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CESAME: THE PERSONAL HISTORY OF LEARNING MATHEMATICS IN THE CLASSROOM. AN ANALYSIS OF SOME STUDENTS' NARRATIVES

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Our paper comes from the work we did in the Association pour le développement du Génie Cognitif, Nice (GECO) and more precisely in a project named CESAME supported by the Institut Universitaire pour la Formation des Maîtres (IUFM) of Nice. CESAME means: « Construction Expérientielle du Savoir, et Autrui dans les Mathématiques Enseignées » and our aim is to interpret what students say or write about their experience of mathematics and more precisely about their experience of the Necessity in mathematics (see the paper by Drouhard presented in the group of CERME). In this paper we shall present the part of our work which concerns the way to « reach » this personal experience, so we shall examine how one can make the students produce narratives about their experience.

We shall develop some theoretical questions, examine what could be efficient methodologies and present and analyse some narratives we have collected.

We make use of some concepts which have been developed by the French didacticians, mainly Brousseau and Chevallard. References of their works are given in the bibliography; moreover we give short illustrations at the end of this paper.

1. First step

At the beginning of the year, we very often ask our students (high school, university or pre-service teachers) the following question: « tell briefly a personal memory, something which happened to you last year or before, on a mathematical subject. This anecdote should have a mathematical content, and concern yourself (do not recall the day when you fell off your chair, and do not cite merely the title of a chapter) ». This question may be formulated in different ways, for example « tell two memories concerning mathematics, one which impressed you positively, the other one negatively ».

The purpose of such a question is to try to catch what students might say about their experience in mathematics and about their relation to mathematics. This is interesting for us, both as researchers and as teachers.

What is the importance for us of such a demand? to cope with this question it appears to be useful to introduce the notion of « didactical time ». The « didactical time » means the time which regulates the way knowledge processes and the way in which knowledge is divided for teaching (Chevallard & Mercier, 1987). The didactical time gives the norms for the process of knowledge, and it organises the chronology of the students' learning. Its management is under the responsibility of the teacher. Generally the students cannot convert the didactical time into a personal history of their own knowledge. That means that in the math classroom there is almost no time for the students to express their own personal relation to time; in a more precise way, we can say that there is no time for them as individuals to say : «I did not know; then I did this..., and now I know ».

Asking a student to recall memories gives her/him the opportunity:

1. to identify an event which is important for her/him in her/his personal history of mathematical knowledge or from the point of view of her/his relations to mathematics.
2. to put words on it and thus to make it exist for her/him, interacting with others, since a narrative is not completely private.

For us researchers, the interest is double in the way that we believe that:

1. the history of knowledge is part of the knowledge
2. mathematical knowledge is not limited to an accumulation of definitions and theorems, but includes knowledge about the nature of mathematics (Assude and al, 1997). When recalling a math event the student identifies what is « mathematical » for him and this is a knowledge about mathematics which, for us, is part of an individual's mathematical knowledge.

2. Time and Narratives

We shall examine here some theoretical questions about narratives, based mainly on the works of P. Ricoeur (Ricoeur P. 1983). As we said before and as we shall see more precisely later, the time in the classroom is « blank » for the student. When they are asked to recall a personal memory concerning mathematics, they do not imagine that they may evoke the way they lived during the time of the class of mathematics, their own, personal history. There seems to be no capitalisation of this temporality as they are not able, or they don't allow themselves to tell the story of their personal knowledge.

The problem our students are facing when confronted with our demand is:

1. what is the meaning of the question : « what happened to me? »?
2. how could I say it?

These two questions are central in our preoccupation because we think that students certainly build stories about the time they are living and about the events in this time. What could be the content of such stories? It should not be a mere accumulation of anecdotes; one doesn't need to re-live or re-evoke the entire situation with all its details to understand the meaning of a mathematical object. Things are certainly more complex: when the story is stored in the memory, the different events are related to one another, some disappear being included into others, the chronology also may be changed. In fact one builds a new story which can be stored but also which can be recalled upon need. Roger Schank (1995) says (translated into English by C. Sackur (sorry for the poor English!)

> *« The creation of a narrative is a process which implies the memory. Why do human beings tell stories? For a very simple reason; because the process itself of creation of the story creates the mnemonic structure in which the gist of the story will remain for the rest of our life. To talk is to remember. Certainly, psychologists have known for a long time that repetition helps memory. But the fact of « telling a story » is not repetition; it is creation, and the fact of creation is a full mnemonic experience.*
>
> *Moreover stories which are most able to be told, and often the most interesting ones to listen to or to read are those related to frustrating experiences. To put our memories into shape, and to learn, we tell stories in which difficulties have been solved in a way or another. Thus we memorise the solutions, and build a new knowledge, which is stored to be used in the future. »*

Our hypothesis is that learning includes developing the ability to recognise in the *present* an «echo » of a *past* story which lies in our memory. This permits us in the *present* to anticipate, to know whay our expectations are, to understand and thus act for the *future*.

In other words, the theoretical bases of narratives lie in this triple representation of *past, present* and *future*. They answer the question of the signification of temporal experience. The *present* is both the time of *memory* and the time of *anticipation*, that is of attention for a project concerning the future.

We shall examine now if we can find in the memories of the students what we are looking for: the expression of temporal construction and use of knowledge and among others the experience of necessity.

3. Typology of memories

We asked a wide variety of students about their memories: high school students, university students, pre-service teachers (primary school) and mathematics pre-service teachers. Most of the narratives we talk about in this paper are collected at the very beginning of the year when the teacher first meets with the students. As one can easily understand by reading the present paper, we, in the CESAME group do think that memory of learning is an important issue in the process of learning mathematics; we then present the work in the math class emphasising this point. This is the reason why we first ask for the narratives and then start talking with the students about what we think is mathematics learning. Later, during the school year we ask for other narratives and one of the subjects of our work is precisely the role these could play in the teaching and learning of mathematics.

We have just began the analysis of some of these memories and we shall present here the very first results so as to illustrate both our aims as researchers and as practitioners. As a consequence of our theoretical background we have constructed a grid for this analysis which permits us to give a typology of the memories and is a tool for interpretation.

Grid for the analysis of the narratives

I.ACTORS

	who is in the story?	who is acting?	who is the interlocutor?	who is thinking?
the student				
another student				
the group				
the teacher				
« someone »				
another				

II. MATHEMATICAL CONTENT

- label

- precise content

- precise statement

III. NON MATHEMATICAL CONTENT

- evaluation

- test

- results of tests

- oral questioning

IV. WHAT ARE THEY EXPERIENCING?

- understanding

- not understanding

- break of « didactical contract »

- epistemological break

- positive experience

- negative experience

V. STYLE OF WORK

- classical

- discussion

- group

VI. CHRONOLOGY

VII. LEVEL OF KNOWLEDGE

The analysis of the first narratives we collected (about one hundred) led us to three observations; the first is that most memories we studied involve institutional matters. We call them « non mathematical contents », such as tests...(see above). Most of these memories relate negative experiences, unfairness or humiliation. A second observation is that no « other » is mentioned except the teacher who is perceived either like an outstanding person or like an humiliating person and very seldom like someone who helps in learning mathematics. The third one is that mathematical contents are rare: there are very few mathematical events.

Here are two examples of narratives produced by 12th graders:

ERIC: *during the lesson on primitives which lasted one week, I used to mix up primitives and derivatives in almost*

all excises. For the test I managed not to make the confusion and I got a mark of 12/20.

I.ACTORS

who is thinking?				
the student	*	*		
another student				
the group				
the teacher				
« someone »				
another				

II. MATHEMATICAL CONTENT

*- label **

III. NON MATHEMATICAL CONTENT

*- test **

*- results of tests **

IV. WHAT ARE THEY EXPERIENCING?

*- positive experience **

*- negative experience **

V. STYLE OF WORK

*- classical **

VI. CHRONOLOGY *not mathematical*

VII. LEVEL OF KNOWLEDGE *nothing*

CAROLINE: *when I was in grade 9, my ambition was to become a math teacher. My teacher proposed that I would teach the lesson on linear systems (2 equations, 2 unknowns). I never had had such a terrible experience: nobody was listening to me. Hence I don't want to become a math teacher anymore.*

I.ACTORS

	who is in the story?	who is acting?	who is the interlocutor?	who is thinking?
the student	*	*		
another student				
the group				

the teacher	*	*	*	
« someone »				
another				

II. MATH

*- label **

III. NON MATHEMATICAL CONTENT

*- oral questioning **

IV. WHAT ARE THEY EXPERIENCING?

*- negative experience **

V. STYLE OF WORK *not classical*

VI. CHRONOLOGY *not mathematical*

VII. LEVEL OF KNOWLEDGE *no*

As a conclusion, we may say that students produce memories without mathematical knowledge (although the demand was explicit on that point), involving no other people but the teacher, without any chronology. There is no « before », no « now », no « after ». Now the question is: either the students never experienced any mathematical event, or they don't remember them (that means that they didn't identify them as mathematical), or they don't feel allowed to talk about them.

These first results show us the importance of institutional liabilities which weight on the students when they are facing an unusual demand. This questions us from different points:

1. is there room in the math class for students to live a personal story about mathematics?
2. what do students allow themselves to recall about their personal relationship with mathematics?
3. what is the role of the history of the construction of knowledge in the knowledge itself?
4. how could we lead students to talk about the experiences they were confronted to in the math class?

From the point of view of the practitioner, these observations give us a clue to make things change. We have to legitimate, from the point of view of the institution, the facts of talking about mathematics, of living mathematical events in the math class... A use of narratives, which would become frequent, could help us in this direction. They could become a didactical tool which would help the student to contact her/his own personal experience of mathematics and identify the nature of her/his relationship with mathematical knowledge. The remaining problem is how to make students produce the narratives and how to use them in the classroom.

4. Some examples of narratives

As we said before, students limit themselves to the institutional aspects, where time is not something to share and is not related to their own work. The school as an institution gives no importance to the way time flows for the students and to the way they live in the classroom.
As an extreme example of this non-existence of a lived and shared time we can cite this narrative of a mathematics pre-service

teacher:

> *« positively: addition*
>
> *negatively: subtraction*
>
> *more seriously, thing that were really important have nothing to do with mathematics. I'm sorry. »*

We can observe here that someone who s almost a math teacher has nothing to tell about her/his experience of mathematics. S/he will not be able to use it while teaching, to have it as a resource when s/he encounters difficulties in her/his teaching. This narrative shows that the memories can be a sign of the nature of the relationship with mathematics. This is important for a practitioner.

We will oppose to this narrative that of another pre-service teacher:

> *« During my first year at the university I had once the satisfaction of solving a problem with a solution which I entirely constructed myself. Then the teacher worked for a whole hour to try to find where was the bug, but never managed to find any. »*

Here we think that we find the existence of a mathematical memory: the student talks about an experience that s/he lived, which had to do with the necessity in mathematics, which involved others, the teachers and the group of students as interlocutors.

Rather than very dull stories about institutional liabilities which are what we most often gather, we shall cite two other narratives which have been collected in the middle of the year. In this class the teacher used to allow the students to express their personal temporality by proposing styles of work which are not classical which we shall not describe here and by discussing the role of memory. So after some month he asked the question: « two or three things which happened since September. I remember that... »

> *« When I was in grade 10, I used to think that if a unction was growing its limit had to be +□ . This year I learned, while studying the chapter on functions, that a function could be growing and have an upper bound at the same time. »*

Here one can find clearly the expression of a lived temporality. This 11th grader expresses the memory of a mathematical event.

Here is another narrative:

> *« I remember failing on the inequality $x < x^2$. I thought it was a true inequality, and now I know that, of course, it is false. ».*

This student, some months after the event took place, a sort of evidence, which is certainly not the result of a convention. This narrative leaves room to time, and we can find something which appears to us like necessity.

Conclusion

A majority of narratives do not include anything related to mathematical content, to other people than the teacher, and to mathematical events. Nevertheless we have shown that some narratives do give room to the expression of time and of necessity. We think that those illustrate a better relationship to mathematical knowledge than the others.

Our further work will tend to go further in the interpretation of these narratives and in the study of some questions such as: what is the role the narratives could play in the classroom? could they lead to a better construction of mathematical

knowledge? how could we make students produce them: how make students build stories for themselves about their life in the math classroom? how to ask to get these stories?

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Notes on didactical situation and didactical contract:

Didactical and a-didactical situations are situations of teaching in the class room; in a didactical situation the intention of teaching some knowledge is explicit: the teacher says: « to day we shall study the concept of derivatives ». In an a-didactical situation the intention is not explicated; the teacher gives some problems to the students; to solve the problems the students will need to learn something which will be later identified as the concept of derivatives. In an a-didactical situation the need for the concept is better perceived by the students.

Didactical contract means the set of more or less implicit rules which regulate the relations between teacher and students in the classroom. Some are institutional and very general rules (the students must sit during the math class), others are very specific

ones linked to mathematics. Breaking the didactical contract produces facts which permit to identify some of the rules: the teacher is not supposed to give a test on a lesson which was studied the year before; a problem should not lead to the answer: « one cannot know ».

Such problems have been studied under the concept of : « the Age of the Captain » at elementary school level. An example is: « in a classroom there are 12 girls and 10 boys, what is the age of the teacher? » Almost no child says: « I cannot know ». More than that if the answer is obviously ridiculous the children will say: « it's your fault, you didn't give me the right numbers! ». The effect of the didactical contract is that any question should have an answer using the data of the exercice and the computing capacities the child recognises her/himself.