THE NATURE OF MATHEMATICS AND TEACHING

Paul Ernest

University of Exeter, UK

What is the relationship between conceptions of the nature of mathematics and teaching? This question, central to research in the philosophy of mathematics education, is the one that I address here. I wish to suggest that a strong relationship exists, linking the personal philosophies of mathematics of teachers to the experience of mathematics their students have in the classroom. This claim fits with the widely accepted view that "All mathematical pedagogy, even if scarcely coherent, rests on a philosophy of mathematics." (Thom 1973: 204). Other articulations stress both that teaching approaches in mathematics incorporate assumptions about the nature of mathematics, and that any philosophy of mathematics has classroom consequences (Hersh 1979, Steiner 1987). Empirical research (e.g. Cooney 1988) has confirmed claims that "teachers' views, beliefs and preferences about mathematics do influence their instructional practice" (Thompson 1984: 125). Thus it may be argued that any philosophy of mathematics (including personal philosophies) has many educational and pedagogical consequences when embodied in teachers' beliefs, curriculum developments, or assessment systems.

The claim I have made is a very simpleminded one, and it raises a number of issues and questions. What philosophies of mathematics are under discussion? Are there other contributing factors? What is the nature of the causal link that relates belief-systems to classroom experiences? What is the simplest model of the claimed relationship that accommodates sufficient complexity to be of use? Before I can elaborate this claim I need to clarify the issues raised in these questions.

Absolutist Philosophies, Images and Values

There is a range of perspectives in the philosophy of mathematics which can be termed 'absolutist'. These view mathematics as an objective, absolute, certain and incorrigible body of knowledge, which rests on the firm foundations of deductive logic. Among twentieth century perspectives in the philosophy of mathematics, Logicism, Formalism, and to some extent Intuitionism and Platonism, may be said to be absolutist in this way (Ernest 1991).

What must be emphasised is that absolutist philosophies of mathematics are not concerned to *describe* mathematics or mathematical knowledge. They are concerned with the epistemological project of providing rigorous systems to warrant mathematical knowledge absolutely (following the earlier crisis in the foundations of mathematics arising from the introduction of Cantor's infinite set theory). Many of the claims of absolutism in its various forms follow from its identification with rigid logical structure introduced for these epistemological purposes. Thus according to absolutism mathematical knowledge is timeless, although we may discover new theories and truths to add; it is superhuman and ahistorical, for the history of mathematics is irrelevant to the nature and justification of mathematical knowledge; it is pure isolated knowledge, which happens to be useful because of its universal validity; it is value-free and culture-free, for the same reason.

The outcome is therefore a philosophically sanctioned image of mathematics as rigid, fixed, logical, absolute, inhuman, cold, objective, pure, abstract, remote and ultra-rational. Is it a coincidence that this image coincides with the widespread public image of mathematics as difficult, cold, abstract, theoretical, ultra-rational, but important and largely masculine? Mathematics also has the image of being remote and inaccessible to all but a few super-intelligent beings with 'mathematical minds'.

A second coincidence is the parallel with Gilligan's (1982) definition of 'separated' values The 'separated' position valorises rules, abstraction, objectification, impersonality, unfeelingness, dispassionate reason and analysis, and tends to be atomistic and thing-centred in focus. These are claimed to be the stereotypical masculine values in western Anglophone countries.

An absolutist view may be communicated in school by giving students mainly unrelated routine mathematical tasks which involve the application of learnt procedures, and by stressing that every task has a unique, fixed and objectively right answer, coupled with disapproval and criticism of any failure to achieve this answer. This may not be what the philosopher recognises as a philosophy of mathematics or the mathematician recognises as mathematics, but a result is nevertheless an absolutist conception of the subject (Buerk 1982). In some cases the outcome is also mathephobia (Buxton 1981).

Fallibilist Philosophies, Images and Values

A more recent position in the philosophy of mathematics is fallibilism, which emphasises the practice of mathematics and the human side of mathematics. Fallibilism views mathematics as the outcome of social processes. Mathematical knowledge is understood to be fallible and eternally open to revision, both in terms of its proofs and its concepts (Lakatos 1976). Consequently this view embraces as legitimate philosophical concerns the practices of mathematicians, its history and applications, the place of mathematics in human culture, including issues of values and education - in short - it fully admits the human face and basis of mathematics (Davis and Hersh 1980).

Fallibilism rejects the absolutist image of mathematics described above as a misrepresentation. It claims instead that mathematics has both a front and a back (Hersh 1988). In the front, the public are served perfect mathematical dishes, like in a gourmet restaurant. Here the impression of absolute mathematics is preserved, but in the back, mathematicians cook up new knowledge amid mess, chaos and all the inescapably associated features of human striving. Fallibilism admits both of these realms: the processes and the products of mathematics need to be considered an essential part of the discipline. For accuracy the false image of perfection must be dropped (Davis 1972).

One of the innovations associated with a fallibilist view of mathematics is a reconceptualised view of the nature of mathematics. It is no longer seen as defined by a body of pure and abstract knowledge which exists in a superhuman, objective realm (Tymoczko 1986). Instead mathematics is associated with sets of social practices, each with its history, persons, institutions and social locations, symbolic forms, purposes and power relations. Thus academic research mathematics is one such practice (or rather a multiplicity of shifting, interconnected practices). Likewise each of ethnomathematics and school mathematics is a distinct set of such practices. They are intimately bound up together, because the symbolic productions of one practice is recontextualised and reproduced in another (Dowling 1988).

Coinciding at least in part with the fallibilist philosophy of mathematics is the vital image of mathematics communicated in many progressive schools and colleges. Mathematics is experienced as warm, human, personal, intuitive, active, collaborative, creative, investigational, cultural, historical, living, related to human situations, enjoyable, full of joy, wonder, and beauty. Influential inquiries into the teaching of mathematics have propounded humanised views of school mathematics incorporating some of these dimensions (Cockcroft 1982, NCTM 1980, 1989, NCC 1989). The weight of informed educational opinion has likewise supported the progressive reform of mathematics in line with such views.

A second coincidence is with the set of values which Gilligan (1982) terms ‘connected’. The ‘connected’ position is based on and valorises relationships, connections, empathy, caring, feelings and intuition, and tends to be holistic and human-centred in its concerns. There are meant to be stereotypically feminine values.

Philosophies of Mathematics, Values and their Relation to Teaching

Drawing together the disparate threads in the above account it is clear that there is first of all, a strong parallel between the absolutist conception of mathematics, the negative popular view of mathematics, and the set of values described by Gilligan (1982) as “separated”. Likewise, a second parallel exists between the fallibilist conception of mathematics, the connected values described by Gilligan (1982) and the humanistic image of mathematics promoted by modern progressive mathematics education as accessible, personally relevant and creative (Cockcroft 1982, NCTM 1989).

The second parallel can be used to improve accessibility and to transform the public and school images of mathematics. But the analysis cannot stop here. For the absolutist image of mathematics is precisely what attracts some persons to it. In my research on student teacher’s attitudes and beliefs about mathematics I found a subgroup of mathematics specialists who combined absolutist conceptions of the subject with very positive attitudes to mathematics and its teaching. However amongst non-mathematics-specialist future primary school teachers I found a loose correlation between fallibilist conceptions and positive attitudes to mathematics and its teaching (Ernest 1988, 1989b). Thus the connections even just between beliefs and attitudes to mathematics are complex and multifaceted.

Many mathematicians also love mathematics just for its absolutist features. It is both consistent and common for teachers and mathematicians to hold an absolutist view of mathematics as neutral and value free, but to regard mathematics teaching as necessitating the adoption of humanistic, connected values. This raises the problem of how the relationship between philosophies of mathematics, values and teaching can be modelled.

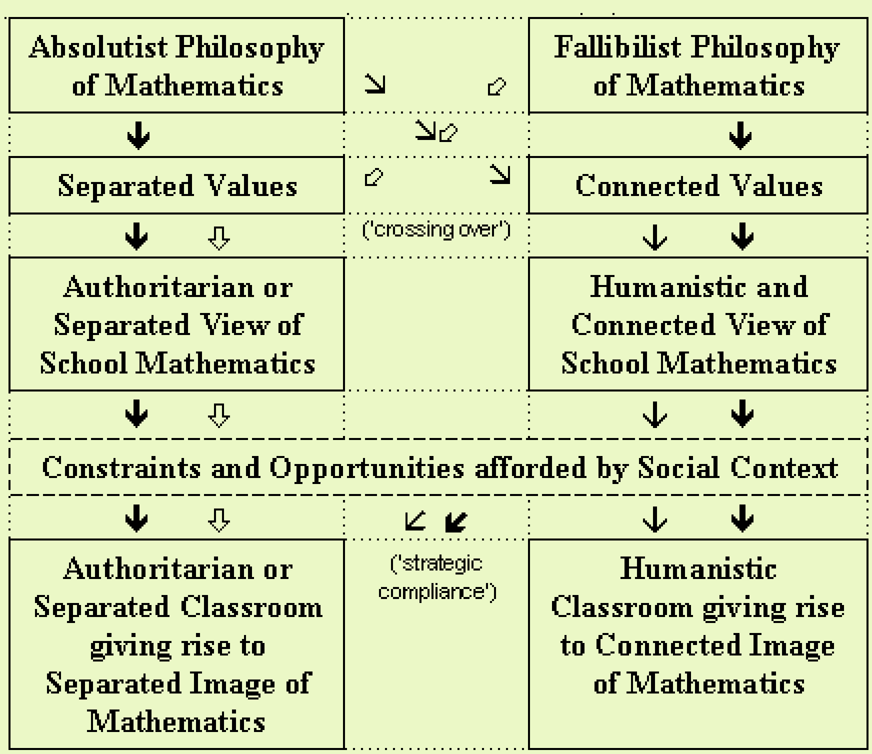
Elsewhere I have argued that teachers' personal philosophies of mathematics, understood as part of their overall epistemological and ethical framework, impact on their espoused conceptions of teaching and learning mathematics. These in turn, subject to the constraints and opportunities of the social context of practice, give rise to the realised theories of learning mathematics, teaching mathematics, and the related use of mathematical texts and curriculum materials in the classroom (Ernest 1989a). Such a model is partially validated by empirical work (Ernest and Greenland 1990).

However classroom consequences are not in general strictly logical implications of a philosophy, and additional values, aims and other assumptions are required to reach such conclusions (Ernest 1991, 1994). Because the link is not one of logical implication, it is theoretically possible to consistently associate a philosophy of mathematics with almost any educational practice or approach. Both a neo-behaviourist or cognitivist (such as Ausubel 1968) and a radical constructivist (such as Glasersfeld 1995) may be concerned to ascertain what a child knows before commencing teaching, despite having diametrically opposite epistemologies (absolutist and fallibilist, respectively). Likewise a traditional purist mathematician and a social constructivist (fallibilist) may both favour a humanised, multicultural approach to mathematics, but for different reasons (the former perhaps to humanise mathematics, the latter to show it as the social construction of all of humanity and for reasons of social justice).

Although there is no logical necessity for, say, a transmission-style pedagogy to be associated with an absolutist, objectivist epistemology and philosophy of mathematics, such associations often are the case (Ernest 1988, 1991). This is presumably due to the resonances and sympathies between different aspects of a person's philosophy, ideology, values and belief-systems. These form links and associations and become restructured in moves towards maximum coherence and consistency, and ultimately towards integration of the personality. (Of course psychological compartmentalisation and 'splitting' are also possible.)

A simplified model may be conjectured which suggests that the value-position of a teacher, curriculum development or school plays a vital role in mediating between personal philosophies of mathematics, and the image of mathematics communicated in the classroom. Figure 1 shows how an absolutist philosophy of mathematics combined with separated, thing-centred or authoritarian values can give rise to an authoritarian or separated view of school mathematics.

Figure 1: Simplified Model of Relationship between Personal Philosophies of Mathematics, Values and Classroom Image of Mathematics.



This, subject to the constraints and opportunities afforded by social context of schooling, often results in a traditional classroom projecting a separated image of mathematics, or, more radically, in an authoritarian classroom also communicating a separated image of mathematics. (The disjunction indicates the non-identity of authoritarianism and separatedness in values, belief systems, and in the classroom,). Similarly, a fallibilist philosophy of mathematics combined with connected, person-centred and humanistic values can give rise to a humanistic connected view of school mathematics. Subject to the same constraints, this can result in a humanistic classroom communicating a connected image of mathematics. These two possible sets of relations are shown by bold vertical arrows. They represent the most straightforward relationships between philosophies, values and classroom images of mathematics.

The figure also illustrates ‘crossing over’: how an absolutist philosophy of mathematics if combined with connected, humanistic values can give rise to a humanistic view of school mathematics. This, subject to social constraints, may be realised as a connected classroom image of mathematics. A deep commitment to the ideals of progressive mathematics education can and frequently does co-exist with a belief in the objectivity and neutrality of mathematics, especially amongst mathematics teachers and educators. Fallibilism has no monopoly on this. In cases like these, connected values are often associated with education and the conception of school mathematics, rather than with academic (mathematicians’) mathematics. This is illustrated in the figure by the thin black arrows. It is theoretically possible that a fallibilist philosophy of mathematics can combine with separated values, resulting in an authoritarian view of school mathematics. This, subject to contextual constraints and opportunities, can give rise to a separated image of mathematics realised in the classroom. This is shown by the outline arrows, and is probably infrequent, because of the common association of fallibilism with progressive pedagogical views in the mathematics education community.

Finally, it has been observed that the various constraints of the social context of schooling can be so powerful that a teacher with humanistic values and a humanistic view of school mathematics is nevertheless forced into ‘strategic compliance’ (Lacey 1977), so that the image of mathematics realised in classroom is a separated one. This may be a temporary consequence of contextual constraints, but if permanent leads to tensions and stress. This state of affairs is indicated in the diagram by the bold and thin arrows deviating left towards a separated classroom image under the impact of the social context. These arrows may originate with either an absolutist philosophy (thin arrows), or a fallibilist philosophy (bold arrows), but in both cases cross over. Empirical research has confirmed that a number of teachers with very distinct personal philosophies of mathematics (absolutist and fallibilist) have been constrained by the social context of schooling to teach in a traditional, separated way (Lerman 1986).

Much work in the philosophy of mathematics education pertains to exploring the link between the philosophies of mathematics implicit in teachers' beliefs, in texts and the mathematics curriculum, in systems and practices of mathematical assessment and in mathematics classroom practices and ethos, and the results with learners. There is a growing literature base, especially on teachers beliefs, images of mathematics and related attitudes (Ernest 1988, 1989a, b, 1991, Hoyles 1992, Lerman 1986). A few researchers have looked at students’ images too (Hoyles 1982, Kouba and McDonald 1991). Whilst much progress has been made, much work remains to be done in the area, and it is clear that the relationships are complex and non-deterministic.

Conclusion

I have sketched three sets of dichotomies: absolutist versus fallibilist philosophies of mathematics, separated versus connected sets of values, mathematics classrooms promoting separated versus connected images of mathematics. There have been two main threads of argument. First, that these dichotomies are all to some extent parallel, so that teachers’ absolutist philosophies of mathematics, separated values, and mathematics classrooms with a separated image of mathematics all work together and reinforce each other. The widespread negative public images of mathematics is presumably the outcome of what I have termed a separated classroom image of mathematics. Teachers’ fallibilist philosophies of mathematics, connected values, and classrooms promoting a connected image of mathematics are linked and mutually reinforcing. Ignoring the mediating factors, this analysis suggests that if we want to change a widespread negative public image of mathematics then we must change the image communicated by the mathematics classroom. This is an unsurprising but important conclusion, which might help to redress some of the gender-related problems for mathematics, and perhaps also for science and technology, remarked in the literature (Walkerdine et al. 1989, Burton 1986) .

The second thread of my argument cuts across any simplistic understanding and acceptance of the first. It appears that the values (plus the conception of school mathematics) rather than the philosophy of mathematics embodied in the teacher or classroom are the dominant factors in determining the image of mathematics communicated in the classroom and hence ultimately the public image of mathematics. Perhaps this is not surprising, since the realisation of such values will embody the type of teacher-pupil relations, the degree of competitiveness, the extent of negative weight placed on errors, the degree of public humiliation experienced in consequence of failure, and other such factors which powerfully impact on the young learners self-esteem and self-concept as a learners of mathematics.

Of course, like any analysis made in terms of dichotomies, the above account is oversimplified, perhaps even simplistic. Human beings do not fall neatly into two boxes. In my analysis of mathematics curriculum ideologies (Ernest 1991) I suggested 5 basic ideological types and distinguished over a dozen components for each ideology, and that model was a simplification. Nevertheless, such simplified models can suggest the way that important theoretical factors impact on the teaching and learning of mathematics and suggest ways forward in terms of both practice and research (Ernest and Greenland 1990).

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