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MAKING SENSE OF MATHEMATICAL MEANING-MAKING: THE POETIC FUNCTION OF LANGUAGE

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ABSTRACT. In trying to make sense of mathematical meaning-making, sections of the mathematics education community have increasingly turned to linguistics as a basis for theorising mathematical discourse. In this paper, we critique the standard interpretation of (Jakobson's) structural linguistic theory which has been used by mathematics educators. From the theoretical perspective we outline, based on the work of Jakobson and Barthes, we re-interpret some examples of mathematical meaning-making.

Introduction

Recent attempts to make sense of mathematical meaning-making have drawn freely on the ideas of metaphor and metonymy. Broadly, there are two main strands of research: one drawing its theoretical basis from the work of Lakoff and Johnson (1980) on conceptual metaphors and metonymies (e.g. Lakoff and Núñez 1996, Sfard 1994), and the other from the work of the linguist Jakobson (1956, 1960) on metaphoric and metonymic relations in 'texts' (e.g. Pimm 1990, Walkerdine 1988). Here, we concern ourselves with the latter strand of research and critique the 'standard' interpretation of Jakobson's linguistic theory, in particular the interpretation of 'metaphor' and 'metonymy' which considers the two as dichotomous.

In the standard interpretation, metonymical relations operate *within* a discourse (intra-domain) while metaphorical relations refer to things outside it (inter-domain). Although this interpretation adequately allows meaning to be thought of as developed through the interplay of metaphoric and metonymic relations, it is a partial interpretation of Jakobson. Two crucial features are missing: (1) how the relations may operate at *any* level in a text, not just at the high level of inter- and intra-domain relations; (2) how the relations exist in a dialectic, informing each other as well as informing together.

We mention here a few theoretical constructs whose meanings we will elaborate in the paper. In (mathematical) texts there are multiple systems of signification whose pairwise dependencies we analyse in terms of *denotation* and *connotation*. However, these multiple systems are not a property of the text itself, but of the text and the reader. Thus we look at how a reader (learner) may come to build a *connoted reading* out of the signs of the text. We suggest that a key mechanism for this is the *poetic function*; and the key to the poetic function is the dialectic of *metaphoric* and *metonymic* relations.

Denotation and connotation

Language can signify something other than 'what it says'. For example, if you are presented with a poem, it does not say anywhere in the text 'this is a poem'. Nevertheless, you attend to the layout of the text, perhaps the regularity of metre, perhaps rhyme; the text therefore signifies 'this is a poem'. This is the traditional understanding of denotation ('what it says') and connotation ('what it does not say').

Barthes (1967) formulates a general semiotic theory of denotative and connotative *systems*. In a semiotic system, signifiers and signifieds are united in the act of signification into signs. His definition of denoted and connoted rests on a relation between the two systems, independent of the nature of the signifieds: the signifiers of the connoted system comprise signs, or collections of signs, in the denoted system. Thus he reformulates 'saying' and 'not saying' in terms of signifying in different, but related, systems:

> I am a pupil in the second form in a French *lycée*. I open my Latin grammar, and I read a sentence, borrowed from Aesop or Phaedrus: *quia ego nominor leo*. I stop and think. There is something ambiguous about this statement: on the one hand, the words in it do have a simple meaning: *because my name is lion*. And on the other hand, the sentence is evidently there in order to signify something else to me. Inasmuch as it is addressed to me, a pupil in the second form, it

> tells me clearly: I am a grammatical example meant to illustrate the rule about the agreement of the predicate (Barthes 1972, pp. 115-6).

The connoted signified ('I am a grammatical example') has here for its signifier a collection of signs ('because my name is lion') in the denoted system. Barthes names the 'collection of signs' the *meaning*, and the signifier the *form* (*ibid*, p. 117). In order to create a meaning within the connoted system (the grammar lesson), the reader has to do two things. First attention must shift away from the meaning deriving from this sentence about a lion and on to the form. Second, the reader must seek the signified of the form.

We will take as a first mathematical example of denotation and connotation the case of some study materials which form part of a common mathematics curriculum in the UK, the School Mathematics Project (11-16). The writing, and computation, of products involving decimals is initially motivated as a representation of repeated addition (SMP 1983a). This becomes problematic when both quantities are non-integer. SMP introduces this latter case in the context of computing costs where the number of items and the cost per unit item are given (SMP 1983b). Before asking the child to work out the cost of 3.7m of gold braid the text says
When you work out the cost of 3m, you do £2.60 **x 3**
When you work out the cost of 4m, you do £2.60 **x 4** (*ibid*, p.2).

Students are being asked to return to previous texts and by attending to the form, construct the connoted sign: ëthis is about a multiplicative structureí. A shift in the site of potential meaning is demanded.In this teaching sequence, the implied role of the text has shifted from representing multiplication, to the object of attention itself. The way such a shifting occurs has been theorised by Jakobson.

Jakobson proposes that language has six functions, each set towards a specific element of the act of communication (Jakobson 1960, p. 357). For example, the referential function of language relates to its capacity to refer to some extra-textual reality. A more than trivial text will rarely fulfil just one function though a particular function may be dominant. The shift of role of the text to 'object of attention' is a result of the dominance of the poetic function. Before we discuss the poetic function in detail, we need to set out Jakobson's theory of metaphoric and metonymic relations. His presentation is characteristically condensed and we have drawn on the elucidations of Lodge (1977) and Hawkes (1977).

Metaphoric and metonymic relations

The workings of metaphoric and metonymic relations are set down by Jakobson in the following terse paragraph:

> The development of a discourse may take place along two semantic lines: one topic may lead to another through either their similarity or through their contiguity. The metaphorical way would be more appropriate for the first case and the metonymic for the second, since they find their most condensed expression in metaphor and metonymy respectively. (Jakobson 1956, p. 76).

Here, 'topic' and 'development' are to be understood extremely broadly: Jakobson is proposing that metaphoric and metonymic semantic development can exist at *all levels* in the text (*ibid*, p. 77). 'Topic' may be the text, a sentence, a word, a combination of words: any discernible 'unit'. Jakobson uses 'metaphoric' for a relation at any level which is based on similarity, and 'metonymic' where the relation is based on contiguity; he reserves 'metaphor' and 'metonymy' for the figures of speech which are the most condensed expressions of such relationships.

A linguistic example of this development 'along two semantic lines', which lies at the heart of structuralist linguistics, is the syntagmatic/paradigmatic polarity. In the syntagm (a technical word meaning 'combination of signs')

'the girl sat on the chair'

the meaning of each word is developed as the sentence is carved out (the *syntagmatic axis*). Thus syntagmatic relations hold between the constituent signs and between the signs and the syntagm, and are therefore relations of *contiguity*. Further, each word's meaning is affected by its relation to other words that could have been chosen (the *paradigmatic axis*) but were not. Thus paradigmatic relations are relations of *similarity* (or dissimilarity, a negation of similarity). Note that paradigmatic and syntagmatic relations hold between signs in the discourse and *not* between signs and some version of a reality 'out in the world'. The meaning of a sign is developed both by its *reference* to some version of reality, and by its *value*: that is, its paradigmatic and syntagmatic relations to other signs in the discourse. For example, 'sat' draws meaning from its contiguity with 'on the chair': a particular way of sitting. It also draws meaning (paradigmatically) from *not* being 'perched', 'lounged', 'crouched', or even 'spat'.

A mathematical illustration. Pimm has written extensively on 'metaphor' and 'metonymy' at the inter/intra-domain level. For example, he has said that activities which develop 'symbolic fluency', such as when children chant a times table, are metonymic; because they focus a child's attention on the "movement 'along the chain of signifiers'" (Pimm 1990, p. 135). But this ignores the fact that, on a different level of topics, there are metaphoric relations present, formed by similarities between the lines of a chant—e.g. '1 times 2 is 2', '2 times 2 is 4', etc.—in the repetition of signs ('times 2') and the regularity of metre. It is these metaphoric relations, generating a sense of movement and rhythm, which, at least in part, cause the text to be metonymic at the level of topics considered by Pimm.

The poetic function in language

The poetic function, whose set is towards the message itself, operates via transgressions of the language system: transgressions that make the text 'strange'. We shall give one example in some detail: the breaking of the syntagmatic/paradigmatic polarity. As remarked before, this is a fundamental feature of language in structural linguistic terms and hence can be expected to be a particularly fruitful site. As we shall outline, the disruption of the polarity shifts attention to form as signifier and to value as potential signified.

One mode of effecting this transgression is by imposing similarity on the syntagmatic axis, where ordinarily (in referential texts) contiguity is expected— this is the principle constitutive device of poetry according to Jakobson (1960, p.358). Rhyme is perhaps the most obvious kind of 'strange' similarity (sounding alike but semantically unlike). In Barthes' phrase (1967, p. 87), rhyming 'corresponds to a deliberately created tension between the congenial and the dissimilar, to a kind of structural scandal.' Jakobson (1960, p. 358) lists other possible strange similarities including, for example, the equalising of word stress with word stress. An alternative mode of breaking the polarity is to impose contiguity on the paradigmatic axis. In the example given by Lodge (1977, p.77), the syntagm 'ships crossed the sea' can be transformed into 'keels crossed the deep' producing two metonymies. The non-logical deletions, e.g. deleting 'ships' instead of 'keels' from the notional syntagm 'the keels of the ships', render the text strange.

Referential reading becomes interrupted and attention is shifted from the (extra-textual) referent of the sign on to the signs themselves. In poetic texts denoted signs become connoted signifiers. In the case that we have discussed, this occurred through disruption of value and thus value may be brought to the reader's attention: value becomes a potential site of meaning**.** The poetic text ceases to be solely a window onto something else, but invites the reader to attend to its own form. *But*, it does not cease being a 'window onto': it depends on the reader's focus. Sites of potential meanings are multiplied, not exchanged one in favour of another. In Jakobson's words: 'The supremacy of poetic function over referential function does not obliterate the reference but makes it ambiguous' (Jakobson 1960, p. 371).

Connoted signs that arose out of transgressions, out of breaking the rules of the denoted system, become themselves 'institutionalised' for the reader as he or she develops the connoted system as a site of meaning. In this sense, the new system may become as familiar, and its signifieds as 'concrete', as the signifieds of the original denoted system.

Two mathematical examples

Our examples of the SMP text on multiplication, and the chanting of times tables, have already offered two illustrations of the re-interpretation of mathematical texts. Those, and the two further examples here, show the potential of our theoretical ideas for the analysis of mathematical texts. We should emphasise that we are not claiming to be able to offer a semiotic system of mathematical discourse. We are proposing interpretations by analogy with examples within literary theory and linguistics; thus our interpretations can only be pointers towards a more systematic mathematical analysis.

<u>Example 1</u>

Consider the two mathematical texts

'2 + 3 ='

'2 + 3 = 1 + 4'

It is well known that often, long after a learner is capable of reacting to a text of the first kind by performing the sum, the second produces bewilderment: the learner finds it 'wrong' or 'meaningless'. Several authors (e.g. Kieran 1981) have pointed out two related reasons why children react as they do. Firstly, children interpret the equals sign as meaning 'do the sum'. That is, they have a procedural interpretation of the equals sign. Even supposing that the children's interpretation can be shifted to some notion of equivalence, a second reason remains: they may have a procedural interpretation of '2 + 3', or any syntagm whose template is 'number-operation-number'. In this case, '2 + 3' will not be seen as the result '5' but as a sum which, if performed, would give the result '5'. So, '2 + 3' cannot be the same as '1 + 4': they are different sums.

In relation to the child's system, the interpretation "'2 + 3' and '1 + 4' both signify 5" is a connoted reading. '2 + 3', a syntagm (recall, a *combination* of signs) in the child's system, is a form, a signifier in the connoted system. Gray and Tall (1994) have written about the 'process-product' ambiguity in mathematical notation, which for them expresses a cognitive 'process-concept' duality, or 'procept'. They posit that a learner's grasp of this 'notational ambiguity' is central to her success or failure in mathematics. From our perspective, the question is: how might the child's entry into the connoted system be facilitated?

One approach may be to tell the child the rules of the connoted system: '1 + 4' is another name for the number 5. But this ignores that, for the child, '1 + 4' is *not* an empty form, it is a syntagm full of meaning. Viewed in this light, the problem is one of denotation/connotation rather than the more general 'ambiguity'; and this highlights an asymmetry of the two systems for the learner. We cannot hope to obliterate the child's denoted sign, 'sums'; and there is evidence that the 'name for a number' approach is not successful (Kieran 1981). The problem is much more difficult: we would need to find ways of building on the child's system, so that she can appreciate a poetic reading of the text: '2 + 3' is equivalent to '1 + 4' because the result of the sum '2 + 3' is the same as the result of the sum '1 + 4'. This reading is a metaphoric relation resting on a metonymy: a sum is like another sum (metaphoric) because their result (metonymy) is the same. Such a reading is not self-evident: the metonymy is non-logical. To comprehend the syntagm as signifier, as formal mathematical discourse would have it, is a matter of enculturation into this discourse. This will not occur through attention to a single text. Enculturation requires that the connoted system be built up by the learner through numerous and diverse activities, with significant attention to poetic readings of texts.

<u>Example 2</u>

Consider the simultaneous equations

$$x + 5(y + 1) = 0$$
$$5y = -(5 + x)$$

If the equivalence of these equations is not noticed and a solution is attempted then an ambiguity concerning equality arises which is very different from the 'process-product' ambiguity of Example 1: the calculation will end up with something like '0 = 0'. If attention is focused on this as a syntagm composed of '0', '=' and '0' then the statement is tautologous. Clearly, it is a transgression of the 'rules' of the denoted system to supply no information. A student could attempt a poetic reading, focusing on '0 = 0' as a form: as a signifier in the connoted system. But as signifier, its signified is 'the equations are dependent', a far from obvious connection. Perhaps the form is recognised, perhaps the signified is known, but it does not necessarily follow that the sign will be constructed.

Conclusion

In this paper we have briefly outlined a theoretical basis for analysing mathematical meaning-making which calls into question the dichotomous relationship between 'metaphor' and 'metonymy'. The implications are far from esoteric: the theory suggests a need to promote connoted reading of texts by learners, and we are beginning to understand how it may help us to elaborate mathematical meaning-making in terms of webbing—an attempt to explain how a learner struggling with a new mathematical idea can draw on supportive knowledge from a range of sites, rather than simply erecting a hierarchy of abstractions (see Noss and Hoyles, 1996).

We are beginning to make sense of the ways in which carefully-designed computer software can offer learners the means to find more direct entry points into the 'connoted' system, by providing a means for expressing meaning in computational action. Conceived in this way, the computer is a rather special kind of tool in which action involves the formal use of language, and where the usual polarities—meaning and precision, informal and formal—do not hold.

We may speculate that there is a link between this work and our current research on mathematics curriculum design for undergraduate science students. To what extent must the structure of mathematics be understood in order for it to be used effectively as a tool in the sciences? What can we say about the changing relationships between mathematical and scientific epistemologies, and the roles of new technologies in mediating these relationships? In the area of 'service' mathematics teaching there is a standard dichotomy that concerns the ways in which mathematics may be learnt. It can be characterised as 'formal' versus 'informal': one either learns the formal mathematics itself and then 'applies' it to scientific situations, what we might call a 'metonymic' approach (mathematical meaning develops within the discourse), or one simply learns to 'use' mathematics informally in science without attempting a 'formal understanding' of it, what we might call a 'metaphoric' approach (mathematical meaning develops with reference to science). We are questioning this dichotomy—a dichotomy which we speculate is an applied consequence of the metaphoric/metonymic dichotomy with which we began.

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