RATIONALES, PURPOSES AND METACONCEPTS IN MATHEMATICAL ACTIVITY

Simon Goodchild

College of St. Mark and St. John, Plymouth, UK

Careful observation of students' activity in a year 10 mathematics class clearly demonstrates the necessity of distinguishing between task objectives, learning objectives and management (or organisational) objectives in the planning of students' learning experiences. The presumption that learning will follow well chosen, well planned and well targeted tasks is not justified. There is widespread agreement that "mathematics is a difficult subject, both to teach and to learn" (D.E.S. 1982: para. 228) and although much knowledge may be gained from experience 'en passant', so to speak, learning mathematics requires deliberate effort. That learning mathematics arises from purposeful activity is taken, in this account, for granted and the focus of attention is upon the goals towards which students' activity is directed. The intention here is to outline a framework for the analysis of secondary school students' activity in mathematics classrooms; presenting a brief account of the theoretical background to research in a year ten mathematics class in which I am currently engaged. This work, which has taken me into almost every mathematics lesson of a year ten class, includes observation, recording the teachers comments to the whole class and protracted, unstructured conversations with students as they are engaged in task. My aim is to explore what sense the students are making of their activity, their understanding of the mathematics and what they perceive their purpose to be. From this information I hope to be able to make inferences about students' goals and possibly relate these to their experiences within the class and to their teacher's objectives.

The central structure of the framework I am using for the analysis is based upon that offered by Lave (1988) described in her account of cognition in everyday practice. Terms, such as *arena* and *setting* as used by Lave are redefined here in the context of activity in mathematics classrooms; however I believe that Lave's model requires extension if it is to provide a complete account of students' activity. Lave develops an account of cognition as being situated in the practices in which people engage, she posits that:

> ... cognition is constituted in dialectical relations among people acting, the contexts of their activity, and the activity itself. [Lave, 1988: 148]

Students enter the *arena* of the mathematics classrooms with a variety of expectations, intentions and rationales for working. Within this arena the setting is created as students engage in activity in the context of tasks presented by their teacher, the students' perception of the purpose of their activity is an important feature of their interpretation of their experience arising from the activity. These two levels of analysis will be identified in this account of students' activity in the classroom; there is, however, a fundamental difference between the everyday practice of Lave's account and classroom practice, as El'konin observes:

> The basic unit (cell) of educational activity is the educational task. ... An educational task differs fundamentally from other types of problems in that its goal and its result consist of a change in the acting subject himself, not in a change in the objects on which the subject acts. [El'konin, D. B.(1961) quoted by Davydov and Markova (1983) : p. 60]

Thus in addition to the inter-subjective social domain of arena and setting it is necessary to consider the intra-subjective domain of the individual and provide an account of how changes in the 'acting subject', or student, may arise and this will be accomplished by considering the student's metaconcepts of mathematics and learning mathematics. Three 'levels' of analysis are thus defined and a description of students' goals and responses at each of these levels is required to provide a comprehensive account of activity in a mathematics class.

The *arena* of the mathematics classroom is constituted by a broad array of influences, ranging from the policies of central government through the interpretation of these by school, department and ultimately the teacher in the classroom. Decisions effecting the arena will include the provision of resources, accommodation and appointment of specialist staff. The arena is ultimately created as the teacher and students meet together in the mathematics classroom and will be crucially influenced by the expectations, attitudes, beliefs and emotional state of all those participating in the classroom. The students' rationale for acting in this arena will influence the manner in which they engage in the tasks presented by their teacher, it will determine the extent to which they apply themselves to those tasks. Mellin-Olsen (1981) describes two rationales for engaging in activity, an S - rationale and an I - rationale.

The S - rationale describes the situation in which students can perceive the activity of the arena as being personally or socially significant to them, when they identify the activity of the mathematics classroom as relating directly to their needs and it makes sense to them in the context of their own lives and social placing. Mellin-Olsen (1987) describes situations in which mathematics may be made meaningful to students, where mathematics arises from their need to address real and pressing problems to which they are able relate, he also describes situations where the S - rationale is absent because of the chasm between the students 'real life' experience and the contexts provided for their work in school. There is evidence arising from my work that students will try to make their own 'personal' sense of activity thus substantiating an S- rationale by imputing a relationship to everyday life which does not, immediately, arise from the task as the following extract illustrates.

> *[Context: SMP 11 - 16 R2, p119 an exercise on solving equations with brackets e.g. "Solve these equations by first multiplying out brackets - 2(n+5)=7n"]*
>
> SG - When you look at that *[2(n+5)=7n]* what do you understand by it?
> Student - I'm not sure
> SG - If I open a geography book, and I look at some of the things I'd see in a geography book, I might see maps there, and I'd say, well, that gives me an idea of what a country looks like, or where various towns and roads are, or if I looked in a history book it might give me a diagram of a castle, and I'd see what castles sort of look like and things like that. When you open up a maths book, you find expressions like these, well how can you, how do you relate that to something in the real world in the same way that you can relate something in the geography or history
> Student - Well if you had like, you have to share of money like, (SG - Yeah) it's like the same thing
> SG - How do you mean?
> Student - Well, if you've got like so much money and you've got to share it between so many people (SG - Yes) it might help you to equal it out

Mathematics is perceived by many as dealing solely with number, the most common experience students have with number is in financial transactions, so it is of little surprise, perhaps, that this student attempts to make this connection in the context of formal, abstract algebraic manipulation. My conjecture is that students may make such connections even when unwarranted because this may be the only way they can make sense of their activity, thereby satisfying their need for an S - rationale. The I - rationale operates when a student engages in the activity of the arena because it is perceived as being instrumental in achieving some future goal. Thus the activity is not seen as having any value for its own sake as with the S - rationale but is related, say, to passing an examination, giving access to employment or further education. Howson and Mellin-Olsen (1986) caution that the I - rationale may take on a disproportionate role and reason that it is important to develop both S - and I - rationales.

> It is the teacher's task to develop both rationales, whilst ensuring that the I - rationale is not allowed to dominate to such an extent that it distorts the pupils' mathematical education. [Howson & Mellin-Olsen (1986): p. 17]

It is possible that the two rationales described by Mellin-Olsen do not provide a complete picture, there may be many different rationales which account for students engaging in a particular arena, I want to introduce what I will label a P - rationale, which describes the situation when students engage in the task presented for no other reason than because that is the practice of the classroom.

Observation of a year ten class for every lesson throughout the course of a school year has provided the opportunity to witness changes in the arena which have been accompanied by changes in the practice of the classroom and changes in student behaviour. Some of these changes have been quite dramatic such as occasions when the regular teacher has been away and the class has been supervised by another teacher, or change of classroom, or different locations in the time-table and, as many teachers will testify, even the weather may effect the arena and the students' rationale. Other changes have been less dramatic for the whole class but significant for individuals whose behaviour has clearly adapted to the practice of the class.

There is in this P - rationale a sense of the notion of the 'generalised other' introduced by Mead (1934) in his articulation of social or cultural learning. Mead offers an account of learning as a process in which learners respond to the expectations of other (significant) people who constitute the social group within which the subject finds him/herself such as with parents or with siblings or with their own peer group. Mead also accepts that inanimate objects such as furniture may also take on a role in the 'generalised other'. As the person develops so they incorporate an internal sense of the expectations of a generalised other, a mental construct of the perceived attitudes of the 'group', Mead writes:

> It is in the form of the generalised other that the social process influences the behaviour of the individuals involved in it and carrying it on, i.e., that the community exercises control over the conduct of its individual members; for it is in this form that the social process or community enters as a determining factor into the individual's thinking. [Mead (1934): p 155]

Howson and Mellin-Olsen (1986) criticise Mead's account by drawing attention to the variety of 'others' to which a person may relate thus not providing a single coherent 'expectation' but the response to this variety may be explained by the 'multiple subjectivities' described in post-structuralist accounts of cognition (Henriques et al (1984)), and thus it may be argued that the person internalises a multiplicity of generalised others. Further Howson and Mellin-Olsen argue that whereas the subject cannot choose his/her generalised other they can develop their own rationale for learning, albeit influenced by the multifarious beliefs and attitudes encountered in the social domain.

When invited to provide a rationale student's rarely give a response of the S - type , occasionally they will respond indicating the presence of an I-rationale, "I'll need it for the examination" but the overriding sense is not one of imminent examinations, but rather an acceptance of the practice, of being expected to engage in the activity of the classroom; in particular 'I'm doing this because Miss told us to.'

The task presented by the teacher and the students' perception of the purpose of their activity creates the *setting*. It might be presumed that the purpose is for them to learn mathematics, to develop new understanding to gain new knowledge, however such a presumption is not justified. It appears that the students, subjects of the research context here, are not, even subconsciously aware of the distinction between everyday tasks and educational tasks as made by El'konin in the extract above. Students perceive the purpose of their activity as 'doing' rather than 'learning'; as the achievement of the task objectives set by the teacher rather than making sense and developing understanding. It is believed that the teacher encourages this production oriented view of activity in the task objectives given to the students as they are motivated with targets such as they have to *complete* a particular piece of work or *do* certain questions in an exercise. Another signal arises from the teacher's explanation of performance in tests - as the result of effort in revising (rather than understanding).

Students may have their own very clear notions of the purpose of their activity and even when a learning experience is encountered a student may not see this as their purpose in the task. The following extract provides an example of an instance when a student suddenly makes sense of the way in which he should be using information but this sense making experience is not perceived as being central to the task.

> *[Context SMP 11 - 16 R2 pp 86 & 87 Plans and Elevations - Buildings. The student here has to use the given Plan view and Side elevation to make a scale drawing of the end wall of a building, I want to know which of the two given views provided the information that the apex of the roof was above the centre of the base.]*
>
> SG - But that [*Side elevation*] doesn't tell you whether it's in the middle or over here or over here.
> Student - Maybe this [*Plan view*] does.
> SG - How does that tell you?
> Student - Because that's a picture, that one there is a picture of the bird's eye view.
> SG - Right.
> Student - And there's the little line or the joint or (indistinct).
> SG - Yes.
> Student - And it's in the middle. SG - And ah right, so if you look at that one it's in the middle.
> Student - Yeah.
> SG - So therefore this highest point must be.
> Student - In the middle.
> SG - In the middle. Does that make sense?
> Student - Yeah, yeah it does!!
> (Indistinct)
> Student - Well, if that's the middle then it's obvious that's got to be in the middle isn't it?

......

SG - OK Right , now you've just said it's obvious it's got to be in the middle if that's in the middle.
Student - Yeah.
SG - But it wasn't obvious to you in the beginning was it?
Student - No, then I clicked.
SG - OK. And then it clicked?
Student - Yeah.
SG - Well that 's important isn't it?
Student - Yeah.
SG - So have you learned something?
Student - Yeah.
SG - What have you learned?
Student - That you got to take information from the other pictures.
SG - Aha OK. What's the purpose of doing all this work?
Student - Measuring correctly.
SG - Aha, anything else?
Student - Um, don't know, just measuring it correctly and make sure you're drawings are correct.

There is no justification to conclude that if a student does not perceive the purpose of their activity as learning then they will not learn. In their effort to accomplish the task set by their teacher students may well be engaged in 'sense - making' and learning a particular practice. The suggestion is, however, that the learning will be 'context bound', and unlikely to be readily available for transfer to other contexts as described by Lave in her account of cognition in practice (Lave 1988). Indeed it has been observed within the class how work on substitution into formulae in which the students had achieved a reasonable measure of success did not enable them, about a week later, to determine the values of the dependent variable *(y)* in linear equations *(y=ax+b)*. Skemp would describe this type of learning as 'instrumental' (Skemp 1976); and the activity is comprised of a sequence of syntactic operations (Skemp 1982). A recurrent feature of many mathematics lessons is the *exercise*, in which students practise particular skills. It is conjectured that if a student engages in the repetitive, routine nature of an exercise with a syntactic, instrumental knowledge then the result may be described at 'blind activity' in the sense used by Christiansen and Walther (1986: p.284), that is activity which does not lead to fresh or renewed understanding, new meanings or the reinforcement of existing knowledge. And as Wheatley remarks:

> "It is possible that students may be so active that they fail to reflect and thus do not learn" [Wheatley, 1992: 536]

And Steffe and Wiegel caution:

> "One should not mistake however children's activity in the use of their schemes for making modifications in those schemes that might constitute learning" [Steffe and Wiegel 1992: 455]

So we now consider the third level of analysis of students' activity in the mathematics classroom which relates directly to the cognitive development of the student. Suppose a student does perceive that the purpose of their activity is to learn, whatever that may mean to them (it may mean to commit to memory in order that material is available for recall in an examination, or it may mean to understand and make sense). It is widely accepted that responding to some form of cognitive challenge is involved in learning. For example, Piaget writes in terms of accommodation and equilibration following the resolution of cognitive conflict and Vygotsky (1978) describes processes of inter-psychological conflict and intra-psychological conflict as steps in the learning process. The constructivist account of learning places considerable emphasis upon reflective activity. Neisser (1976) offers a model of the way this reflective activity may operate, he describes 'the perceptual cycle' in which the subject's *schema* directs his/her *exploration* thus sampling the *object*, from which sense must be made, and developing understanding or knowledge of the object is applied to modify the *schema*. It is evident from my work with students that conflicting schema and experience do not necessarily result in modifications of schema but may be accepted as a vagary of mathematics. As in the following case where the student has been confronted with the task of finding the gradient of four of sloping railway tracks presented on a square grid. In two cases, those with small gradient, he notes that he can measure along the slope and this will be the same as the horizontal distance between the ends of the slope, in the other two cases, where the slope is greater this is not the case. He acknowledges that the distance along the slope should be greater:

> *[Context: SMP 11 - 16 R2 page 50, Railway gradients. The student has measured the distance along the track and the horizontal distance between the two end points and 'found' them both to be nine centimetres]*

SG - Nine, does it surprise you that the length from that line there is exactly the same as the length up there? Does that surprise you?
Student - Yeah, a bit because I would've thought that there but if you add that black one *[the track]* down onto there [*horizontal line of grid*] it'd been longer.
SG - Yeah, I would have thought that too. Does that, is it the same with all of them?
Student - No, that one (b) there's not. (SG - Right) And that one (c) there's not. That one (d) there is. (SG - Right) Them two there are different.
SG - Do you think that, that really is the same, that length there really is the same as that length there?
(pause 18 seconds)
Student - It must be really.
SG - Why must it be?
Student - Because it's from there to that line there (SG - Yeah) it's nine centimetres (SG - Yes) and then you measure up there from there to there, it's nine centimetres.
SG - Well that's what you got from the ruler isn't it?
Student - Yeah.
SG - Does it make sense?
Student - No. Because them two there if you done the same as done there, on there it's totally different it's that there is smaller along the bottom than it is (SG - Yeah) going up the hill.
SG - Does that bother you?
Student - I just think, figure out one way to work this out and then you go on to the next one and then you have to find another way to work it out.
SG - Does that make sense?
Student - No.

At this level of analysis of students' mathematical activity it is necessary to consider their metaconcepts of mathematics and learning mathematics. In the illustration above the student reconciles the apparent conflict by accepting that in different circumstances one has to adopt different methods or explanations. This may not be an unreasonable response to the situation. It is fairly common in mathematics to ascribe meaning to an object with respect to the context in which it appears. One such, elementary, example is the ordered pair (1, 2) which may be interpreted as a point, a vector, or even an open interval for the domain of a function, etc. Inconsistency and incoherence in mathematics appear to be accepted by this student and thus he does not seek a relational, semantic understanding (Skemp 1976 and 1982).

In the foregoing the illustrations used may be interpreted as depicting students' unhelpful or dysfunctional attitudes and beliefs towards mathematics, however, it is not my intention to suggest inferences regarding what should be the desired goals, i.e. I am not arguing here for a particular 'best' balance of rationale, or a 'proper' sense of purpose, or a 'proper' metaconcept. The research in which I am engaged seeks to explore, analyse and describe, and whereas an attempt is made to correlate the objectives of the teacher (given both explicitly to me or the class, or implicit through the presentation and discussion of tasks) and the nature of the tasks presented with the goals adopted by the students, and it may be possible to draw attention to students' idiosyncratic meanings and alternative conceptions, the structure of the research requires that any value judgements, unless substantiated elsewhere, must remain at the level of conjecture.

Returning to Lave's account of cognition, she argues that the various factors and levels of subjects and social contexts, are in dialectical relationship and any change or development in one component is likely to influence the whole system. Thus Lave dismisses the notion of 'goals' arguing that these may be interpreted as being in one direction only, she prefers to use the word expectations. Changes in a student's metaconcepts may, indeed, influence their rationale for engaging in activity and the account given here is certainly not intended to suggest otherwise. Neither is it the intention here to consider issues of causation and influence and any such inferences need to be explored carefully.

If the argument for these three levels of analysis and the goals adopted by students at each level is accepted then it must follow that attempts at curriculum development should address these goals at all levels. One curriculum development project which does attempt this is the 'Awareness of learning, reflection and transfer in school mathematics project' (Bell et al 1994) which is concerned to develop students' awareness of the components of mathematical activities, mathematical content, types and purposes of mathematical tasks, purposes of different ways of working and general learning principles. The message of the analysis of mathematical activity which I offer above is that sustained effort in this type of development must be pursued if we really want to see improvements in the learning of mathematics.

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