THE SIGNIFICANCE OF HISTORY AND OF NON-ABSOLUTIST PHILOSOPHIES IN MATHEMATICS EDUCATION

Franceso Speranza

University of Parma, Parma, Italy

Three problems about the role of history in education

1. Most scholars agree about the significance of history in education:. But how can we justify it and what kind of history is suitable?
2. Contrast Lakatos's (Kantian) aphorism:
"Philosophy of science without history of science is empty, history of science without philosophy of science is blind" (Lakatos 1971, p.102)
with Ernest's critique:

> Lakatos did not sufficiently establish the legitimacy of bringing the history of mathematics into the heart of his philosophy of mathematics (Ernest 1991, p. 39).

3. Current research in mathematics education in many countries involves a perspective that takes into account of history and philosophies of mathematics. But what are the reasons for such a phenomenon? What is the meaning of the interconnections between philosophy, history and education? In the next section, I give my views on the possible role of history in the philosophy of mathematics - or more generally of science. However, first of all, we must identify *what kind of philosophy of mathematics* could have an intersection with history and education.

Non-absolutist philosophies and the role of history: why philosophy without history is empty

Absolutist philosophies of mathematics (typically logicism and formalism) or of science aim at presenting mathematics or science in an aprioristic frame. These philosophies do not take into account the development of mathematics (and of science) at all its levels: they do not explain why and how scientists construct their theories. For instance, why is the idea of the limit fundamental for modern analysis? Why do we choose a certain axiomatic system for elementary geometry? What is the meaning of the structuralist organization of mathematics? Are there revolutions in mathematics? What is the role of mathematics in science?

Our philosophy of mathematics must have such an epistemology. It must take into account what *really* happened: the paradigm which includes 'case studies' is particularly meaningful. Thus we are naturally led on the historical ground.

Our philosophy of mathematics must also pay attention to the 'cultural dimension' of mathematics, and to the needs of mathematics education (Gonseth's *principe d'idoneité*). Perforce it must be a "non-absolutist philosophy".

Furthermore any two absolutist philosophies are normally incompatible; non-absolutist philosophies, on the other hand, can have many connections. Some fundamental ideas in mathematics and science can be traced back to the Galilean-Newtonian methodology, and more clearly to Gauss's and Lobacevskij's reflections about non-Euclidean geometry, which were developed by Riemann, Helmholtz, Mach, Clifford, Poincaré, Enriques, Bachelard, Gonseth, ...: nearly all these scientists-philosophers were deeply interested in the history of science.

For Enriques, science and history of science are inseparable:

A dynamic vision of science naturally leads on the ground of history. The rigid distinction among science and history of science is based on the idea of the latter as pure literary erudition." (Enriques and Chisini, 1915-18)

For Bachelard, the meaning of scientific concepts cannot dispense with the epistemological ruptures provoked by the *epistemological obstacles* that humanity must overcome in the construction of science - the science of past times is an unavoidable part of *our* science.

Lakatos's quasi-empiricism, the standard bearer of this philosophical tradition, gives a particular role to the connection between formal and informal theories. In this way we are led to introduce the temporal dimension in our epistemological analysis. Ernest's social constructivism links the development of individual knowledge with the social context - the "creative circle" (Ernest 1991, p. 85) develops also in time.

Indeed notable gnosiological (in particular, epistemological) problems can be traced back to antiquity and their 'solutions' depend on the development of science. For example:

a. The problem of universals: general ideas exist independently from concrete entities (Platonic realism), or are inherent to them (Aristotle), or are only *flatus vocis* (nominalism), or are (at least partially) free constructions of our minds (Ockham's conceptualism). A logicist is a realist; an "absolute" formalist is a nominalist; a non-absolutist philosopher is essentially a conceptualist.
b. The concept of space, which can be analyzed by means of some noteworthy contrapositions: for instance, it must be considered only as a system of relations among objects or geometrical figures, or else as an autonomous entity, which is prior, so to say, to its contents. Aristotle and Euclid are oriented towards the first conception, today the second is predominant.
c. The atomism: if we interpret every mathematical entity as a set (of numbers, of points, etc.), we bring a kind of atomism into the heart of mathematics.
d. The *querelle* of rationalism and empiricism: what is the reliable source of knowledge? After the contraposition of Greek philosophy, modern science brought about a strict synthesis of both trends. An understanding of these problems is strictly connected with history. However I find counter-evidence for the importance of history (in the philosophy of mathematics and science) in a tenet of Italian neo-idealists, perhaps the most anti-scientific philosophers. They generally gave a particular emphasis to history: but they denied the possibility of a historical perspective for the theory of knowledge:

> historical matter has nothing to do with gnosiology (...) that defines cognition apart from its historical process (Gentile quoted in Ciliberto 1981, p.160)
>
> ...categories are timeless ... it isn't even true that there is any progress in the history of philosophy, if this means a passage from a category to a higher one, or from an attempt to a higher one (Croce, quoted in Ciliberto 1981, p.160).

In contrast, the philosophy of science that we are studying maintains that a theory of knowledge must develop in connection with the development of science:

> We have no certainty which could be considered independent from every progress of knowledge. (Gonseth 1937)

Non-absolutist philosophies normally do not trace a sharp boundary between philosophy of mathematics and philosophy of science; indeed, a certain interest in some classical philosophical traditions is perceived to be useful (another reason for appreciating history).

Nevertheless some ideas of 'absolutist' philosophies could be considered. The reasons of a realist epistemology can lead us to Popper's 'third world': a relevant attempt at a compromise between realism and constructivism (but other ideas on this subject could be developed). A non-absolutist philosophy rejects any reduction of mathematics to a pure play of symbols. It does, however, take into account the formalist organization of a theory in order to distinguish between an informal argumentation and a formal reasoning, between the syntactic and the semantic levels, and as an introduction to the theory of formal languages and to theoretical computer science.

Why history without philosophy is blind

What kind of history is suitable for the philosophy and for the didactics of mathematics? Certainly we need reliable accounts of past events - a "history of facts". But this is not sufficient, we need also a "history of ideas" . The latter is sometimes regarded with suspicion by some historians, because of its conjectural character. I propose some reflection about this point based on the following:

a. Pre-Euclidean mathematics is known only by means of quotations of later authors; as is the case with pre-Platonic philosophy. But are our sources trustworthy?
b. Important ancient works, such as Euclid's *Elements*, have been philologically reconstructed only in recent times. This meant that intermediate authors had at their disposal different versions. What was the effect of these different accounts? And one ought to bear in mind that the meaning of important terms in ancient mathematics is a debatable issue (see, for example, Szabò 1978).
c. The idea of 'axiom' has radically changed: maybe from pre-Aristotelian to post-Aristotelian mathematics, certainly with the advent of non-Euclidean geometries, it is very different now in a logicist, formalist, or neo-empiricist environment.
d. A historian must choose what events, and *a fortiori* what relations amongst the events, are meaningful. S/he must have some criterion for her/his choices.
   Then history must have, at least partially, a theoretical base. It is noteworthy that some of the most eminent philosophers of science developed a 'philosophy of history of science'. Enriques has written:

> The history of (general) philosophy, if it aims at giving an useful assistance to the problems of knowledge, must be integrated with the history of science, such as it is conceived by the scholars pursuing at the development of thought, beyond the scientists' lives. (Enriques 1906, p. 44)
>
> It is necessary to *construct* the history of scientific thought (...). History also, as a physical theory, is built *a priori*, at least up to a certain point, with the possibility of changing our construction, when it doesn't correspond to texts, documents, proofs." (Enriques and De Santillana 1932, p. 9)

In their opinion, Parmenides was influenced by the trends in Greek mathematics after the 'crisis of incommensurable quantities'. Moreover, some passages could be interpreted as assertions of the ideal character of geometrical entities.

A history of science can then develop also by means of (Lakatosian, we could say) 'bold conjectures'. For Bachelard, history of science has a peculiar character: it must explain also the progress of science:

> The progress of scientific thought is demonstrable, is proved, its proof is even a pedagogical element, which is indispensable for the development of scientific culture. Progress is namely the very dynamic of culture and history of science must describe this dynamic. (Bachelard 1951, I, p1)

The dynamic nature of the history of science as perceived by Bachelard is related thus by Tiles:

> Bachelard (claims) that the form of history of science that is relevant to the philosophy of science (...) can be considered independently of the wider discussion of the nature and role of history in general. (The history of science) is not something that can be written once and for all. On this view science is in a continual dialogue with its own history. (Tiles 1984, p. 13-14).

Lakatos proposes a study of the development of science based on *rational reconstructions* of history, that is on history as it would be, if (in the philosopher's opinion) scientists would behave in a 'rational' way:

> In writing a historical case study, one should, I think, adopt the following procedure: (1) one gives a rational reconstruction; (2) one tries to compare this rational reconstruction with actual history and to criticize both one's rational reconstruction for lack of historicity and the actual history for lack of rationality. Thus an historical study must be preceded by a heuristic study: history of science without philosophy of science is blind. (Lakatos 1970, p. 52-53)

Further

> ...philosophy of science provides normative methodologies in terms of which historian reconstructs 'internal history' and then provides a rational explanation of the growth of objective knowledge (Lakatos 1971, p. 102)

What kind of philosophy can we recommend to a historian? Obviously, a historically-oriented one; a philosophy which is open to change its theses. In short, a non-absolutist philosophy.

Why history and philosophy are important for mathematics education

We can now try to give an answer to the questions about mathematics education, which were posed at the beginning of this paper.

The significance of history for mathematics education is in the philosophy of mathematics on which it is based: history is (or will be) a teaching of philosophy by means of concrete examples.

As to the significance of philosophy of mathematics for mathematical education, it is firstly important when we are confronted with an important decision where purely 'technical facts' do not provide any direction. For example, when we consider questions such as:

a. What type of geometry is suitable for pupils at a pre-formal stage?
b. How can mathematical methods interact with physics (for instance, infinitesimal calculus and mechanics)?
c. What approach to logic is advisable in different school phases?

Moreover, our way of thinking is influenced by implicit philosophies whose origin is in the history of mathematics and in its teaching. For instance, a form of 'Platonic' philosophy of knowledge (whether we really owe it to Plato or not, as some scholars doubt) and of Aristotelian methodology are often present in 'common mathematical sense'. Then a relevant task for philosophy and history is to bring to light these implicit philosophies, in order to construct a more rational frame for our knowledge.

On the other hand, mathematics education offers meaningful hints to philosophy of mathematics. For instance, the above-mentioned questions a), b), c) can be (and in my opinion are indeed) the starting-points for epistemological researches.Moreover, in the light of a non-absolutist philosophy, mathematics coan be an 'ideal laboratory' for the progress of a general theory of knowledge, as happened in Greece in the fifth and in the fourth centuries BC, or in Europe in the seventeenth century.

Cognitive psychology can also interact with epistemology, as Piaget showed (whatever opinion one has about his theory). The former needs a philosophical frame, but it can also give to the latter relevant insights: a theory of knowledge must take into account also the genetic approach. Clearly, all these insights naturally lead us on the ground of non-absolutist philosophies.

In this system of disciplines there is no hierarchy, no discipline has an *a priori* role in comparison with the others. Perhaps some will be surprised by these perspectives of interactions. Let me end by remarking that the interactions between mathematics, science and technology, which today appear so "natural", date back only to few centuries ago. Before the modern age, they were separate fields. I hope that in the near future the "virtuous circle" of disciplines will appear natural to all people.

References

Bachelard G., 1951, L'activité rationaliste de la physique contemporaine, P.U.F., Paris
Ciliberto M., 1982, Scienza, filosofia e politica: Enriques e il neo-idealismo italiano, in Pompeo Faracovi O. (ed.) Federigo Enriques, Approssimazione e verità, Belforte, Livorno 1982
Enriques, F., 1906, Problemi della scienza, Zanichelli, Bologna (Engl. transl. Problems of Science, Open Court, London-Chicago)
Enriques F., Chisini O, 1915-18, Lezioni sulla teoria geometrica delle equazioni e delle funzioni algebriche, Zanichelli, Bologna
Enriques F., De Santillana G, 1932, Storia del pensiero scientifico, Zanichelli, Bologna
Ernest P., 1991, The Philosophy of Mathematics Education, Falmer Press, London
Gonseth F., 1937, L'unité de la connaissance scientifique, in Actes du Congrès Descartes (IX Congrès Int. de Philosophie) Hermann, Paris
Kragh H., 1987, An Introduction to the Historiography of Science, Un. Chicago Press, Chicago
Lakatos I., 1970, Falsification and the Methodology of Scientific Research Programmes, in Phil. Papers, Cambridge U.P., Cambridge 1978, v. 1, p. 8-101
Lakatos I., 1971, History of Science and its Rational Reconstructions, in Phil. Papers, Cambridge U.P., Cambridge 1978, v.1, p. 102-138
Szabò A., 1978, Anfange der Griechische Mathematik, Akadèmiai Kiado, Budapest (Engl. transl. The Beginnings of Greek Mathematics, Reidel, Dordrecht)
Tiles M., 1984, Bachelard, Science and Objectivity, Cambridge U. P., Cambridge

---

Maintained by Pam Rosenthall
email comments and suggestions
Last Modified: 23rd October 1996