POME

**PHILOSOPHY OF MATHEMATICS EDUCATION JOURNAL 10 (1997)**

# ISSUES RAISED BY A SAUSSURIAN ANALYSIS OF CLASSROOM ACTIVITY

**Simon Goodchild**

University College of St Mark and St John, Plymouth UK.

In my analysis of year ten students activity, following Ernest (1993), I apply a Saussurian model as one tool for interpretation.

Skemp (1982) discusses two modes working mathematically, surface level/syntactic and deep level/semantic. Mathematics is presented and communicated with symbols and words; the symbol systems, in particular, have been invented to model the underlying meanings. Skemp argues that it is possible to operate at the level of the symbols, on the syntax of the subject without entering the meanings of those symbols. Syntactic understanding is, therefore, more accessible because the syntax of the mathematics is realised in the process of communication, it lies on the 'surface' of the information as mathematics is communicated. However, Skemp notes that it is necessary for the learner to actively explore the deeper level, semantic meanings. The value of mathematics lies in the way it is able to model the world and although syntactic operations are accessible, simple and routine the aim of mathematics education is to develop understanding at the deeper level so that the generality of mathematical models may be appreciated.

In an account of mathematical activity as a sequence of semiotic transformations Ernest (1993) reveals that it is not possible to simply partition syntactic and semantic modes of working mathematically in the clear cut way that Skemp appears to suggest. Ernest argues that a mathematical task consists of transformations of signs which exist in the form of signifiers or symbols and on meanings, that is interpretations or images of the signified.

> Each step ***Sk*** ð ***Sk+1*** is a transformation of signs which can be understood on two levels. Drawing on Saussure's analysis, each sign ***Sk*** (=**Sk/Ik**) is a pair made up of a signifier **Sk** ('**S**' for *symbol*) and a signified **Ik** ('**I**' for *interpretation* or *image*). So the completed task can be analyzed as in the example shown (below)
>
> A completed mathematical task as a semiotic transformation
>
> **Level of Signifiers: S1 ® S2 ® S3 ® S4 S5 ® S6 ® S7 ® S8 ® S9**
>
> | | | | | | | | |
>
> **Level of Signifieds: I1 I2 I3 I4 Þ I5 Þ I6 I7 I8 I9**
>
> (Ernest, 1993: 240)

In the above figure each 'S' represents a signifier, this might be an algebraic expression, an equation or other representation, each successive 'S' arises out of the previous 'S'. Each 'S' is vertically linked to its realisation in the subject's conception, whereas the 'S's are public, observable and common the 'I's will be the result of each subject's own interpretation and there is no guarantee that the interpretation will be the same from subject to subject. Progress through a mathematical task will take the form of steps along this chain representing transformations of signs, as shown in the diagram above, some steps occur at the level of the signifiers, and the interpretations are 'carried' so to speak, at other times the transformations take place at the level of interpretations and the signifiers are carried.

Ernest writes:

> transformations take place on one of the two levels (signifier and signified), or both together. … the levels of signifier and signified are relative; they are all the time in mutual interaction, shifting, reconstructing themselves.
>
> (Ernest, 1993: 241)

If as Skemp observes the activity progresses with little attempt to develop a semantic understanding then the activity exists mostly at the level of the signifiers, in which case students might be following 'rules without reason' which is characteristic of instrumental activity as described by Skemp (1976).

Methodology

My analysis is based upon data gathered over the course of one school year in which I joined nearly every mathematics lesson of a year ten class. My time with the class was spent observing the activity of teacher and students; recording all the teacher's remarks to the whole class; recording some of the teacher's interventions with individual students; observing students at work and, principally, engaging students in unstructured conversations based on my observations of their activity in the tasks set by their teacher. (The research is reported in detail in Goodchild (1997)).

**Interpretation**

My subsequent interpretation of the large amount of data collected revealed very little evidence of mathematical activity as semiotic transformations in the manner described by Ernest. For the most part students appeared to work either at the level of signifiers or at the level of signifieds. Work at the level of signifiers would be described in Skemp's terms as surface level or syntactic, that is following the rules of operating on symbols. Alternatively if working at the level of signifieds then students would appear to be operating, mentally, on some iconic representation. There very little evidence of crossing between levels, either to make sense of transformations of signifiers or to facilitate complex tasks on the signifieds.

One of the rare occasions in which a student does appear to bring the two levels together is provided by a student Oscar who is surprised by the result of a calculation. In the following extract Oscar has worked out the circumference and area of a circle, diameter 4 cm, and he notices the answers are the same, he expects them to be different and this causes him to reflect. The conversation is initiated by me noting my observation which I interpret as surprise:

I Did something surprise you there?

Oscar Yeah, they're both the same, that and the circumference is the same as the area

I So do you think that's right?

Oscar It must be because all the formulas all right

(55/940428 t.u. 50-53)

The two answers at first surprise him, which causes him to review his working and convince himself that they are correct because 'all the formulas (are) all right'. Later in this conversation he is able to explain why in this case the answers are the same and reason that it will not happen in any other case than circles with radius two. In this episode Oscar appears to reflect first on the meaning, or signified - the numerical answers for the two dimensions are the same and this surprises him. His response is to check his working, that is at the level of signifiers and verify that he has not made any errors. Later in answer to my directed questions he is able to offer an explanation why the symbolic process provides the particular answers in this unique case. In this respect I believe Oscar reveals his versatility in moving between signifier and signified and I conjecture that he is conscious of a deep level, semantic conception.

However, I want to conjecture that for many students mathematical tasks are not experienced as semiotic transformations as described by Ernest. Firstly I want to conjecture that the signifiers or symbols are not perceived as signifying *something* but take on a concrete reality of their own, that is they are perceived by the student as signifieds. When a student works, as Skemp would describe 'syntactically' she is working on the objects of mathematical activity which do not necessarily have any deeper meaning. Secondly I want to conjecture that students *expect* mathematics to consist of working out coded sequences which do not necessarily have any meaning.

The following brief extracts from conversations with Gary are offered to support these conjectures:

I Does that make sense to you?

Gary Yes, somewhat, I'm just getting it from the book of .. I don't know . . .

I OK

Gary It's the same as we did that

I Right, so can you try to explain to me so that I can understand what you've done there as you understand it?

Gary . . . Um, no I can't, I don't know how you do that I just did it, I'm just on the same lines as I did this here, that's what I'm doing

(06/931118 t.u. 17-22)

Seven months later …

I What makes a good explanation for you?

Gary If it's put into simple terms, it's sort of split up into little parts, and it sort of like gives you a code to follow like that.

(67/940614 archived data t.u. 1512, 1513)

It may be asked, if the signifiers of mathematical activity become the signifieds then what, in the Saussurian analysis takes the place of the signifiers? I think the answer to this question might be found in the above extracts of conversations with Gary, the signifiers are what we otherwise refer to as indexicals (pronouns such as 'it' and 'that').

The central issue of concern to me is that we need to address the *expectations* of students and their *awareness* of what constitutes mathematical activity. If students expect a *good* teacher to make things easy for them and provide *good* explanations then there is a pressure upon teachers to fit into Gary's requirements, to put it 'into simple terms, split it up into little parts, give a code to follow'. Further, if students are not aware of the relationship between signifier and signified in mathematics and accept the signifier as a mathematical object devoid of meaning then they are unlikely to engage in both levels of the transformation of signs and thus they will not experience *proper* mathematical activity.

**References**

Ernest, P (1993) Mathematical activity and rhetoric: a social constructivist account. In *Proceedings of the Annual Conference of the International Group for the Psychology of Education*, Tsukuba: Japan, pp. 238 - 245.

Goodchild, S. (1997) An Exploratory Study of Year Ten Students' Goals in the Mathematics Classroom. Unpublished PhD dissertation, University of Exeter.

Skemp, R. R. (1976) Relational understanding and instrumental understanding. *Mathematics Teaching*, no. 77, pp. 20 - 26.

Skemp, R. R. (1982) Communicating mathematics: surface structures and deep structures. *Visible Language,* vol. 16, no. 3, pp. 281 - 288.

Simon Goodchild

staffsg@lib.marjon.ac.uk

---

*Maintained by Pam Rosenthall*
*email comments and suggestions*
*Last Modified: 13th November 1997*