SKOVSMOSE, O. TOWARDS A PHILOSOPHY OF CRITICAL MATHEMATICS EDUCATION, DORDRECHT: KLUWER, 1994.

Reviewed by
Paul Ernest
University of Exeter

This is a very important book for the international mathematics education community, and beyond, and deserves an in-depth critical review. This is what I attempt to offer below, albeit, inevitably, from my own perspective.

Mathematics is generally agreed to fulfil social needs, to provide the skills relevant to everyday life and work in industrial and developing societies, as well as the basis for further study in mathematics, science and technology. The selection of content and the mode of teaching in mathematics are often claimed to be driven by relevance to these needs. This much is widely agreed. However, what is often overlooked, perhaps less so today, is that 'relevance' and 'need' are not neutral, objective judgements, but are based on the perspective of the judge, and the aims at which the judgement is directed. Any such judgement, however much integrity is involved, is determined by what the maker of judgements considers to be appropriate or right.

Naive utilitarian judgements are often (mistakenly) perceived to be beyond ideology. In contrast a radical view of the relevance of mathematics, and consequently of the aims of mathematics teaching, is that it should foster critical mathematical literacy and empower students to become a critical citizens in modern society. Critical mathematical literacy involves a number of components that need to be unpacked. First, it necessitates having a sound knowledge and understanding of a significant subset of school mathematics. Second it involves the confident possession of the process skills of applying mathematical knowledge independently to pose and solve problems in a wide variety of contexts; in finding out any missing facts or acquiring any skills needed in the process; and in evaluating the solutions critically. Third, it necessitates the ability to identify, interpret, evaluate and critique the mathematics embedded in social and political systems and claims, from advertisements and government pronouncements to the means of decision-making and the distribution of resources employed in society.

The goal of critical mathematical literacy is the empowerment of learners both as individuals and as citizens-in-society. This is achieved by developing mathematical power both to overcome barriers to higher education and employment and thereby to increase economic self-determination; and to foster critical awareness and democratic citizenship via mathematics. Ultimately, the aim is social change via the empowerment of the citizenry towards participating more fully in and subsequently working towards a more just and democratic society.

These aims concern more than the content of the mathematics curriculum, or even the problems addressed. They also necessitate a reformed pedagogy which in my view requires the use of a questioning and decision making learning style in the classroom. The pedagogy must include discussion, permitted conflict of opinions and views, the challenging of the teacher, the questioning of content and the negotiation of shared goals. The learners should pose their own problems and initiate their own projects, at least some of the time. Learning materials should include socially relevant projects, authentic social statistics, accommodate social and cultural diversity, and use local cultural resources.

To some, this discussion of the politics of mathematics education will ring alarm bells. Is such an approach in danger of becoming a propagandising and political misdirection of the young? Is this at risk of resulting in the imposition of a regime of 'political correctness' in mathematics? As someone who has taught contentious issues in the mathematics classroom and lecture hall let me suggest that the contrary is the outcome. Any attempt to impose a politically or ideologically slanted view invariably results in dispute and heated argument rather than in passive acceptance. I have long known that the best way to teach anti-racist and anti-sexist mathematics is to adopt a racist and sexist position (which I cannot bring myself to do), to tap into both a sense of justice and resistance to authority. I am certain that the freethinking youth of today, unless oppressed by an authoritarian regime in society or in the classroom, cannot easily be propagandised by any approach that encourages discussion and dissent.

Such undeniably political aims for mathematics teaching, coupled with contentious social issues and teaching styles in the classroom,

are clearly controversial. To date I know of no country where such aims and approaches have had full official sanction, because they involves a critical stance towards the social *status quo*, whatever the political persuasion of the government. Thus during the years when the British government introduced the National Curriculum, whilst legislating for a more utilitarian school mathematics it simultaneously asserted the irrelevance of any political or ideological aspect to mathematics. Margaret Thatcher made her opposition (and incomprehension) of critical numeracy and literacy clear in her address to the Conservative Party Conference, in October 1987, claiming that "children who needed to count and multiply were learning anti-racist mathematics - whatever that might be. Children who needed to be able to express themselves in clear English were being taught political slogans."

Ironically, at the very same time the Danish government was agreeing to fund a research initiative on 'Mathematics Education and Democracy in Highly Technological Societies'. The aim of this project was to explore the political significance of mathematics in schooling, and to use it as a vehicle for critical thinking and for the furtherance of democracy and social justice. The major fruit of this initiative, or at least that which is most visible to an international audience, is the present book.

'Towards a Philosophy of Critical Mathematics Education' is an important book. It makes the case that in advanced technological societies the ability to critique the uses of mathematics in supporting and rendering invisible the ideologies of the controlling (and other) interest groups is a vital skill, if democracy is to be preserved. A central part of Skovsmose's argument concerns the 'formatting power' of mathematics. He argues that in a complex technological society much of our lives are structured by our interactions with technology and related social systems. These in turn hinge on 'realised abstractions' (theoretical concepts and systems) - often largely mathematical - which are invisibly (or less often visibly) woven into the fabric of the technological and social systems that structure and control our lives. The regulation of our lives by measured time and timetables, by tax laws, and by computer forecasting models, for example, illustrates such formatting power. Thus to be in control of our lives and our societies - via voting and the exercise of democratic rights - necessitates being critically aware of and having power over the realised abstractions of mathematics as it exercises its formatting power. 'Realised abstractions' are the social manifestations of mathematical ideas, which he calls 'thinking abstractions'. As these notions indicate, he introduces a range of theoretical notions in his analysis and critical discussion.

In his critique of technological society and the role of mathematics within it, Skovsmose draws primarily on the critical theory of the Frankfurt School. This includes, most notably, the works of Marcuse, Adorno and Habermas, although he also draws on other philosophers and critics of technology, such as Ellul. His educational sources 'begin' with Adorno's 1966 article 'Education after Auschwitz'. He also cites a substantial body of work by German and Scandinavian scholars on critical approaches to mathematics education in the 1970s which is virtually unknown in the English speaking world. In addition, he draws upon Freire and Giroux. He acknowledges some existing work on critical mathematics education, but mostly in footnotes as he works out his own argument independently.

One of Skovsmose's key claims is that to learn mathematics critically, students need to engage in mathematics-based projects which focus on its social applications. Here arises one of the first dilemmas of the book. In chapter 4 (pages 62-73) he describes a school-based project undertaken by two co-operating teachers with their classes of schoolchildren (ages not specified, but presumably middle-school-aged). The project concerned 'economic relationships in the world of the child', with three topics in twelve separate units exploring:

1. pocket money,
2. child benefit allowance, and
3. the costs of equipping a youth club.

This project activity was apparently a great success, as a progressive teaching approach. But from the perspective of critical mathematics education, only one unit appears to pose questions that critique the economic, social or political status quo. Unit 3 covers shopping costs, pocket money, and asks "What is a reasonable salary? Is it right that some people earn more money than other people? What are the reasons for differences in salary?" This limited focus on such questions raises a question. What is the difference between *practical or utilitarian mathematics*, taught by means of projects to help students to apply their skills to social and 'real world' situations, and *critical mathematical activity*? The former, it might be argued, helps prepare students as consumers and perhaps workers, rather than as critical citizens. The latter should be able to scrutinise and challenge published 'mathematical facts', rather than just make 'wise buys'. Skovsmose himself asks to what extent the project relates to democratic competence, critical mathematical literacy and critical citizenship His answer, drawing on the work of Wagenschein and others, is that exemplarity is needed to introduce the appropriate themes embodied in the particular. But it would seem to me that beyond this, critical reflection and discussion of the particular cases drawn for study is needed. This would seem to me makes the difference between critical and

non-critical education. Judging from the account of the project, there was insufficient discussion and critical reflection to properly exploit the project work in this way.

Two more projects which are described in Chapter 5 are called ‘Golfpark’ and ‘Constructions’. The first concerns a piece of open land next to a school in Aalborg, Denmark. The students of the school were asked to investigate the piece of land and to consider ways of developing it to the benefit of the community and without detriment to the environment. The student investigations went beyond the piece of land but returned to the issue of how best to develop it. In making models, imagining alternatives which were sometimes tenuously linked to the starting problem (this was a project that particularly exploited imaginative thought) students used mathematical tools and skills, but often without awareness that they were doing mathematics. Likewise in the other project ‘Constructions’ the students built structures and used technological devices such as Control Lego and the computer language Logo. But at the end of the project the question can be posed (and is posed by Skovsmose and the teachers involved in the projects): To what extent did the students learn mathematics?

In recognition of the fact that the mathematical concepts, knowledge, and skills employed in the projects were not made explicit and overtly developed or reflected upon, Skovsmose introduces the theoretical idea of ‘mathematical archaeology’. This is the action of ‘digging out’, identifying and discussing the actual use made of mathematics, which becomes hidden by merging invisibly with the overt subject matter. This is an important point, and a telling lesson for ‘real problem solving’, applications and modelling which have been fashionable in school mathematics for the past decade or so. For many instantiations of these latter approaches happily claim to teach mathematics in context, but never make explicit what is the mathematics content in an application. Psychological research on the transfer of learning is quite clear. Unless what is learnt is made explicit, the chances of it being transferred and applied in a new context by the learner are remote. So there is an important lesson which emerges from Skovsmose’s analysis of the weaknesses of these projects, and from his idea of a mathematical archaeology. However a further question may be posed. To what extent is the mathematics education he proposes a *critical* mathematics education? Granted that being aware of the mathematics in a project is a more effective way of learning applied mathematics, to what extent does it help democracy? Being able to identify mathematics in contexts would seem to me to be a necessary but not sufficient skill to qualify as critical mathematical literacy.

Skovsmose indicates his awareness of this problem by following his chapters on the projects mentioned by a chapter (number 6) on ‘Reflective Knowing’. Here he distinguishes three competencies which together compose critical mathematical literacy, which he terms mathemacy. These are mathematical competence (mathematical knowledge and skill at all levels), technological competence (technology knowledge and know-how, especially in the application of mathematics) and reflective knowing. This is first defined in terms of the ability to evaluate and discuss critically the social and ethical aspects of the aims and consequences of technological plans and activities. Subsequently it is elaborated in terms of knowing (as opposed to knowledge) directed at the problems and uncertainties in the mathematical modelling process and in particular those issues hidden by the technical language of such activity (the Wittgensteinian ‘language games’ involved in modelling, as he puts it).

Having critiqued the projects he illustrated, Skovsmose goes on to describe further projects ‘Family support in a micro-society’, ‘Our community’ and ‘Energy’. In the first project, students devised different ways of distributing family income supplements to 24 families for whom they had written cameos. This activity raised the issue of the algorithms or rules for the distribution of support, and the reconciliation of the principles according to which the support was distributed and what was technically (algorithmically) feasible. Because different groups of children devised different distributions, their discussions and dialogue forced the children to justify their own group’s approaches and critically engage with those of other groups. This factor was vital in raising the discussion from the technical level to that of reflective knowing.

The ‘Energy’ project involved a class of 15 year olds who addressed a number of practical problems such as the energy values and costs of foodstuffs, physical activity, farming (both pig-rearing and barley growing), energy in the home, and global energy issues. Their investigations included farm visits and the collection of energy data in their homes. They used formulas, graphs and other mathematical means. In discussing this project, Skovsmose goes on to describe how the mathematical competence, technological competence and reflective knowing are independent and how some of these can be exercised or be present in the absence of the others. This discussion illustrates the way he has used his examples throughout: to illuminate, critique and develop his developing notion of mathemacy.

In the final two chapters Skovsmose reflects on the notions of critical mathematics education and knowledge that he develops in the book. Building on his introductory discussion of the contributory concepts of ‘crisis’, ‘critique’ and ‘emancipation’ he adds the dimension of reflexivity. Critical mathematics education requires that students are intentionally involved in their ‘learning activities’. Adding the goal of epistemic development, he argues that this can only take place (in the desired way) when the learner is consciously

pursuing her or his own goals in the process. Discarding (externally) directed and blind activity, Skovsmose argues that mathemacy requires the pursuit of self-directed activity. This independence is reflected in being able to make well-grounded independent judgements and to do so with authority; to provide adequate justification for them. This involves knowledge, and Skovsmose completes his treatment with a chapter on knowledge and knowing. Here he criticises foundationalist accounts of knowledge as absolutely grounded bodies of truth. Critique, which Kant wielded in a once and for all attempt to clear the ground for knowledge (like the doubt wielded by Descartes, although he does not mention this example) cannot be put to rest. Instead, he claims, a fallibilist view of epistemology must keep critique alive alongside an open concept of knowledge.

Skovsmose offers his own take on epistemology. Building on the theory of speech-acts, he argues (something like) that to claim to know a proposition P legitimately is to promise to be able to justify P, and to have grounds which can be exhibited in so doing. The proposal is interesting and worthwhile, but some aspects of the formulation have a subjectivist ring, when something more interpersonal is intended. I will leave readers to make up there own minds about the force of these epistemological proposals.

I have one final criticism of the book. It neglects a great deal of literature that could have been used to buttress or parallel the arguments. First of all, as I mentioned above, there is an extensive mathematics education literature on critical mathematics education and on its political dimensions. This could have been used to extend some the arguments and to provide examples of projects outside of Denmark. This would locate the publication more firmly in international thought in the area. There is a great deal of work on problem posing, investigatory work, mathematical modelling, gender, race, ability, class and mathematics education, ethnomathematics, history and philosophy of mathematics and its bearing on mathematics education which is relevant here but which is all but omitted. More indications of the connections, links and parallels would have strengthened the monograph, and resulted in a more definitive work. Second, emerging both from critical theory and the sociology of knowledge there has been a major growth of work in social studies of science and in science, technology and society. This too could provide a link if not an underpinning theoretical framework for this project. There is little indication of the existence of this literature. Lastly, one could also argue that there is a neglect of the philosophy of education, in which issues of aims, democracy and citizenship have long been discussed.

But this criticism is perhaps one of style or genre. Skovsmose cleaves to the older philosophical style of only citing those authors that play a major role in developing the ideas, with an occasional indication of further examples of parallel studies. He makes no attempt to be all-encompassing or encyclopaedic as modern scholarship often strives to be. Perhaps this should not be signalled as a weakness.

Overall, Skovsmose does the educational community a valuable service in publishing this book. It treats a very important area of education. It simultaneously recounts his reflective thoughts about the nature of critical mathematical literacy and the role of mathematics in democratic education, and exemplifies concrete projects carried out with school children with the aim of achieving his theoretical goals. It is not only an exposition of critical education, but a self-critical inquiry, as I indicated above. In the monograph the author extends the theoretical basis for a critical philosophy of mathematics education, as the book title promises. He also indicates some parameters of what is possible, and in doing so exemplifies how mathematics can contribute to education for global awareness, economic understanding, critical citizenship, and other cross-curricular themes, without compromising his vision of a just society. In making this contribution he is part of a characteristic Nordic tradition of research in mathematics education, which includes others such as M. Niss and S. Mellin-Olsen. This tradition emphasises the political dimensions of mathematics education, especially critical citizenship and critical mathematics education, with an emphasis on social justice issues communicated in an unstrident and unselfconscious way not always achieved by Anglophone researchers. In the present troubled times in which a market-forces ideology is forcing a narrowly utilitarian education on the schoolchildren of Britain, this book and its concern with values and justice is like a breath of fresh air. It rejects the old progressive-utilitarian division, and offers instead a different vision of what ‘relevance’ can mean in education.

Maintained by Pam Rosenthall
email comments and suggestions
Last Modified: 5th November 1996