PHILOSOPHY OF MATHEMATICS EDUCATION JOURNAL 10 (1997)

A SEMIOTIC AND ANTHROPOLOGICAL APPROACH TO RESEARCH IN MATHEMATICS EDUCATION

Juan D. Godino and Carmen Batanero

University of Granada

The posing and solution of research problems in Mathematics Education requires the integration and adaptation of theoretical contributions made by different sciences and technologies interested in human cognition. In this paper, a theoretical model, intended to articulate the epistemological, anthropological, and psychological facets involved in teaching and learning mathematics, is presented. In this model we reconceptualize some basic constructs, such as mathematical object, meaning and understanding, as well as the study of their mutual relationships. Two interdependent dimensions (personal and institutional) are distinguished for these constructs, and a research agenda for Mathematics Education, based on the notions of 'semiometry' and 'ecology of meaning', is also outlined. This model is based on the ontosemantic theory of mathematics described in Godino and Batanero (1994; in print), which acknowledges a fundamental role to problem situations and to the actions of people and institutions for building mathematical knowledge.

Epistemological, psychological and anthropological assumptions about mathematical knowledge

A research agenda for Mathematics Education must solve dilemmas deeply rooted in its reference disciplines, as well as identifying their complementarities, integrating and coordinating different trends and contributions. It must particularly bridge the gap between the Frege's Platonism and the conventionalism of Wittgenstein, the individualism of Piaget and the collectivism of Vygotsky, between thought and mathematical language, between subjective and objective knowledge. To sum up, it must try to coordinate the epistemological, psychological, and anthropological dimensions involved in the processes for creating and diffusing mathematical knowledge.

An explicit theory about the nature and origin of mathematical objects, which makes socio-relativism strongly supported by the recent philosophical tendencies compatible with the realistic experience so firmly rooted in the intuition of mathematicians, is called for to overcome the aforementioned dilemmas. Such a theory would allow a model of mathematical understanding to be elaborated that could take into account the institutional and sociocultural factors involved therein, in addition to psychological processes.

In this article, we present a synthesis of the first results from a project that tries to formulate a theory of mathematical objects and their meanings, from the perspective of Mathematics Education (Godino and Batanero, 1994; in print), and some new supports and developments therefrom.

Our theoretical model is based on the following epistemological, cognitive, and anthropological assumptions about mathematics

i. Mathematics is a human activity involving the solution of problematic situations (external and internal), from which mathematical objects progressively emerge and evolve. According to constructivist theories, people's acts must be considered the genetic source of mathematical conceptualization.
ii. Mathematical problems and their solutions are shared in specific institutions or collectives involved in studying such problems. Thus, mathematical objects are socially shared cultural entities.
iii. Mathematics is a symbolic language in which problem-situations and their solutions are expressed. The systems of mathematical symbols have a communicative function and an instrumental role.
iv. Mathematics is a logically organized conceptual system. Once a mathematical object has been accepted as a part of this system, it can also be considered as a textual reality and a component of the global structure. It can be handled as a whole to create new mathematical objects, widening the range of mathematical tools and, at the same time, introducing new restrictions in mathematical work and language.

These assumptions take into account some recent tendencies in the philosophy of mathematics (Tymoczko, 1986; Ernest, 1991) and Mathematics Education (Cobb, 1989; Chevallard, 1992; Steffe, 1993; Ernest, 1994).

Synthesis of an ontosemantic theory for Mathematics Education

In this section, we shall present a synthesis of the notions introduced in our theoretical model; we refer the reader to our previous papers where a more detailed presentation is provided. In Godino and Batanero (1994) the notion of *arithmetic mean* is used to illustrate this theory and different backgrounds and connections with other authors are described. In Godino and Batanero (in print) *the statistical concept of association* is used as an example, and a proposed research 'problematique' is more widely described.

Our theoretical model starts out from the notion of *problem-situation*, which is taken a as primitive idea. For any given person, a problem-situation is any circumstance in which he/she must carry out mathematising activities (Freudenthal, 1991), that is,

- building or looking for possible solutions that are not immediately accessible;
- inventing an adequate symbolization to represent the situation and the solutions found and to communicate these solutions to other people;
- justifying (validating or arguing) the proposed solutions;
- generalizing the solution to other contexts, problem-situations and procedures.

A class of mutually related problem-situations, sharing similar solutions, processes, or representation will be referred to as *a problem field*. This construct is closely related to those regarding *semantic field* (Boero, 1991), and *subjective experience domain* (Bauersfeld, Krummheuer & Voigt, 1988).

The subject performs different types of *practices,* or actions to solve a mathematical problem, to communicate the solution to other people or to validate or generalize that solution to other settings and problems. The subject's knowledge arises as a consequence of that subject's interaction with the field of problems, which is mediated by institutional contexts.

Two primary units of analysis to study the cognitive and didactic processes are the *meaningful practices,* and the *meaning of an object*, for which we postulate two interdependent dimensions: personal and institutional. A practice is meaningful for a person (respectively, for an. institution) if it fulfils a function for solving the problem, or for communicating, validating, or extending its solution. Meaningful practices are considered as a situated expressive form and involve a problem-situation, an institutional context, a person, and some semiotic instrumental tools mediating the action.

This notion of practice is used to conceptualize *mathematical objects*, both in their psychological and epistemological facets (personal and institutional objects). Mathematical objects -abstractions or empirical and operative generalizations (Dörfler, 1991)- are considered as emergents from the systems of personal (institutional) practices made by a person (or within an institution) when involved with some problem-situations.

The system of meaningful prototype practices, i.e., the system of efficient practices to reach the goal aimed at is defined as the *personal (institutional) meaning of the object.* It is considered to be the genetic (epistemological) origin of personal objects (institutional objects). It is linked to the field of problems from which this object emerges at a given time and it is a compound entity.

The "meaning of a mathematical object" (concept, proposition, theory) could be compared to the "encyclopedia of uses" of the term or expression denoting that object, although this "encyclopedia" is related here to a person or institution and uses are conceived as meaningful practices. The meaning of an object should be considered as a compound entity (Putnam, 1975; Bunge, 1985), in which different kinds of elements may be distinguished:

- extensional elements (prototype situations in which the object is used);
- intensional elements (different characteristic properties and relationships with other entities);
- expressions and symbolic notations used to represent situations, properties, and relationships.

The compound nature of meaning is opposed to the unity nature of the object, and it allows us to focus, from another point of view, the design of teaching situations and the assessment of subjects' knowledge.

To sum up, we postulate a relativity of the emergent objects, intrinsic to different groups of people and institutions involved in the field of problems, and also depending on the available expressive forms. This assumption could be useful to explain the adaptations (or transpositions) and mutual influences that mathematical objects undergo when transmitted between people and institutions.

Relationships and connections with other theoretical frameworks

We recognize that our ideas have been closely inspired by the epistemological assumptions of Brousseau' Situations Theory (1986), the pragmatic semantics by Wittgenstein (1953) and the anthropological approach to the Didactic by Chevallard (1992). We also share with Cobb (1989; 1994), Bartolini(1991) and Bauersfeld (1992) the pragmatic theoretical objectives of coordination of constructivist and sociocultural perspectives in Mathematics Education. Human activity and social interaction concepts, as well as the search for complementarities are the basis of our systemic approach to the Theory of Mathematics Education (Steiner, 1987). Furthermore, the notion of meaning (Bruner, 1990) and negotiation of meaning (Bauersfeld, Krummheuer and Voigt, 1988) -in the heart of the microculture generated in the mathematics class, and between this and the wider mathematical culture- underlie the socioconstructivist, relativist and pragmatic ontosemantic on which we propose to base research into Mathematics Education.

Our theoretical model is explicitly based on the notion of *object*, with clear realistic connotations, which might apparently contradict the constructivist assumptions adopted. However, we think that constructivism does not imply antirealism. The metaphor of the ontological object (Johnson, 1987) is an essential and unavoidable resource of thought, even for mathematics (Sfard, 1994), since mathematical entities are intuited as realities independent of mind, as they have most of the characteristics of real or perceptible objects, except for been immaterial. As Bereiter (1994; p. 22) asserts: *"They have origins, histories; they can be described and criticized, compared with others of their kind. They can be found to have properties that their creators or previous generations were unaware of"*.

The theoretical model postulates a kind of sociocultural realism, not an absolutist monism. The mathematical noosphere (Morin, 1991; Godino, 1993) is conceived to be constituted by a rich variety of worlds amongst which complex ecological relationships are established. The non-objectivist theory of knowledge (Edwards & Núñez, 1995) and those enactivist developed by authors such as Maturana and Varela (1987), Lakoff (1987), Johnson (1987) (embodied cognition, experientialism), etc., could be also coordinated with our ontosemantic model for mathematical knowledge.

Elements for a model of understanding in Mathematics Education

From the ontosemantical ideas summarized in the previous Section, we identify the following consequences that should be considered in order to elaborate a theory on understanding mathematics (Godino, 1996).

Institutional and personal dimension

According to our pragmatic and relativist conception of mathematics, a theory of mathematical understanding must recognize the dialectical duality between the personal and institutional facets of knowledge and its understanding, to be useful and effective to explain teaching and learning processes.

The definition of understanding by Sierpinska (1994) as the '*mental experience of a subject by which he/she relates an object (sign) to another object (meaning)*' emphasizes one of the senses in which the term 'understanding' is used, well adapted for studying the psychological processes involved. Nevertheless, in mathematics teaching the term 'understanding' is also used in the processes of assessing students' learning. School institutions expect subjects to appropriate some culturally fixed objects, and assign the teacher the task of helping students to establish the agreed relationships between terms, mathematics expressions, abstractions, and techniques. In this case, understanding is not merely a mental activity, but it is converted into a social process. As an example, we may consider that a pupil "understands" sufficiently, for example, the concept of *function* in secondary teaching and that he/she does not understand it, if the judgement is made by a university institution. Furthermore, from a subjective sense, understanding cannot merely be reduced to a mental experience but it involves the person's whole world (Johnson, 1987).

Systemic and dynamic nature of understanding

Since, in our theoretical model, we start out from the notions of object and meaning, personal understanding of a concept is "grasping or acquiring the meaning of the object". Therefore, the construct 'meaning of an object' is not conceived as an absolute and unitary entity, but rather as compound and relative to institutional settings. Therefore, the subject's understanding of a concept, at a given moment and under certain circumstances, will imply the appropriation of the different elements composing the corresponding institutional meanings.

Furthermore, recognizing the systemic complexity of the object's meaning implies a dynamical, progressive, though nonlinear nature of the process of appropriation by the subject (Pirie & Kieren, 1994), due to the different domains of experience and institutional contexts in which he/she participates. We agree with Glasersfeld (1989) when he states: "The process of accommodating and tuning the meaning of words and linguistic expressions actually continues for each of us throughout our lives. No matter how long we have spoken a language, there will still be the occasions when we realize that, up to that point, we have been using a word in a way that now turns out to be idiosyncratic in some particular respect" (p. 133).

Human action and intentionality

Our theoretical model also includes, as the primary unit of analysis, the notion of *meaningful prototype practice*, defined as the action that the person carries out in his/her attempts for solving a class of problem-situations and for which he/she recognizes or attributes a purpose (an *intentionality*). Consequently, understanding the object, in its integral or systemic sense, requires the subject, not only the semiotic and relational components, but to identify a *role* -an *intention* (Maier, 1988)- in the problem solving process for the object.

Assessment of understanding

We conceive the assessment of understanding as the study of the correspondence between personal and institutional meanings. The evaluation of a subject's understanding is relative to the institutional contexts in which the subject participates. An institution (educational or not) would say that a subject "understands" the meaning of an object - or that he/she 'has grasped the meaning' of a concept, if the subject is able to carry out the different prototype practices that make up the meaning of the institutional object.

It is also necessary to recognize the *unobservable construct character* of personal understanding. Consequently, an individual's personal understanding about a mathematical object may be deduced from the analysis of the practices carried out by the person in solving problematic tasks, which are characteristics of that object. Since, for each mathematical object, the population of such tasks is potentially unlimited, the analysis of the task variables and the selection of the items to design evaluation instruments become of primary interest. The construct 'meaning of an object', in its two dimensions, personal and institutional, might be a useful conceptual tool to study the evaluation processes, the achievement of the '*good understanding*' (Sierpinska, 1994), and the institutional and evolutionary factors conditioning them.

Semiometry and ecology of meanings: A research agenda for Mathematics Education

The consideration of the meaning of mathematical objects as systems of practices and the discrimination between personal and institutional meaning introduces in the didactical 'problématique' the study of the structure and characterization of these theoretical entities.

i) A primary class of didactic research studies must be orientated towards the determination of institutional meanings, especially the meaning within mathematical institutions. We have to research into the characteristic uses of mathematical concepts, propositions and theories and to identify their different representations. This reference meaning may be compared with the meaning of mathematical objects in teaching institutions. We can also study the conditioning factors producing the development and changes of these meanings.

ii) The theoretical system we have described in this paper also allows us to focus, from a new perspective, the problem of assessing mathematical knowledge. According to our theory, a subject's cognitive system (his/her conceptual and procedural knowledge, his/her intuitions, representation schemas, ...), that is to say, the network of personal objects at a given time, is an organized and complex totality. The distinction we have stated between the domain of ideas or abstract objects (personal and institutional) and the domain of meanings, or system of practices from which such unobservable objects emerge, is used to clarify the problem of looking for the correspondence between both domains, i.e., the problem of assessing institutional and personal knowledge.

As a consequence of our theorization the observable nature of social practices allows us to determine the *field of problems* associated with a mathematical object, as well as its institutional meaning, with the help of a *phenomenological* and *epistemological* study. The analysis of the task variables for this field of problems provides a first criterion to structure the population of possible tasks. From this population, a representative sample could be drawn to guarantee content validity for the assessment instrument. These two elements: *field of problems* and *task variables* thereof shall provide the first reference points in the selection of relevant evaluation situations for assessing subjective knowledge.

Ecology of meanings

The problems involved in studying the evolution of institutional meanings of mathematical objects could be modelled with the help of the ecological metaphor (Godino 1993): A particular object performs a function in different types of institutions and it is needed to identify the necessary and/or sufficient conditions that allow this object to play its role in these institutions.

The notions of institutional object and meaning are intended to be used as conceptual instruments in this ecological and semiotical analysis of mathematical ideas.

The type i) and ii) studies described would constitute the institutional and personal 'statics of meanings' in this ecological metaphor. Its aim would be to find the 'state and control variables' of meaning, considered as a system, at a particular moment in time. These studies of the static aspects of meaning should be completed with dynamic studies, which we are going to describe below.

iii) The study of changes that the institutional meaning of a mathematical object undergoes to become knowledge to be taught in different teaching institutions (curricular design, mathematical textbooks, ...) would constitute the dynamics of institutional meanings (didactical transposition (Chevallard, 1985), ecology of meanings).

iv) Another fundamental problem in this category is the construction of adequate institutional meanings referring to a mathematical object for a specific school level, i.e., curricular design. According to the theorization proposed, teaching should be based on the presentation of a representative sample of problems and other elements of the meaning of mathematical objects, taking into consideration the time and resources available.

v) The meaningful learning (relational or significative) of the subject can be modelled as a sequence of 'acts of understanding', or acts of overcoming obstacles (Sierpinska , 1994). The characterization of these acts and the identification of the mechanisms which produce the obstacles (Artigue, 1990) are central themes in the dynamics of personal meaning of mathematical objects. Metaphorically, the study of teaching and learning processes could be viewed as the study of the effects on personal meanings of 'shocks' of didactical sequences, which hold the elements of meanings.

In the same way, a part of the characterization of the dynamics of personal meaning would be the study of the evolution of students' conceptions, i.e., the transformation of personal meanings as a consequence of instruction.

Coordination of theories in Mathematics Education

The practical utility of a theoretical model, such as the one we have presented, cannot be shown by an example. On the contrary, it would be necessary to carry out a coordinated plan of research, in which we are still engaged. Nevertheless, we can bring forward some consequences of our model, which are of interest for the general orientation of the research into the Mathematics Education.

i) From this theoretical system it can be deduced that didactic research should preferentially pay attention to the study of the complex relationships between the institutional meaning of mathematical objects and personal meanings built by the subjects. Teaching and learning processes occur in the heart of several institutions, in which mathematical knowledge takes specific meanings, which condition those learning processes. Although mental processes also conditioning students' learning, the centre of attention for didactic research should not be the students' minds, but the cultural and institutional contexts in which teaching takes place.

ii) The systemic nature of the *meaning of an object* (concepts, propositions, theories) allows us to orient the sampling selection processes of teaching and assessment situation: The institutional meaning play the role of a reference universe for these sampling processes. Furthermore, the components that we have incorporated into the meaningful practices, and the suggested category of such practices, permit us to focus attention on the dialectic object-situation and institution-mediating tools when studying cognitive and didactic processes.

iii) The distinction between the personal and institutional dimensions, for both objects and the meanings, allows us to articulate the epistemological, psychological, and anthropological facets in human cognition processes, and, therefore, in mathematics education processes. An important part of the didactic research 'problématique' may be formulated by means of two basic ideas: semiometry, i.e., the characterization of the personal and institutional meanings, and ecology of meanings, i.e., the study of the interrelationship between both meanings.

iv)The theoretical system intends to articulate different cognitive and epistemological approaches by building an ontosemantic link between them. Among these approaches, we can quote: social constructivism, activity theory, situated cognition, ethnomathematics, cognitive and didactic anthropology, theory of didactic situations, etc. However, the level of agreement and complement between these theories and our model will require further study in the future.

In short, the main utility of the ontosemantic theory outlined and the socio-epistemic relativism postulated, even for mathematical knowledge, is based on their potential for integrating different theories. Moreover, it provides a framework to formulate or to reorientate research questions in Mathematics Education.

Acknowledgement

This research has been supported by the DGICYT grant PS93-0196, (M.E.C., Madrid).

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Last Modified: 11th November 1997