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PEIRCE'S SIGN AND MATHEMATICS EDUCATION: AN INTRODUCTION

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My purposes in this brief presentation are to outline some of the principal features of the sign as conceived by Charles Sanders Peirce and to sketch the relationship between Peirce's sign and mathematics education. To this end, I will:

a. summarize Peirce's notion of sign and the associated semiotic process,
b. draw some parallels between the ideas of Peirce and Vygotsky (1978),
c. explain how Peircean semiotics supports a full conception of constructivism, and
d. link Peircean semiotics to classroom instruction.

I hope to show that Peirce's formulations are very suited to the conception of, and instruction in, mathematics education.

Although mathematics as a discipline can be described as a socially-constructed entity that has become an important part of cultural knowledge, my focus at this time is on the individual learner and how s/he makes sense of mathematics in the context of her/his own life and cultural setting. Although the distinction between collective and individual knowledge should not be a simple one to make, I shall try to maintain that separation here.

The Sign According to Peirce

About 1897 Charles Peirce (1839-1914), who is often considered to be the primary creator of modern semiotics, described semeiotic (Peirce used the English word form that was closest to the original Greek) as "the...formal doctrine of signs" (Hartshorne & Weiss, 1932, p. 134). Today, semiotics is defined as the doctrine, or study, or science of signs. Less formally, semiotics is explained as the study of all systems of signs (including such disparate subjects as mathematics, architecture, language, music, and dance) and symbols (which are special cases of signs in Peirce's system), as well as the study of how signs are used in making meanings and messages.

Peirce's best known definition of sign reads as follows (Hartshorne & Weiss, 1932, p. 135):

A sign, or representamen, is something which stands to somebody for something in some respect or capacity. It addresses somebody, that is, it creates in the mind of that person an equivalent sign, or perhaps a more developed sign. That sign which it creates I call the interpretant of the first sign. The sign stands for something, its object. It stands for that object, not in all respects, but in reference to a sort of idea, which I have sometimes called the ground of the representamen.

Throughout his writings, Peirce expanded on the assorted elements and trichotomies that compose the irreducible sign triad of object, sign (or representamen), and interpretant. Although the debate continues 100 years later about how to interpret this definition of the sign and its dynamic processes, my condensed interpretation of a single sign function reads as follows. From an Immediate Object, that part of the Dynamical (i.e., "real") Object which determines the sign based on the Immediate Interpretant (an existing representation derived from past experience), a Sign is created which determines the Dynamic Interpretant (the actual semiotic effect, or reaction, produced by the sign). This process is a never-ending one of semiosis that progresses toward the Final Interpretant, which is the result of the ultimate sign producing its full effect on the mind. Any single experience can be seen as enhancing the quality or depth of the resulting interpretant which in turn creates a more developed sign. Let us turn to a concrete example: in the mathematics classroom, a teacher may reach for a metre stick to underline a number line concept. Depending on students' prior experiences with metre sticks, they may perceive a weapon, a flat cane, a measuring unit, or a form of numerical representation. However, for all of these students, increased experience with the metre stick in a mathematical context will alter in an ongoing way

their perceptions of what that object represents to them. Hopefully, the metre stick will gradually become a sign of mathematical activity imbued with mathematical meaning that continues to develop through life.

It is important to note that Peirce emphasized the irreducible nature of the functioning sign even though he developed trichotomies of relationships within the sign elements themselves. Thus, the relationships: (a) of the sign with itself (or, some would say, with its ground) are the qualisign, sinsign, and legisign to represent firstness, (b) of the sign with its object are the icon, index, and symbol to represent secondness, and (c) of the sign with its interpretant are the rheme, dicisign, and argument to represent thirdness. In recent years, many scholars have tried to isolate the trichotomies, especially the icon, index, and symbol, even though this action destroys the integrity of the sign. Dissociating the trichotomies in this way is akin to studying the effects of water by examining the properties of hydrogen or oxygen.

I think that Peirce's concept of the sign and its continuous development within the context of different cultural systems such as language and mathematics makes eminent sense. This view accepts that individuals see and understand mathematics very differently from one another depending on prior experiences.

Peirce and Vygotsky

The making of meanings by individuals has been a focus of study throughout the history of western thought, despite the substantial influence of logical positivism and behaviourism during the past century. One of the most currently influential lines of scholarship derives from the Russian sociohistorical school of thought that is represented most substantially by Lev Vygotsky (1896-1934). Vygotsky is associated with at least three main premises (cf., Cole, 1990; Smith, 1995; Vygotsky, 1978; Wertsch, 1985a, 1985b). The first of these is that humans are linked with the physical world and each other by processes of cultural mediation. This mediation is achieved by using both the psychological tools or signs (e.g., mathematics and languages) and the technical tools (e.g., hoes and hammers) of the surrounding culture. A second major premise is that cultural mediation and, therefore, human psychological functions are historical phenomena which undergo continuous and lifelong change. Vygotsky's third premise is that psychological functions arise from practical activity in specific contexts. Lave and Wenger (1991) have advanced this perspective to embrace what they term "legitimate peripheral participation".

From a Peircean perspective, Vygotsky's psychological and technical tools are signs or systems of signs. In addition, Vygotsky's premise that cultural mediation and human psychological functioning are ever-changing historical phenomena is entirely consistent with Peirce's view that evolving signs, or meanings, undergo continuous change and development, or semiosis, in the context of one's sociocultural environments. The third Vygotskian premise, that human psychological functions arise from practical activity in specific contexts, is entirely consistent with tenets of pragmatism, the philosophical base of Peircean semiotics.

Of course, Vygotsky and Peirce differ in a number of specific details and emphases. Nevertheless, some of Vygotsky's major contributions, such as the zone of proximal development, lend themselves to analysis from within the Peircean framework. Indeed, establishing such links is an important area for further research (e.g., Oppizzi, 1997).

Peirce and Constructivism

At the present time, constructivism is a "buzz word" with substantial currency in academic and teaching circles. Numerous recent books and articles on mathematics and science teaching, in particular, have addressed assorted elements of constructivism (e.g., Davis, Maher, & Noddings 1990; Hills 1992). But what is "constructivism"? How rational is the construction of knowledge? What is being "constructed", and to what ends, in the constructivist process (Smith, 1997)? How is constructivism related to Peirce's semiotic process?

Recently, Epstein (1994) provided one means of understanding the full nature of constructivism. Epstein proposed that there are two systems for understanding the world: the "experiential" and the "rational". These systems are parallel and interacting and each is comprised of constructs that bear on both the self and the world. The experiential system, with a long evolutionary history, is described as holistic, affective, composed of associationistic connections, able to respond immediately, capable of being experienced passively and preconsciously, and subject to context-specific processing. Learning in this system is characterized by affect, pictures, parables, and stories. Religion communicates well with this system, as do superstitions, fears, and advertising. In the experiential system, schemata are constructed which consist mainly of generalizations from emotionally significant past experiences and which are organized into an overall adaptive system. The generalizations are formed by and occur through prototypes, metaphors, scripts, and narratives.

On the other hand, the rational system, with a short evolutionary history, is described as analytic, logical, composed of logical connections, oriented toward delayed action, capable of being experienced actively and consciously, and characterized by cross-context processing. Learning in this system is deliberate, effortful, and rational, with a heavy dependence on language. This system is constantly influenced out of awareness by the experiential system. In the rational system, beliefs are constructed from cultural sign systems such as language and mathematics. Thus, when the individual is taken as the unit of analysis in a constructivist process, Epstein's model holds that constructions occur on two levels and that these constructions are very different in nature, influence, and scope. Most academic endeavours emphasize constructivist products of the rational system and deny or downplay products of the experiential system.

How does Peircean semiotics link with this more complete view of constructivism? Peirce's sign embraces both systems, the experiential and the rational, in its definition and elaboration as suggested above. Recall that elements which constitute the sign involve firstness (involving feelings or sensations), secondness (the hard facts of experience or encounters with brute reality), and thirdness (the rational thoughts that are neither the qualities of firstness nor the facts of secondness, but which incorporate both prior elements of phenomena). Accordingly, we can understand both constructivism in its full sense and Peircean semiotics as emphasizing learning process over learning product by promoting sign development that is both contextually-sensitive and inextricably linked to prior relevant experiences. In this view, mathematics education can be supported as a contextually-tied activity involving the promotion of semiosis. The teacher's role changes from transmitting information and seeking predetermined responses to co-participating with students in activities that promote meaningful learning. Speaking semiotically, this form of teaching promotes at the individual level an ongoing development of the sign and an increased competence in the sign system that is mathematics.

Peircean Semiotics Applied to Mathematics Education

As the foregoing indicates, developing signs for mathematics is linked to continued and meaningful exposure to mathematical concepts and, more importantly, mathematical activities. However, mathematics and logical-mathematical thinking constitute just one of at least eight modes of knowing, or knowledge domains, or intelligences (Gardner, 1983, 1995). Further, as most teachers know, many students have much more extensive experience (i.e., have better developed signs) in one of these alternative modes of knowing. The challenge for the teacher in this case is to promote learning and meaning in mathematics instruction by linking mathematics to the learner's preferred or stronger intelligence. This process is known as transmediation (Siegel, 1995), which may be defined as the generative process of translating meaning from one system of signs to another (e.g., from music or language to mathematics). In this process, one seeks to construct analogous meanings in sign systems which are different from the sign system(s) used to deliver the original messages. Transmediation may occur more easily if teachers use different modes of instruction and permit different forms of class assignments. Several other suggestions for the classroom mathematics teacher follow.

Acknowledge the central role of group activities in meaning-making. Although the emphasis here has been on development of the sign in individuals, the making of meaning is also guided by broader cultural agendas that may be either explicit or implicit in nature. In school, teachers should recognize the value of cooperative learning among peers with the realization that knowledge is social, negotiated, and distributed.

Provide a variety of tasks for a variety of domains. As described above, humans learn in a variety of ways. Thus, mathematical concepts are not, and should not be confined to, abstract linear tasks. Teachers can take advantage of students' deeper knowledge (i.e., more developed signs) in other ability systems by representing a fuller scope of mathematics and by appealing to transmediation. For example, learners with few mathematical strengths might be encouraged to perform first in linguistic, musical, spatial, or bodilykinesthetic realms before attempting to represent the parallel meaning in logical-mathematical form. This procedure was used by Griss (1994), who reported the use of creative movement as a means of teaching topics in such diverse areas as mathematics, science, literature, and social studies.

Teach for understanding. Teaching for understanding is consistent with the main premises of semiotics and constructivism. This approach acknowledges the multidirectional nature of learning and meaning-making that must be achieved by all learners, including the teacher. Several assumptions follow from this perspective on learning: (a) the teacher encourages multiple perspectives on a task, (b) both individual and group activities are used, (c) different classroom roles are assumed by both students and teacher, with the latter modeling continuous learning, (d) subject matter is related directly to students' lives and engages them actively in meaningful contexts, (e) process and continuous self-assessment rather than product alone are emphasized, and (f) the ideas of students are taken seriously and honestly attended to. For example, Haas and LoPresto (1994) reported on a project in which students explained the solutions to complex mathematical problems before a panel of judges. Efforts such as these support students' development of

competency in the signs of mathematics by providing authentic tasks that are made meaningful.

Acknowledge individual interests and strengths. In the foregoing discussion, it was noted that learners should be encouraged to make meaning within their preferred or stronger sign systems. The often-cited expression "start where the child is" still holds.

Summary

In this presentation, I have introduced Peirce's sign and its relationship to some key premises of Vygotsky and of comprehensive constructivism. I have tried to underline that, to be consistent with Peirce's semiotic approach, mathematics education should be seen as extending well beyond a simple rational undertaking to involve the embodied mind in the context of cultural activity. Hopefully, this stance will be perceived as a liberating rather than constricting one for teachers of mathematics.

References

Cole, M. (1990). Cognitive development and formal schooling: The evidence from cross-cultural research. In L. C. Moll (Ed.), Vygotsky and education: Instructional implications and applications of sociohistorical psychology (pp. 89-110). Cambridge: Cambridge University Press.

Davis, R. B., Maher, C. A. & Noddings, N. (Eds.) (1990). Constructivist views on the teaching and learning of mathematics. Journal for Research in Mathematics Education Monograph, 4.

Epstein, S. (1994). Integration of the cognitive and the psychodynamic unconscious. American Psychologist 49, 709-724.

Gardner, H. (1983). Frames of mind: The theory of multiple intelligences. New York: Basic.

Gardner, H. (1995). Reflections on multiple intelligences: Myths and messages.

Phi Delta Kappan, 77, 200209.

Griss, S. (1994). Creative movement: A language for learning. Educational Leadership, 51(5), 78-80.

Haas, N. M., & LoPresto, S. (1994). Panel assessments: Unlocking math exams. Educational Leadership, 51(5), 69-70.

Hartshorne, C., & Weiss, P. (Eds.) (1932). Collected papers of Charles Sanders Peirce, Vol. 2: Elements of logic. Cambridge, MA: Harvard University Press.

Hills, S. (Ed.) (1992). The history and philosophy of science in science education, Vol. 2. Kingston, ON: Queen's University, The Mathematics, Science, Technology and Teacher Education Group and the Faculty of Education.

Lave, J., & Wenger, E. (1991). Situated learning: Legitimate peripheral participation. Cambridge: Cambridge University Press.

Oppizzi, L. (1997). Exercice, transmission et acquisition des connaissances cliniques chez des practiciens de la médicine chinoise. Lausanne, Switzerland: University of Lausanne, Institute of Psychology.

Siegel, M. (1995). More than words: The generative power of transmediation for learning. Canadian Journal of Education, 20, 455-475.

Smith, H. A. (1995). Cultural psychology and semiotics: Confronting meaning in educational practice. Canadian Journal of Education, 20, 407-414.

Smith, H. A. (1997, June). Situating constructivism: Psychological and world view considerations. Paper presented at the meeting of the International History and Philosophy of Science and Science Teaching Group, Calgary, Alberta, Canada.

Vygotsky, L. S. (1978). Mind in society: The development of higher psychological processes. Cambridge, MA: Harvard University Press.

Wertsch, J. V. (1985a). Vygotsky and the social formation of mind. Cambridge, MA: Harvard University Press.

Wertsch, J. V. (Ed.). (1985b). Culture, communication, and cognition: Vygotskian perspectives. Cambridge: Cambridge University Press.

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