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RESPONSE TO 'COMMENTARY ON DEHAENE' BY GEORGE LAKOFF

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Rafael Nunez kindly forwarded George Lakoff's Commentary on Dehaene to me (and others), and it raises deep and interesting issues to which I cannot resist responding. So I don't get your attention under false pretences let me say that what I offer below is a defence of the social constructivism which Lakoff critiques.

Like George Lakoff I am waiting impatiently for a book, but is the one Rafael Nunez and he himself are writing on their research on the cognitive structure of mathematics. I have as yet only seen the earlier draft 'The Metaphorical Structure of Mathematics: Sketching Out Cognitive Foundations For a Mind-Based Mathematics', but it holds great promise for a semiotic theory of mathematics. If we momentarily view the conceptual structure of mathematics as a many storied sky-scraper, each floor at a greater level of abstraction than the one below, this semiotics would locate the basement in embodied knowing - metaphors that arise from being in and of the world. As Piaget, Bruner, Varela and others have realized, our knowing is at some level rooted enactively in our knowledge of moving in and interacting with the world around us. Lakoff and Johnson had the wisdom to extend the range of metaphors derived by these means to a much richer repertoire than that allowed by Piaget and Bruner. These metaphors then, are in part, the signifieds of mathematical expressions, thus founding mathematics on a semiotic basis. They also form interdomain links, both horizontal and vertical. Thus, as Lakoff says, mathematics is based in both image-schemas, the universal primitives of spatial relations, and conceptual metaphors, described cognitively as cross-domain mappings.

Lakoff goes on to conclude, like Dehaene, that mathematics arises out of our brains and bodies. I agree, but would wish to add, it also arises from our conversations and inter-personal relations, and these are an essential part of it (in the sense of being ineliminable). We are human beings, not animals, and as humans we are profoundly shaped by our interdependencies, contexts and cultures, which overlay and run through our bodily (and brain) experiences.

It appears as Dehaene, Lakoff and others say, we have an inbuilt capacity for counting, just as we do for language and for recognizing human faces, clearly vital skills for becoming a social being.

Lakoff claims that cartesian coordinates, imaginary numbers, fractional dimensions, and mathematics as a whole are imaginative constructions of human beings. He agrees with Dehaene that this requires a nonplatonic philosophy of mathematics that is also not socially constructivist.

"Such a philosophy of mathematics is not relativist or socially constructivist, since it is embodied, that is, based on the shared characteristics of human brains and bodies as well as the shared aspects of our physical and interpersonal environments. As Dehaene said, pi is not an arbitrary social construction that could have been constructed in some other way. Neither is e, despite the argument that Nunez and I give that our understanding of e requires quite a bit of metaphorical structure. The metaphors are not arbitrary; they too are based on the characteristics of human bodies and brains."

This is why I feel impelled to comment. A social constructivist philosophy of mathematics can be relativist (in a weak sense) without regarding mathematics (or any area of knowledge) as arbitrary. It can accept that there are constraints in the brain, body, physical environment, and culture, but that even so mathematical knowledge is emergent and underdetermined (i.e., not logically necessitated and predicted) by these constraints. There are degrees of freedom in it; more than are commonly recognized.

Like Lakoff's experientialist philosophy or "embodied realism", an appropriately formulated social constructivism is not platonic or objectivist. It also shares the view that the correspondence theory of truth does not work for mathematics. Lakoff suggests that mathematics is at least in part, invented not discovered, and I want to argue that this is also the position of the variety of social constructivism that I sketch below. We both challenge the absolutist view of mathematics as universal, objective and certain.

In my forthcoming book Social Constructivism as a Philosophy of Mathematics (SUNY Press, 1997), I argue that not only is mathematics fallible, but it is created by groups of persons who must both formulate and critique new knowledge in a formal 'conversation' before it counts as accepted mathematics. Knowledge creation is part of a larger overall cycle in which mathematical knowledge is presented to learners in teaching and testing 'conversations' in schools and universities, before they themselves can become mathematicians and participate in the creation of new knowledge. This perspective offers a middle path between the horns of the traditional objective/subjective dilemma in knowledge. According to social constructivism, mathematics is more than a collection of subjective beliefs, but less than a body of absolute objective knowledge, floating above all human activity. Instead it occupies an intermediate position. Mathematics is cultural knowledge, like the rest of human knowledge. It transcends any particular individual, but not all of humankind, like art, music, literature, religion, philosophy and science. Our knowing may be partly grounded in embodied knowing, but as Vygotsky says, this gets woven into our language usage and transformed into higher level thought. Only through interpersonal conversation and living cultures can we come to know and create mathematics. And, according to social constructivism, only through conversation and cultural emergence does mathematics come into being and develop.

This is a fallibilist view that tries to account for mathematics naturalistically, that is in a way that is true to real world practices. Unfortunately, social constructivism is too often caricatured by opponents as claiming that mathematics may be part or all wrong; that since mathematics is not absolutely necessary it is arbitrary or whimsical; that a relativist mathematics, by relinquishing absolutism, amounts to 'anything goes' or 'anybody's opinion in mathematics is as good as anybody else's'; that an invented mathematics can be based on whim or spur of the moment impulse; and that if social forces are what moulds mathematics then it must be shaped by the prevailing ideology and prejudices of the day, and not by its inner logic.

However these claims and conclusions are caricatures. Social constructivism does not mean that some or all of mathematics may be false (although Gödel's incompleteness results mean that we cannot eliminate the possibility that mathematics may generate a contradiction). Instead, it denies that there is such a thing as absolute truth, which explains why mathematics cannot attain it. Truths in mathematics are never absolute, but must always be understood as relative to a background system. Unlike in physics, in which there is just one world to determine what is true or false, mathematics allows the existence of many different interpretations. So an assumption like Euclid's Parallel Postulate and its denial can both be true, but in different mathematical interpretations (in the systems of Euclidean and non-Euclidean geometries). Mathematicians are all the time inventing new imagined worlds without needing to discard or reject the old ones.

A second criticism levelled at social constructivism is that if mathematics is not absolutely necessary then it must be arbitrary or whimsical. Relativist mathematics, the criticism goes, by relinquishing absolutism amounts to 'anything goes'. Therefore an invented mathematics is based on whims or spur of the moment impulse. For example, Roger Penrose asks, are the objects and truths of mathematics "mere arbitrary constructions of the human mind?" His answer is in the negative and he concludes that mathematics is already there, to be discovered, not invented.

Plausible as this view seems at first, it is often argued on mistaken grounds. Mathematicians like Penrose (in the Emperor's New Mind) often contrast necessity with arbitrariness, and argue that if relativist mathematics has no absolute necessity and essential characteristics to it, then it must be arbitrary. Consequently, they argue, anarchy prevails and anything goes in mathematics. However, contingency, not arbitrariness, is the opposite of necessity. Since to be arbitrary is to be determined by chance or whim rather than judgement or reason, the opposite of this notion is that of being selected or chosen. I wish to argue that mathematical knowledge is based on contingency, due to its historical development and the inevitable impact of external forces on the resourcing and direction of mathematics, but is also based on the deliberate choices and endeavours of mathematicians, elaborated through extensive reasoning. Both contingencies and choices are at work in mathematics, so it cannot be claimed that the overall development is either necessary or arbitrary. Much of mathematics follows by logical necessity from its assumptions and adopted rules of reasoning, just as moves do in the game of chess. This does not contradict fallibilism for none of the rules of reasoning and logic in mathematics are themselves absolute. Mathematics consists of language games with deeply entrenched rules and patterns that are very stable and enduring, but which always remain open to the possibility of change, and in the long term, do change.

The criticism that relativism in mathematics means that "anything goes" and that "anybody's opinion is as good as anybody else's" can be countered by using William Perry's distinction between the positions of Multiplicity and Contextual Relativism. Multiplicity is the view that anyone's opinion is valid, with the implication that no judgements or rational choices among opinions can be made. This is the crude form of relativism in which the opposite of necessity is taken as arbitrariness, and which frequently figures in 'knockdown' critiques of relativism. It is a weak and insupportable 'straw person' position and does not represent fallibilism. Contextual Relativism comprises a plurality of points of view and frames of reference in which the properties of contexts allow various sorts of comparison and evaluation to be made. So rational choices can be made, but they always depend on the underlying contexts or systems. Social constructivists adopt a parallel position in which mathematical knowledge is always understood relative to the context, and is evaluated or justified within principled or rule governed systems. According to this view there is an underlying basis for knowledge and rational choice, but that basis is context-relative and not absolute.

This position weakens the criticism from absolutists that an invented mathematics must be based on whims or spur of the moment impulses, and that the social forces moulding mathematics mean it can blow hither and thither to be reshaped accorded to the prevailing ideology of the day. The fallibilist view is more subtle and accepts that social forces do partly old mathematics. However there is also a largely autonomous internal momentum at work in mathematics, in terms of the problems to be solved and the concepts and methods to be applied. The argument is that these are the products of tradition, not of some externally imposed necessity. Some of the external forces working on mathematics are the applied problems that need to be solved, which have had an impact on mathematics right from the beginning. Many examples can be given, such as the following. Originally written arithmetic was first developed to support taxation and commerce in Egypt, Mesopotamia, India and China. Contrary to popular opinion, the oldest profession in recorded history is that of scribe and tax collector! Trigonometry and spherical geometry were developed to aid astronomy and navigational needs. Later mechanics (and calculus) were developed to improve ballistics and military science. Statistics was initially developed to support insurance needs, to compute actuarial tables, and subsequently extended for agricultural, biological and medical purposes. Most recently, modern computational mathematics was developed to support the needs of the military, in cryptography, and then missile guidance and information systems. These examples illustrate how whole branches of mathematics have developed out of the impetus given by external needs and resources, and only afterwards maintained this momentum by systematizing methods and pursuing internal problems.

I welcome the emergence Lakoff's embodied philosophy of mathematics, since it joins with social constructivism in opposing that absolutist views of mathematics that have dominated the field.

However I hope have shown that a better case can be put for the relativist views of social constructivists; that they make space for the mind, body, and the social, in feeding into the creation of mathematics. For although social constructivists believe that mathematics has a contingent, fallible and historically shifting character, they also argue that mathematical knowledge is to a large extent necessary, stable and autonomous. Once humans have invented something by laying down the rules for its existence, like chess, the theory of numbers, or the Mandelbrot set, the implications and patterns that emerge from the underlying constellation of rules may continue to surprise us. But this does not change the fact that we invented the 'game' in the first place. It just shows what a rich invention it was. As the Giambattista Vico said, the only truths we can know for certain are those we have invented ourselves. Our bodies, minds and our culture feed into that.

Maintained by Pam Rosenthall
email comments and suggestions
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