying my true science if you entangle me with your reality." The pure-versus-applied tension is aggravated among mathematicians (and slanted so that most resolutions are in favor of the pures) by this metaphysical pecularity.

Applications provide outreach to other departments in the university community. Without applications, mathematics becomes isolated, and this can be dangerous as demonstrated by the fate of the mathematics department at the University of Rochester. The department was downsized because of low numbers of students and minimal linkage to other departments (http://www.ams.org/index/ams/rochester/roch-ap1.html, April, 1999):

> Effective teaching of calculus is an essential ingredient of a quality undergraduate educational experience at Rochester, particularly given the large proportion (over 70%) of first year students who enroll in the calculus sequences. Although arguments could be made that graduate students in Math play a key role in calculus

instruction, much like the role that graduate students in English play in basic-level writing courses, the dwindling numbers of Math graduate students undercut one rationale for retaining a Ph.D. program in Math. There are other ways to service our need for calculus instruction, including the hiring of non-research (adjunct) faculty and/or the redirection of other qualified faculty from other disciplines.

For these reasons, we do not believe that continuation of the Ph.D. program in Math is justified. Linkages with other departments and programs are minimal, as is grant income (generally true of Math departments). We believe that a refocused department that emphasizes quality calculus instruction (to a smaller undergraduate student body), attention to majors and minors, and individual research excellence, will best serve the overall needs of the College. A reduction in steady-state faculty size over time from 21 to 10 FTEs, with additional non-tenure-track teaching faculty who staff much of the elementary calculus sequences, can achieve these goals.

Teaching Data Analysis

This conflict between mathematics and statistics is reflected in teaching as well. According to David Moore, mathematicians should not teach statistics because of the tendency to focus on the mathematics instead of the data analysis. Tukey (1962) is in agreement (p 11):

> Teaching data analysis is not easy, and the time allowed is always far from sufficient. But these difficulties have been enhanced by certain views which have been widely adopted, such as those caricatured in :
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> (a1) "avoidance of cookbookery and growth of understanding come only by mathematical treatment, with emphasis upon proofs".
>
> (a2) "It is really quite intolerable for the teacher then to have to reply, 'I don't know'."
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> (a3) "Whatever the facts may be, we must keep things simple so that we can teach students more easily".
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> (a4) "even if we do not know how to treat this problem so as to be either good data analysis or good mathematics, we should treat it somehow, because we must teach the students something".

Cobb (1991) provides solutions to improve a typical statistics course in a mathematics department:

> Almost any course in statistics can be improved by more emphasis on data and concepts, at the expense of less theory and fewer recipes. To the maximum extent feasible, calculations and graphics should be automated.
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> Any introductory course should take as its main goal helping students to learn the basics of statistical thinking. [These include] the need for data, the importance of data production, the omnipresence of variability, the quantification and explanation of variability.

Yet virtually all statistical textbooks of recent years have fairly routine problems introducing statistical software as an add-on to the primary focus of routine computation. Most statistics courses typically use examples which can be computed using a hand calculator. Rarely is the course centered around the use of statistical packages. Textbooks still reflect this type of course. While most texts do have computer applications, it is invariably as an add-on instead of an integral part of the course. For example, here is a typical problem found in Mendenhall (1993):

> The data in table [1] were obtained from an experiment that was conducted to compare the mean power level readings (in watts) on a type of military electronic tube by two identical pieces of test equipment. The power output for each of 10 military electronic tubes, randomly selected from production, was measured by both pieces of test equipment. The objective of the experiment was to detect a difference in mean power level readings for the two pieces of test equipment, if, in fact, a difference exists.

Table 1.

Tube Number	Tester 1	Tester 2
1	2563	2556
2	2665	2479
3	2460	2426
4	2650	2619
5	2610	2617
6	2657	2491
7	2529	2590
8	2427	2466
9	2448	2516
10	2480	2428

A problem like this is totally divorced from the reality of most students. That textbooks remain fairly traditional is attested to by the number of editions of each book currently in publication. It is far better to bring in real datasets and use a statistical software package to analyze the outcomes. Students need to see real data with all its problems. I tend to use data that I have encountered in the course of my own statistical research. Therefore, this data tends to be in the arena of medical decision-making In addition, students are required to devise their own statistical experiment, including data collection. Students must define their experiment within the first two weeks of the semester. Then the course becomes data-driven, so that the necessary statistical techniques are provided to the students to complete their experiments. I require students to find and evaluate statistical information in the statistical literature (Cerrito, 1999). There are many rich data sources available. In particularly, the Journal of Statistical Education maintains an archive of such sets (http://www.amstat.org/publications/jse/archive.html, April, 1999). It was necessary to develop course packets to teach courses with actual research experiences at an elementary level (www.math.louisville.edu/~pbcerr01). This course is currently offered in the Department of Mathematics. This is the first course where students are exposed to the actual practice of statistical experimentation. It works best when the course concentrates on a handful of databases, formulating appropriate questions as each new statistical method is presented.

This type of course is very much in line with the recommendations of the Boyer Commission on Undergraduate Education (1998):

> The research universities need to be able to give to their students a dimension of experience and capability they cannot get in any other setting, a research experience that is genuine and meaningful. They should turn out graduates who are well on the way to being mature scholars, articulate and adept in the techniques and methods of their chosen fields, ready for the challenges of professional life or advanced graduate study. Research universities have unique capabilities and resources; it is incumbent upon them to equip their graduates to undertake uniquely productive roles.

Statistics in Mathematics Departments

Undergraduate Curriculum

If data analysis is separate from mathematics, should statistics be taught in mathematics departments? Should every university have a separate statistics department? It is true that without a statistics department as a focus, statistics tends to be taught in every department which has a need of data analysis. Unless the department itself is prepared to be flexible in its definition of scholarship, the answer should be no. According to Moore (1998, p97):

> Data analysis, statistical graphics, data production, and even the somewhat arcane reasoning behind "statistical significance" are mismatched with the mathematical content needed by potential math majors. Yet "statistics" that ignores these topics isn't a responsible introduction to statistics. Statistics in a mathematics core curriculum is an oxymoron.

However, Watkins (1998, p99) disagrees, stating:

> In the best of all possible worlds, statistics would not be considered mathematics. However, in this world we must blur the distinction for the benefit of our students. Statistics must be part of the mathematics core because the students who take the core need statistics and there is no other place to get it.
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> The statistics now being taught is taught in mathematics departments because the vast majority of colleges and universities have no statistics department to teach it.

This statement reflects the reality that although statistics should not be taught by mathematicians, it frequently is. Many departments of mathematics do hire statisticians in their faculty. However, the expectation is that they will be mathematicians, to the detriment of any statistician.

Technology Usage in Mathematics Departments

Once technology is introduced into the classroom, data analysis can be taught using the full range of computer hardware and statistical software. There is remarkable agreement among statistical educators that statistics education should automate routine computations (http://www.amstat.org/publications/jse/index.html, April, 1999). However, there is little agreement among mathematics faculty as to how much technology should be used, and how it should change the curriculum. Many mathematics faculty actively resist the use of technology. A report by the National Research Council (1991) states:

> The way mathematics is taught at most colleges-by lectures-has changed little over the past 300 years, despite mounting evidence that the lecture-recitation method works well only for a relatively small proportion of students. Moreover, the syllabi of many undergraduate mathematics courses and the template-style textbooks are detached from the life experiences of students and are seen by many students are irrelevant.
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> Nothing in recent times has had as great an impact on mathematics as computers, yet in most college courses mathematics is still taught just as it was 30 years ago -as a cerebral, paper-and-pencil discipline for which computers either are irrelevant or can be ignored. Computers serve mathematics these days as indispensable aids in research and application. Yet only in isolated experimental courses has the impact of computing on the practice of mathematics penetrated the undergraduate curriculum.

There is strong resistance to change in either curriculum or in the adoption of technology. In my own department, one faculty member wrote a letter to the entire faculty. He protested the adoption of computer labs in calculus instruction (which our department has been using for only 2 years):

> A similar analysis applies to the [calculus] labs. Because the labs are neither tightly controlled nor synchronized, the value they add to [calculus] is not plain. The cost to the students IS plain -- one hour per week. Thus the cost outweighs the value... which is equivalent to a price increase [since] it is now a requirement for the major. The Maple Labs in [calculus] are a detriment to the department because their value has not exceeded their cost (as perceived by the students).

Although I would agree that poor instruction is a detriment to students, the answer is to improve the instruction-not to eliminate it. This same attitude is present when mathematics faculty investigate statistics instruction:

> Many students in [statistics] have little background in mathematics. Therefore, a good textbook and lectures consistent with this textbook are essential for helping students learn complicated statistical ideas.

This attitude is in direct contrast to that of applied statisticians (Smith, 1998, www.amstat.org/publications/jse/v6n3/smith.htm):

> A radical reform of introductory statistics classes has been advocated by many, often motivated by observations similar to Hogg (1991): "students frequently view statistics as the worst course taken in college." Some statisticians believe that the goals of an introductory statistics course should be redirected from mathematical technique to data analysis. Others advocate changes in pedagogy, replacing passively received lectures with hands-on activities.
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> To help students develop statistical reasoning, a traditional introductory statistics course was modified to incorporate a semester-long sequence of projects, with written and oral reports of the results. Students test scores improved dramatically, and students were overwhelmingly positive in their assessment of this new approach.

If mathematics departments hire statisticians, they must give them free reign to teach statistics in their own way, using applications instead of theorems and proofs. Diversity in instructional techniques must be actively embraced instead of resisted. At the same time, there must be consistency across sections of the same course. Therefore, mathematicians who teach the statistics courses must continue to include substantial data analysis in the curriculum, allowing the statisticians to lead in the curriculum development.

Statistics Faculty in Mathematics Departments

Mathematics Department must allow for diversity in its faculty. Davis (1994) outlined well the conflict between pure and applied mathematicians. Statisticians are at the absolute far end of the applied spectrum; only mathematical statisticians are somewhat in the mainstream of mathematical research. At first glance, it would appear that the University of Louisville has accepted a diverse model of research. Boyer (1990) proposed that the concept of research be expanded to the concept of scholarship, presenting four different categories. Applied statistics (or as Tukey insists, data analysis) would fit into the category Boyer identifies as the Scholarship of Integration: building bridges between disciplines, or interdisciplinary research. Data analysis requires data and the data are usually collected in a separate discipline. Four years ago, the administration at the University of Louisville adopted the Boyer model for all faculty in all schools and departments. All types of scholarship would be equally valued.

Unfortunately, the theory of scholarship was not practiced. I can speak from personal experience. Ten years ago I was hired into the Department of Mathematics at the University of Louisville. My degree is in mathematics; my doctoral thesis on the very theoretical topic of probability measures defined on topological semigroups. Since graduation, my research area has become increasingly applied in the field of biostatistics. At this point in time, 75% of my assigned workload consists of collaboration with medical researchers.

However, this activity has not been accepted in the mathematics department. Four years ago, the Department selected a new Chair. He refuses to acknowledge that any applied statistics is legitimate research in a mathematics department:

> As a member of the Department of Mathematics, it would be assumed that your statistical contributions would not just be basic mathematics applied or integrated to some other discipline, but in fact should be highly sophisticated statistical models which would be of publishable quality in its own right, appearing in refereed journals-particularly those journals which subscribe to high standards of the mathematical sciences and its applications.

It was made clear that collaborative work published in medical journals do not apply toward promotion in a mathematics department. Fortunately, he was overturned on grievance appeal. However, I have had to expend much energy within the appeal structure of the university just to get my data analysis accepted as legitimate, quality research.

Thus the situation in my own department greatly parallels the situation at the University of Rochester (http://www.ams.org/index/ams/rochester/notices-axj.html, April, 1999):

> The administration has said that one of its reasons for choosing to cut the Mathematics Department was that it had very few interactions with other departments. The question of how much interaction there has been is a matter of some dispute. The Mathematics Department has collected a number of examples of joint research between its faculty and faculty in other departments. The topics range from ultrasonic medical imaging to cryptography. In addition, students and faculty from other departments regularly attend graduate classes in the Mathematics Department.
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> By contrast, the picture painted by the administration is one of an isolated Mathematics Department. As part of the formulation of the Renaissance Plan, Aslin and Phelps conducted interviews with seventy -five faculty--three members from each of the twenty-seven departments on campus (in a couple of departments, fewer than three faculty were interviewed--one of these was the Mathematics Department). Aslin says they asked science and engineering faculty specifically about linkages to the Mathematics Department. ``To be quite frank, we found very few," says Aslin. The issue seems to be that the interactions were ad hoc--a single research project or a specific course rather than an institutionalized program with high visibility. The interactions ``were faculty A with faculty B because they had taken the initiative to form some sort of intellectual link," he explains. ``But they were not nearly as robust as the kinds of interactions we saw between other departments outside of mathematics." And, Aslin claims, this was not because other departments have no interest in mathematics. ``It turns out that they have sought those linkages external to the University of Rochester."

Only when the department faculty agreed to substantial change and more outreach to other departments, did it receive support from the administration (http://www.ams.org/index/ams/rochester/home.html, April, 1999).

Most mathematics departments exist in this same isolation; most statistics departments do not. Therefore, if statistics is to remain in mathematics departments, and receive reasonable consideration and high standards for students, mathematics departments must become more accepting of data analysis.

Discussion

Statistics should be taught to mathematics majors, either as a required course in a statistics department, or in the mathematics department itself. However, it should be taught so that the students learn statistical thinking. This can only be done in a "hands-on" curriculum. Students learn statistics by doing statistics.

In order for this to happen, mathematics departments must allow for diversity in their departments. There must be a diversity of ideas, of practice, and of faculty. The diversity must fully encompass academic discipline, and departments must become more accepting of applications. The administrative structure must allow for different types of mathematical research by its faculty, including applied collaboration with other departments. The faculty must be permitted to be diverse in their classroom instruction as their own specialities have developed the necessary pedagogy, and that diversity must be accepted by faculty in the department outside of the speciality which developed the curriculum.

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Patricia B. Cerrito

Department of Mathematics

University of Louisville

Louisville, Kentucky 40292

502-852-6826

502-852-7132

pcerrito@louisville.edu