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REVIEW OF THE ELECTRONIC EDITION OF *THE COLLECTED PAPERS OF C. S. PEIRCE*

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The Collected Papers of Charles Sanders Peirce, volumes 1-6 edited by Charles Hartshorne and Paul Weiss published in 1931-1935, and volumes 7-8 edited by Arthur Burks published in 1958 (Cambridge, MA: Harvard University Press) have been produced in electronic edition on CD-Rom as an indexed and searchable database.

This is one of the philosophical works in *The Past Masters Series* of Bibliographic Databases produced by InteLex Corporation. The corporation has a Web site at http://www.nlx.com , and an E-mail address of sales@nlx.com . You can get specific information about this database and the rest of the series at: http://www.nlx.com/cat/peirce.html and http://www.nlx.com/text/catalog.html for the rest of the Past Masters series, which includes the published works of Wittgenstein (mainly in German).

On the C. S. Peirce CD-Rom, in addition to the 8 volumes of collected papers there is a Peirce Bibliography and a useful and lively Editorial Introduction by John Deely which locates it in current semiotic and Peirce scholarship.

Charles Sanders Peirce is one of the key founders of semiotics, and a great philosopher, mathematician, and logician. He is widely regarded as the most original thinker the United States of America has produced to date. His ideas are deep and perhaps difficult, especially as he is fond of inventing his own technical terminology. This is not mere jargon, as it represents a wide and deep conceptual framework which is far from fully explored and utilised. Peirce is a clear thinker and lucid writer of the highest order. Having Peirce's work on CD-Rom is a great boon. It means that the widely scattered references to say 'mathematics' or 'sign' can be traversed easily, and key passages located.

It has now become acknowledged that semiotics is the central hinge on which all of Peirce's thought turns. As John Deely says in his introduction "the principal optic through which Peirce early and ever-after came to view the problems of philosophy [is] the optic of "semiotic", as he called it".

It would be foolish and arrogant of me to try to summarise Peirce's work in this review, so instead I let him speak for himself. Of course my not so invisible hand lies in the choice of passages. Some indication of the nature and range of his thought is given by the following quotes, which struck me as intriguing, memorable or important.

There are three kinds of signs which are all indispensable in all reasoning; the first is the diagrammatic sign or icon, which exhibits a similarity or analogy to the subject of discourse; the second is the index, which like a pronoun demonstrative or relative, forces the attention to the particular object intended without describing it; the third [or symbol] is the general name or description which signifies its object by means of an association of ideas or habitual connection between the name and the character signified. (Peirce, *Collected Papers*, Vol. 2, par. 243.)

A sign stands for something to the idea which it produces, or modifies. Or, it is a vehicle conveying into the mind something from without. That for which it stands is called its object; that which it conveys, its meaning; and the idea to which it gives rise, its interpretant. The object of representation can be nothing but a representation of which the first representation is the interpretant. But an endless series of representations, each representing the one behind it, may be conceived to have an absolute object at its limit. The meaning of a representation can be nothing but a representation. In fact, it is nothing but the representation itself conceived as stripped of irrelevant clothing. But this clothing never can be completely stripped off; it is only changed for something more diaphanous. So there is an infinite regression here. Finally, the interpretant is nothing but another representation to which the torch of truth is handed along; and as representation, it has its interpretant again. Lo, another infinite series. (Peirce, *Collected Papers*, Vol. 3, par. 339)

The … reader [might] suppose that indices have exclusive reference to objects of experience, and that there would be no use for them in pure mathematics, dealing, as it does, with ideal creations, without regard to whether they are anywhere realized or not. But the imaginary constructions of the mathematician, and even dreams, so far approximate to reality as to have a certain degree of fixity, in consequence of which they can be recognized and identified as individuals. In short, there is a degenerate form of observation

which is directed to the creations of our own minds--using the word observation in its full sense as implying some degree of fixity and quasi-reality in the object to which it endeavours to conform. Accordingly, we find that indices are absolutely indispensable in mathematics; and until this truth was comprehended, all efforts to reduce to rule the logic of triadic and higher relations failed; while as soon as it was once grasped the problem was solved. The ordinary letters of algebra that present no peculiarities are indices. So also are the letters A, B, C, etc., attached to a geometrical figure. (Peirce, *Collected Papers*, Vol. 2, par. 305).

As for algebra, the very idea of the art is that it presents formulæ which can be manipulated, and that by observing the effects of such manipulation we find properties not to be otherwise discerned. In such manipulation, we are guided by previous discoveries which are embodied in general formulæ. These are patterns which we have the right to imitate in our procedure, and are the icons par excellence of algebra. The letters of applied algebra are usually tokens, but the x, y, z, etc., of a general formula, such as (x+y)z = x z + y z, are blanks to be filled up with tokens, they are indices of tokens. Such a formula might, it is true, be replaced by an abstractly stated rule (say that multiplication is distributive); but no application could be made of such an abstract statement without translating it into a sensible image. (Peirce, *Collected Papers*, Vol. 3, para.364)

In mathematical reasoning there is a sort of observation. For a geometrical diagram or array of algebraical symbols is constructed according to an abstractly stated precept, and between the parts of such diagram or array certain relations are observed to obtain, other than those which were expressed in the precept. These being abstractly stated, and being generalized, so as to apply to every diagram constructed according to the same precept, give the conclusion. (Peirce, *Collected Papers*, Vol. 2, par. 216)

We form in the imagination some sort of diagrammatic, that is, iconic, representation of the facts, as skeletonized as possible. The impression of the present writer is that with ordinary persons this is always a visual image, or mixed visual and muscular; but this is an opinion not founded on any systematic examination. If visual, it will either be geometrical, that is, such that familiar spatial relations stand for the relations asserted in the premisses, or it will be algebraical, where the relations are expressed by objects which are imagined to be subject to certain rules, whether conventional or experiential. This diagram, which has been constructed to represent intuitively or semi-intuitively the same relations which are abstractly expressed in the premisses, is then observed, and a hypothesis suggests itself that there is a certain relation between some of its parts--or perhaps this hypothesis had already been suggested. In order to test this, various experiments are made upon the diagram, which is changed in various ways. This is a proceeding extremely similar to induction, from which, however, it differs widely, in that it does not deal with a course of experience, but with whether or not a certain state of things can be imagined. Now, since it is part of the hypothesis that only a very limited kind of condition can affect the result, the necessary experimentation can be very quickly completed; and it is seen that the conclusion is compelled to be true by the conditions of the construction of the diagram. This is called "diagrammatic, or schematic, reasoning." (Peirce, *Collected Papers*, Vol. 3, par. 778.)

Another operation closely allied to generalization is abstraction; and the use of it is perhaps even more characteristic of mathematical reasoning than is generalization. This consists of seizing upon something which has been conceived as a {epos pteroen}, a meaning not dwelt upon but through which something else is discerned, and converting it into an {epos apteroen}, a meaning upon which we rest as the principal subject of discourse. Thus, the mathematician conceives an operation as something itself to be operated upon. He conceives the collection of places of a moving particle as itself a place which can at one instant be totally occupied by a filament, which can again move, and the aggregate of all its places, considered as possibly occupied in one instant, is a surface, and so forth. (Peirce, *Collected Papers*, Vol. 1, par. 83.)

Peirce has little to say directly about learning or education, but he has much that is profound to say about mathematics, knowledge, and logic and he invents many of the tools and concepts of modern semiotics single-handed. His analysis of the semiotics of mathematics is powerful and surprising. Furthermore, his fascinating account of mathematical understanding as the apprehension and manipulation of mental images has much to offer the psychology of mathematics, but which has yet to be fully exploited.

To anyone wanting to explore the thought of Peirce, this CD-Rom is a godsend. There are many rich diagrams in the text on the CD-Rom, and in Windows they appear on the screen effortlessly, so this is not a mere collection of ASCII text.

Maintained by Pam Rosenthall
email comments and suggestions
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