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LOGIC AS SEMIOTICS - PEIRCE'S EXISTENTIAL GRAPHS AND POSSIBILITIES FOR MAKING MORE SENSE IN LEARNING LOGIC.

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Charles Sanders Peirce is known for his contributions to a number of fields. Specifically he is associated with Semiotics, Logic and Pragmatism and it is clear that for Peirce there are inextricable links between all three:

> Logic, in its general sense, is, I believe I have shown, only another name for semiotic, the quasi necessary or formal, doctrine of signs. (Buchler (1958) page 98.)

It is this connection, between logic and semiotics that I want to explore in this article, specifically within the context of teaching logic.

Peirce's early work centred around the development of a semiotic which would act as a consistent philosophical basis for the world. The perspective he offered in his "new list of categories"(1867) was an attempt to offer an alternative to the Kantian view from a this semiotic perspective. It is here that he first introduces the notion of thirdness, a concept that is to be found in various manifestations throughout his work.

By now much of the substance of Peirce's early work should be familiar to readers and I shall not repeat it here. Brief synopses can be found in Vile and Lerman (1996) Radford and Grenier (1996) or in general terms in Eco (1979). What is of interest here will be his later work in which, from a semiotic underpinning, he attempted a development of logic aimed at clarifying reasoning and formulating easily understandable general principles of reasoning:

> Now although a man needs not the theory of a method in order to apply it as it has been applied already, yet in order to adapt to his own science the method of another with which he is less familiar, and to properly modify it so as to suit it to its new use, an acquaintance with the principles upon which it depends will be of the greatest benefit. For that sort of work a man needs to be more than a mere specialist; he needs such a general training of his mind, and such knowledge as shall show him how to make his powers most effective in a new direction. That knowledge is Logic. [1]. page 198.

Such a grand design, but how was it achieved? Through a semiotic perspective. Peirce applied his notions about signs and their nature, along with his knowledge of logic (in which he had great prowess), to the problem of the representation of reasoning, resulting in a form of diagramming which he called existential graphs. For Peirce his graphs, and actions upon them represented relationships and reasoning in a most clear form and unambiguous form. Peirce considered deductive logic to be the study of process, and an empirical science and he saw his graphs as providing a context for experimentation.

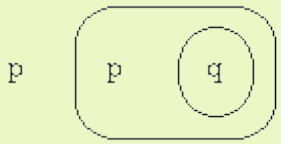
> One can make exact experiments upon uniform diagrams; and when one does so, one must keep a bright outlook for unintended and unexpected changes thereby brought about in the relations of different significant parts of the diagram to one another…… Just so, experiments upon diagrams are questions put to the Nature of the relations concerned. (4.530)

(Because of the nature of the writings of Pierce most of his work was never published during his lifetime, although much of it is available in collected volumes. The standard reference is to a collection of 8 volumes " The collectedpapers of Charles Sanders Pierce" 19931-1958; Cambridge; harvard.it. References are usually given in a standard form (volume number, paragraph number) referring to this collection.)

I am inclined to agree with Zeman (1997) who suggests that in his graphs Peirce is aiming at an iconic, transparent, representation of relationships, a sign that he has identified with clarity and with connection (with an object) through resemblance. In fact, Zeeman goes on to say, the notion of iconicity is directly connected with the mathematical idea of (one to one) mapping and that Peirce aims through his graphs to "map" the important features of mind, and hence externalise reasoning.

Without further ado let us see a simple example of Peirce's Existential graphs to help contextualise this discussion. To illustrate, let us examine *Modus Ponens,* represented in traditional propositional logic as (p Ç (p Þ q)) Þ q . Logically p Û q is equivalent to ~(p Ç

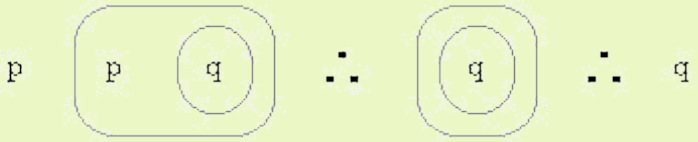
~q). Existential graphs always represent logic denoted in 'and' and 'not' form, thus the antecedents of *Modus Ponens* can be represented by the following existential graphs:



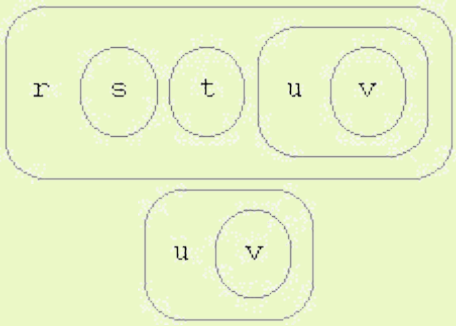
With the co-existence of each propositional element describing the 'AND' relationship and the 'rings' being equivalent to ~(...), the 'not' relationship. These rings are known as *negative contexts*. There are a few other terms (Peirce was fond of his neologisms) that it is necessary to explain, and a few rules to grasp. For example:

- Elements not within any negative contexts are in the *outermost context*:
- An element is dominated by another element if the dominated element is ringed by a negative context and the dominating element is outside that boundary.
- Any dominated element that matches a dominating element can be 'rubbed out' or *deiterated*.

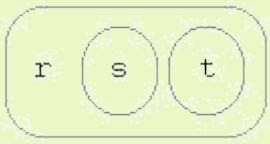
So in the graph above, as p is dominated by its matching p in the outermost context, it can be deiterated leaving q surrounded by two rings alone. After removing p within the single negative context level, the resulting graph simply states ~(~q) which, by *double negation*, results in q, this all sounds complicated until we see the following existential graphs



I want to suggest that existential graphs are a more visual and holistic way to view logical reasoning, and although clearly the system is isomorphic to the more usual, more symbolic, approach to predicate logic it has a certain degree of 'perspicacity', particularly in terms of meanings which is obscured in the more traditional approach. This is more evident in more complex examples: For instance consider the graphs:



all this could be deiterated to yield:



In traditional propositional form this is just an example of resolution using MP but I believe that there is a sense in which this representation is 'more obvious'. (see Polovina and Heaton (1991) for a more detailed discussion of existential graphs and of their enhancement, conceptual structures).

There has been much recent interest in Peirce logic (Searle et al (1997)) and applications of the graphical approach have been seen to be of value in areas such as information systems (Raban (1997)) and accounting (Polovina (1997)) But also Peirce's categories of logical reasoning, Deduction, Induction and Hypothesis (abduction) have proven useful as tools of interpretation in mathematics education (Saenez-Ludlow (1997), Vile (1996)). It would seem that a less abstract approach to logical reasoning would provide possible route to overcoming problems in learning and doing logic.

Logic is the backbone of mathematics and computer science, yet it is clear from the limited research in this area that many intending mathematicians and computer scientists have very limited logical facility. For example Barnard (1995) has shown convincingly that simply negating a statement can be problematic and the recent London Mathematical Society report (Howson 1996) confirms that students ability to prove is very poor. In my own teaching I have found that many students have problems with understanding the

nature of implication (Vile and Polovina (1997)). The reasons for this are as yet unknown although there is evidence to suggest that the degree of symbolism and abstraction in logic (Dubinsky *et al.* (1998)) and in mathematics in general (Vile (1996)) cause conceptual difficulties.

Peirce suggested that " [through a] System of diagrammatization … any course of thought can be represented with exactitude" (1906, p492), and most of the work he carried out in the later part of his life was in working towards ways of achieving this exactitude. I must admit to being a relativist and as a result having to disagree with Peirce on this point, nevertheless I do believe that his work was not in vein, for although it is not possible (in my world view at least0 to represent or map the ';mind' exactly, the very process of constructing a map (in the form of an existential graph) helps me to construct a meaning in action. In my experience, logical connections appear to me as I draw and later as I take a step back and view my system as a whole. Resolution is then a simple mater of 'abducing' through Peirce's generic laws. I can certainly see what Zeeman means when he refers to the iconic nature of existential graphs and I can see in the graphs themselves a manifestation of Peirce' semiotics.

So, why not try Peirce graphs as a system for logical exploration with students, of course that is what my colleague and I intend to do. We expect (because that is how we feel about existential graphs ourselves when we are using them) to find that students can 'see' the logic and have a more holistic and intuitive feeling for resolution and deduction. But we are not sure, what we do know is that do students have problems in logical reasoning and that as yet Peirce logic has not been tried as a context for overcoming these problems. Certainly the approach fits within a semiotic of mathematics education in that 1) Peirce graphs themselves are a result of semiotic enquiry into the nature of mind and relations and 2) the apparent iconicity of the graphs indicates a less arbitrary, and hence less complicated meaning-set necessary for comprehension. We shall see!

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Maintained by Pam Rosenthall
email comments and suggestions
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