EDITORIAL: TEACHING AND THE NATURE OF MATHEMATICS

Dennis Almeida and Paul Ernest

University of Exeter, UK

.

Introduction

The papers that follow are devoted to the questions What is mathematics? and What does the nature of mathematics have to do with its teaching in schools? The issue is interpreted in different ways by the different authors who vary from global, overall discussions of these issues to detailed discussions examining pupil goals, conceptions and proof practices, showing what are the outcomes of assumptions about the nature of mathematics embedded in school mathematics teaching. In this introduction we offer some reflections on the background to these concerns in British mathematics education and elsewhere. We also point to some important issues that are relevant but are not fully represented in the collection.

It is also worth remarking on the relatively novel nature of the concerns expressed in this volume. Certainly prior to the 1970's the issue of the nature of mathematics was not on the agenda for discussions about mathematics education. It was acknowledged that mathematics had changed from traditional mathematics, which went up to the end of the 18th century, and which still dominated all GCE Advanced level mathematics syllabuses until the 1970s, to modern mathematics, which began in the second half of the 19th century and which filtered through into the Modern Maths in schools in the 1960s and 1970s. So some changes in mathematics the form of progressive advances in content were previously acknowledged.

However in the past 5 or 10 years, it has come to be recognised that mathematics is not a fixed, unitary, absolute body of knowledge that changes only by growth at the periphery. Advances in the history and philosophy of mathematics, the sociology of knowledge, and post-modernist thought (see for example Ernest 1994) have shown that the myth of the unchanging nature of mathematics is probably held in place by the use of a single term 'mathematics' for several diverse domains of knowledge and discursive practice. School mathematics and the research mathematician's pure mathematics are wholly different areas of study. Controversy has erupted over the natures of both of these domains: the first is the subject of political contestation; the latter of philosophical dispute (Ernest 1991).

Perhaps the key issue that arises in a consideration of the relation between teaching and the nature of mathematics is how do questions, controversies, views, conceptions and philosophies of mathematics impact on the classroom. The answer is complex, and the papers in this collection address this theme from a number of directions. One immediate link, which is still speculative rather than proved, concerns the parallels that might be drawn between certain views of mathematics held by teachers or embodied in the mathematics curriculum, and certain classroom teaching styles or approaches. Thus a generation ago it might be claimed that the mathematics curriculum was dominated by a 'formalist' perspective of mathematics with some emphasis on Euclidean geometry and formal deductive proof. This situation suggested a correspondence between what might be termed a formalist academic perspective of mathematics and the school mathematics curriculum. Progressive reforms in the mathematics curriculum, such as those recommended in the Cockcroft Report (1982) and reflected in the introduction of investigational work in the GCSE examinations in mathematics, and in the inclusion of mathematical processes in National Curriculum mathematics, have eroded this correspondence. Consequently the school mathematics curriculum no longer corresponds to one perspective of mathematics - it corresponds instead to a whole range of perspectives. In the statutory and non-statutory aspects of the National Curriculum in mathematics there is evidence of non-absolutist and learner-centred notions in mathematics (Using and Applying Mathematics, Handling Data), of the use of information technology and computers in mathematics, of an informal perspective on mathematical reasoning and proof, and of the (limited) accommodation of social factors in the learning of mathematics. However other less progressive perspectives can also be discerned in the British National Curriculum with its insistence on paper and pencil algorithms for subtraction and division to be mastered by all before students progress to more challenging ideas. School mathematics may have changed, at least in part, from being formal knowledge transmitted by the teacher, to accommodate both a range of perspectives of knowledge and a range of teaching and learning styles from transmission or imposition by the teacher to learner involvement and negotiation.

The central issue discussed in this issue of Perspectives is that the nature of mathematics and its relation to teaching. One of the claims made in some of the papers is that a new wave of 'fallibilist' philosophies of mathematics have been gaining ground. These propose a conception of mathematics as human, corrigible, historical, value-laden and changing. The suggestion that mathematics itself is not neutral but laden with the values of the people and culture in which it was created is not only controversial, but of far-reaching and fundamental significance for the teaching of mathematics. In the area of gender and mathematics, for example, it is regarded as possibly the key issue (Walkerdine, 1988). The adoption of a value free, neutral, absolutist stance towards mathematics is to embrace the ‘separated’ values associated with stereotypical masculinity, whereas accepting a fallibilist view is more congenial with a gender- and culture-fair approach to mathematics teaching.

A cultural view of mathematics suggests opening up the classroom to the outside world. The consequent multicultural approach incorporates mathematics from European and non-European sources such as finger counting techniques in Roman times, an investigation of number words used by Lincolnshire shepherds, board games from Ghana and Nigeria, probability activities from Brazil, Rangoli patterns, mathematics from the ancient Indian Vedas, geometry in Islamic art (Bailey and Shan, 1991)

It is not necessary to adopt a fallibilist philosophy of mathematics to be concerned with the social, cultural and political dimensions of mathematics education described here, although it can lend support. A fallibilist perspective provides a powerful additional source of arguments for the social responsibility of both mathematics and its teaching. It also fits well with the emerging constructivist views of learning in mathematics and science education. But all of these benefits can be had without this philosophical commitment.

What is harder to accommodate without a fallibilist view of mathematics is a non-Eurocentric view of the history of mathematics. Our present mathematics tradition is commonly ascribed as originating in the 3rd Century BC with the publication of Euclid’s Elements. The Greek mathematical tradition involved

i. discovering a result
ii. verifying the result
iii. explaining the result
iv. communicating to and persuading others of (i), (ii) and (iii), and
v. systematising results into a deductive system.

It now seems that mathematical traditions other than the Ancient Greeks also attended to (i), (ii), (iii) and (iv). Modern mathematical historians (Joseph, 1992; Man-Keung Siu, 1994; Beggren, 1990) point to strong evidence for this in the ancient Chinese, Indian, and Indo-Persian mathematical traditions. Reading the history of mathematics in any culture gives the distinct impression that mathematics is a continuous process involving a social dialectic - in other words mathematics is not clinical, pure and absolute. Beggren, in describing the way in which Islamic mathematics functioned, says that “We have seen....that satisfactory proofs arise from a historical process, where rigour is achieved not by a stroke of genius but by a dialogue of mathematicians with each other and with their forbears.” This is not unlike the description of modern mathematics given by the mathematical philosopher, Imre Lakatos (1976), in his seminal contribution to fallibilism: *Proofs and Refutations*.

One of the issues not addressed by most of the papers in this collection, but which arises from the above discussion, is that of the political dimensions of mathematics education. This is, of course, controversial, especially to those for whom mathematics remains a neutral value-free body of knowledge. Such a view places mathematics (and physical science) in a different position from other subjects in the school curriculum such as history and social sciences. From our perspective we regard this as untenable. As Shan and Bailey point out “Teachers of history, geography, social studies have been engaged in teaching politically controversial and sensitive issues for years........(and)...politics is always present in our teaching, but is not perceived as political when supportive to the dominant ideology". In our view it is an advantage for the learner if mathematics is seen not as an inert and abstract body of knowledge but knowledge which is dynamic which learners can use to explain and analyse both practical (design and technology, physical sciences) and social problems (unemployment, defence, pollution, government expenditure on education) statistical.

Our view of the aim of mathematics teaching is that it should foster critical mathematical literacy and thus empower students to become a critical citizens in modern society. This involves having a sound knowledge of a significant subset of school mathematics and the confident possession of the process skills of applying mathematical knowledge independently to solve and pose problems and evaluating the solutions critically. However it also necessitates the ability to interpret and critically evaluate the mathematics embedded in social and political claims and systems, from advertisements to government pronouncements (Ernest 1991).

References

Bailey, P. and Shan, S. J. (1991) *Multiple Factors: Classroom Mathematics for Equality and Justice,* Stoke-on-Trent: Trentham Books.
Beggren J. L. (1990) Proof, Pedagogy, and the Practice of Mathematics in Medieval Islam, in Hanna and Winchester (1990).
Cockcroft, W. H., Chair (1982) *Mathematics Counts*, London: HMSO.
Ernest, P. (1991) *The Philosophy of Mathematics Education*, London, The Falmer Press.
Ernest, P. Ed. (1994) *Mathematics, Education and Philosophy: An International Perspective*, London, The Falmer Press.
Hanna, G. and Winchester, I. (eds) (1990), *Creativity, Thought, and Mathematical Proof*, OISE, Toronto.
Joseph G. G. (1992) *The Crest of the Peacock: Non-European Roots of Mathematics*, I B Tauris.
Lakatos, I (1976) *Proofs and Refutations*, Cambridge University Press.
Man-Keung Siu (1993) Proof and Pedagogy in Ancient China: Examples from Liu Hui's Commentary on Jiu Zhang Suan Shu, *Educational Studies in Mathematics, 24*
Shan, S. J. and Bailey, P. (1991) *Multiple Factors: Classroom Mathematics for Equality and Justice,* Stoke on Trent, Trentham Books.
Walkerdine, V. (1988) *The Mastery of Reason*, London, Routledge.

Maintained by Pam Rosenthall
email comments and suggestions
Last Modified: 18th October 1996