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ABSOLUTISM OR NOT ABSOLUTISM - WHAT DIFFERENCE DOES IT MAKE?

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Teachers of mathematics deal in the currency of mathematical knowledge, and as a result are likely to have epistemological opinions, views on the nature of mathematical knowledge. According to Vergnauld (1990):

> "The most commonly shared epistemology is the belief that mathematical activity consists in the discovery of timeless truths (Platonism) independent of culture, and that it is mainly a matter of logical reasoning" (page 28).

Ernest (1991) has presented a critique of this and related 'absolutist' perspectives, broadened to include all the views in which mathematics is perceived as a logical, deductive system, the:

> "realm of certain, unquestionable and objective knowledge" (page 3), which "consists of certain and unchallengeable truths" (page 7).

Drawing on many sources, Ernest shows that such perspectives can be and have been challenged, in favour of the "fallibilist" view that:

> "mathematical truth is fallible and corrigible and can never be regarded as beyond revision and correction" (page 18).

A teacher of mathematics can now in a sense choose between diametrically opposing views on the nature of mathematical knowledge. I intend in this article to explore the extent to which epistemological beliefs have pedagogical corollaries: if a teacher becomes persuaded to reject absolutism, would she teach differently? *Should* she teach differently?

Are there 'absolutist' and 'fallibilist' teaching styles?

There have not been extensive studies relating teaching approach to epistemological view (see Roulet 1992 for an exception). Nevertheless an association has grown up between beliefs about mathematical knowledge and particular characteristics of teaching, as follows:

Absolutist teaching		Fallibilist teaching
Behaviourist approach	1	Constructivist approach
Clear and coherent presentation	2	Self discovery
Pupil practice and exercises	3	Real world examples and problems
Emphasis on 'content'	4	Emphasis on process
Discouragement of discussion	5	Encouragement of discussion

The impression of a teaching style associated with an epistemological opinion on the nature of mathematics may have arisen from casual but widespread anecdotal evidence, or from the reflections of leading thinkers about the implications of beliefs, or both. However, any inclination of teachers with a particular epistemological view to teach in the ways listed does not mean that the teaching is somehow forced upon them by that opinion, even if argued through in a 'logical' way by theorists. There may be other hidden factors, such as the teachers' views on children and learning, which those with a similar approach to teaching share, based perhaps on the broader educational opinions of theorists operating within 'traditions' of shared values.

If the epistemological view is considered in isolation, the association is not forceful.

1. Even if every "behaviourist" is found in practice to have absolutist views, the denial of an absolute origin for mathematical truth

is still perfectly consistent with behaviourism, because fallible knowledge can still be useful. Insofar as the mathematics to be acquired consists of reliable and useful behaviours, the fallibility of mathematics may be seen to be irrelevant. Equally the idea that knowledge is constructed in the person can be readily accommodated to an absolutist view of mathematical knowledge. The perceived certainty of the truths does not in itself suggest how they are learned.

2. For similar reasons it would seem quite appropriate to present clearly and coherently those mathematical ideas which are deemed to be useful, even if they are thought fallible; and the praxis of self-discovery can be used in the hope of discovering absolute truths, in the way proposed by Constance Kamii (1986) for example.
3. The skills of mathematics can be considered important, even by those with a reluctance to consider its truths absolute and certain. Similarly, real world examples and problems can be seen as important applications of absolute truths, which should be taught alongside them.
4. Whatever one's views on the nature of mathematics, the content and the processes of mathematics are both integral to it. An emphasis on one over the other is surely a distortion of the perspective rather than a consequence of it.
5. The teacher who said "What is there to discuss? The facts can stand for themselves," might be presumed to be absolutist in his/her view. However, other absolutists, within the Piagetian tradition for example, actively value children's discussion in its role of promoting the resolution of 'cognitive conflict' in the "right" direction.

The association of teaching styles with philosophical beliefs can be exaggerated. There is no intrinsic reason why a teacher with an absolutist view of mathematics should not encourage children to articulate their perceptions or should not employ eliciting language to try to develop the child's own understanding of the problem, or why a teacher with fallibilistic views should not teach the children to utilise specific efficient processing methods and approaches.

"Fallibilistic teaching" may appeal as a blanket term to refer to styles of teaching which have been self consciously developed from a fallibilistic perspective, such as the quasi-empirical approach proposed by Lerman (1983) to stand in contrast to a Euclidean approach, or the constructivist approach described by Von Glaserfeld (1991) and his followers. It would be misleading, however, to suggest that the acceptance or rejection of absolutism as a view on the nature of mathematics in itself suggests a teaching style.

The absolutist view on mathematical ideas

The effect that an absolutist view or its rejection has on teaching is not, in my opinion, because a view on the nature of mathematics compels a particular teaching style, but because a view on the nature of knowledge suggests an approach to learning. A belief in mathematics as a deductive system with certain objective knowledge has as a corollary a view on the nature of knowledge in the person, and in particular the knowledge of the child, which affects how teaching is approached.

The most commonly shared view in this respect is that mathematical ideas are thought to exist as objects of some kind outside people. When someone is said to be having mathematical ideas it is taken to mean that they are participating in the truths of mathematics; that they are sharing in the greater reality. How such a thing occurs may be thought of in different ways: as a direct understanding of an ideal form, perhaps, or deriving truths by deductive means from 'self evident' premises, or in some other way. In each case, when the ideas of mathematics are taken to exist somehow outside the person, and learning mathematics means acquiring the ideas, coming to share the agreed and mutual understanding which is mathematics, I will refer to it as an 'absolutist' perspective on mathematical ideas. The 'absolutist' stance is that there are "right" ideas, which are learned, and "understanding" is synonymous with having "right" ideas.

In an 'absolutist' account of learning the word "concept" can be and is used both to refer to an aspect of mathematics itself and to refer to children's understanding. Given the indivisibility of "right" ideas, it is a consequence of the view that "the student either has that concept or not, there are no partial stages" (Lerman 1989 p220) and there is an "assumption that concepts are acquired all at once" (Ernest 1991 p240) which are then manifested in all applicable circumstances. Seeing concepts in such a binary fashion (off or on) can lead to a teacher with an absolutist view supposing that there can be no misunderstanding of mathematics, only failure to understand, and bring an approach to teaching which cannot respond to the child's perspective, can only repeat the "right" idea over and over in the hope and expectation that it will eventually "click".

The non-absolutist view on mathematical ideas

To reject an absolutist perspective on mathematical ideas is to say that:

> The individual construction of concepts and their relationship is personal and idiosyncratic, even if the outcome may be

shared competencies" (Ernest 1991 p241).

In this view mathematical ideas do not exist as objects outside people, but exist only in the thinking of individuals, which are not directly accessible to others. Understanding is a matter of adequacy in thinking, not correspondence to an external reference, as there is no external reference. "Adequacy" might be defined as ideas that constitute a conception that leads to correct conclusions, and so adequacy might arise from different ideas and different conceptions about the mathematical situations (the diagrams and symbols and words) with which the person is confronted. There can be variation between individuals in their ways of understanding a mathematical situation, and understanding related to a given aspect of mathematics may take different forms. As a result misunderstanding can be thought of as ideas that constitute an inadequate conception, one that leads to incorrect conclusions. Lack of understanding could then mean either:

a. confused ideas that do not amount to a conception, in that they do not lead to any conclusions, or
2. the absence of any ideas.

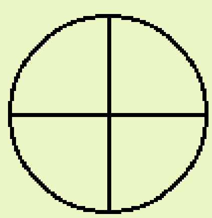
The view does not imply that mathematics is invented by each child, as there is a collection of the expressions of other people's conceptions available to act as a stimulus, and these external manifestations of conceptions largely define the subject of mathematics both at school and beyond - the various attempts, past and current, to come to adequate conceptions of a set of situations, expressed in the language of mathematics. However one of the core notions of this view is that these expressions of ideas are the only mathematical objects, and exist as symbol strings, not as ideas.

Describing children's understanding

The use of the term "concept" to refer to the object of teaching and learning makes immediate sense only from an absolutist perspective in which the concept word refers both to the child's idea and to the notion in mathematics. The rejection of mathematical objects in the sense of "right" ideas strips the term "concept" of its reference. It is then not meaningful to refer to "the concept of perimeter" when trying to characterise a child's understanding or one's intentions for it. At the very least we have to adjust it to "a conception of 'perimeter'", and mean "some notion (yet to be described) concerning the area of experience and language the understanding of which I refer to as 'perimeter' and which I take to be characteristic of the area of experience and language which other people also refer to as 'perimeter'". Labelling children's thinking with concept names suggests that we know how they are conceptualising the situations that face them, which implies absolutist assumptions. The non-absolutist will accept that we cannot ever fully understand a child's thinking in mathematics, any more than anywhere else, but try to get as much insight as possible about it, by using descriptive terms and metaphor.

For example, the following two samples are taken from children's work about fractional numbers:

When Richard (age 8) was shown this picture



and asked to say which of a range of fractional numbers it represented, he said
"It fits three tenths. Five, ten, fifteen, twenty, twenty five, thirty, and three tens are thirty."

Dayle (age 8) read 2/6 as "two sixes of a thing" and 4/8 as "four eights of a thing", and felt that 2/5 and 1/10 had the same value

> "because two fives is ten and one times ten is ten."

Thought of in concept terms, one can only say that neither child understands fractions, but it is clear that they were thinking about what was facing them, and coming to terms with it in their own way. Their ideas were not adequate, but they were not non-existent, and we might call them "misunderstandings" of the situation.

One might, for example, suggest that Richard saw the shaded disc as a clock face, and may have supposed that the numbers (3/6;

1/3; 3/10 and so on) were yet another new way of writing times, as he had been doing the previous day.

Dayle seemed to see the numbers in a way that suggested multiplication to him, perhaps because of the symbols used for multiplication in his classroom, or perhaps because of mis-hearing "two tenths" and its like in the past.

Whatever the reason, merely to say that each child does not have the concept of a fraction is to lose all the possible benefits that may accrue from trying to understand how they thought of the situation they faced, that might be used to help them develop an adequate conception of it.

Equally, when a child correctly describes situations involving fractional numbers it is very limiting to say "She understands fractions" and leave it at that. The implicit assumption that mathematical knowledge is general, acquired once and ever available in application, is part of the 'absolutist' perspective, and can lead to fragmented teaching and misleading record keeping. If I am to characterise a child's understanding within the area of 'fractions' helpfully I should consider her response to a wide range of challenges, not suppose that adequacy to one situation implies adequacy to others, or that there is one idea which she either possesses or not.

A broader approach is used already in other subjects. It would not be considered sufficient in the teaching of geography, for example, to say "She understands volcanoes" - rather one asks what the child understands about volcanoes, trying to get a handle on the way she conceptualises the forms, features and processes which are part of the study of volcanoes.

The teacher who rejects the absolutist perspective on children's ideas in mathematics will accept the children's conceptions in their own terms, and by listening to what children say and looking at what they do, find ways to refers to understanding other than by the use of concept words, and as a result may be better placed to promote understanding, to develop adequate thinking.

Teaching towards adequacy

In teaching that is geared towards adequacy the important question to ask of the children's mathematical ideas is whether they are sufficient to the demands and practices of the situations with which they are faced, rather than whether the children's ideas are the "right" ones. Different ideas around a mathematical topic can be accepted, and a rigid pursuit of one alternative need not prevail. The widespread assumption that mathematics has to be learnt correctly from the very beginning can be questioned, and a piecemeal approach to the acquisition of the "right" view, initially just the easy bits that even the younger child can cope with, can be challenged. Similarly the requirement for the "right" meaning of every word in the mathematics lessons, which prevents conversations from even occurring because of the teacher responding wholly on the basis of the intended meaning ("You do not say that, you say this"), can be loosened, and opportunities to develop the children's conceptions through natural language can be exploited.

Even in this view, however, there is still a case for the teacher trying to communicate her own conceptions. To try to communicate a more adequate alternative to the student's existing conception is part of teaching in all subjects. The efforts of the teacher to communicate her ideas and to persuade the children to adopt them are important mechanisms for developing better ideas in the children. The difference is that the teacher would offer her useful perceptions and conceptions, she would not impose them. She would say "Look at it like this because it is helpful to do so", not "Look at it like this because it is right." She would also be unlikely to suppose that anything that the child is asked to consider is "self-evident".

The challenge is not to let the notion of the "right" conception get in the way of learning: to accept that every child may have ideas which can be usefully described, not merely tagged as named concepts, and to develop every child's ideas to support reasonable mathematical practices. There will not be an attempt to transmit the "right" idea, but given that the ideas of the teacher will be more sophisticated than those of the child, an attempt by the teacher to communicate her own ideas may be helpful in improving the ideas of the children.

A possible effect of the rejection of an absolutist perspective on mathematical ideas on teacher attitude to mathematics teaching

One of the consequences of an absolutist conception of mathematical ideas can be an effect on the confidence of teachers. Those teachers who believe that there are "right" ideas and one's job is to teach them, yet do not think that they themselves have them or are not sure, do not feel authorised to act. "How can I teach them the concept if I do not have it myself?" A fear of not meeting the ideal, which arises directly from a view of what mathematics is and mathematical ideas are, de-skills the teacher, and tempts her to concede authority to others, who may be called 'mathematicians' and be deemed the "experts". The authors of published schemes, for example, have been thought of in this way, and thus charged with a responsibility that they are not in a position to fulfil.

To deny absolute truths as an ideal and to re-define the goal as improvement towards broader adequacy offers the possibility of the teacher recovering both her confidence and her responsibility for learning. Any teacher whose ideas are more adequate than those of the children is in a position to help the children in her charge. Her ideas do not have to be "right". She does not have to see herself as a mathematician; and rather than conceding responsibility to "experts" who are not in a position to act, she can accept the responsibility for her own ideas. The ideas of a teacher are more important than the ideas of the author of a text, even if the latter is a mathematician, because the teacher can build on her insights into the child's conception to connect the child to a new way of seeing the situation.

Of course one consequence of this is that there is an attendant responsibility on the teacher to work on her own ideas along with those of the children. However, this is already accepted in geography and science, and can become so in mathematics.

Absolutism or not

I believe that the rejection of an absolutist perspective on mathematical knowledge "in the person" is of greater significance to teaching than "fallibilism" as such, and can bring consequences to the classroom far beyond those attached to the denial of an absolutist perspective on mathematical knowledge "in the world". Indeed, when considering the nature of knowing rather than the nature of knowledge, 'absolutist' can no longer be contrasted with 'fallibilist'. It is not necessary to think of mathematics as unchallengable truth to consider mathematical ideas as objects which exist outside people. The fallibility of mathematical truth does not imply a particular view on how mathematics is understood. It is possible to adopt a fallibilistic view on the nature of mathematics while retaining an 'absolutist' view on the nature of mathematical ideas, saying perhaps that the "right" ideas are the socially agreed ideas. Roulet's (1992) fallibilist subjects, for example, speak of "the concepts being taught" with concepts "introduced and developed". A fallibilist may still conceptualise a body of knowledge in mathematics, consider it to exists outside human minds, and see it as the subject of teaching and learning, even if that knowledge is understood to be ultimately fallible.

The denial of the existence of an external reference point for ideas might bring about a very different kind of mathematics classroom, in which the teacher is assisting each child to develop a personal body of knowledge that is adequate to the situations of mathematics (including its applications) with which they are likely to be faced. Although it may be convenient to continue to categorise the 'body of knowledge' of mathematics in conventional ways, this will not then be interpreted as a description of target ideas. The list of topics for teaching would be those aspects for which the children's developing conceptions should become more adequate. The notion of a body of knowledge as concepts, facts or ideas would be replaced by (or re-conceptualised as) a set of represented situations, to a subset of which each child should be adequate in their thought and practice. Different subsets would characterise each school's mathematics, and decisions about what to include or not at each phase of education could be taken on pragmatic grounds, on the basis of anticipated practices. This might end the soul searching about what "really" counts as mathematics, which accompanies a notion of ideal conceptions towards which teaching is to be geared. Mathematics would perhaps lose some of its special flavour, and no longer be tainted by the tang of "right" ideas.

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email comments and suggestions
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