PHILOSOPHY OF MATHEMATICS EDUCATION NEWSLETTER 8

This Newsletter is the publication of the

PHILOSOPHY OF MATHEMATICS EDUCATION NETWORK

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AIMS OF THE NEWSLETTER

The aims of this newsletter are: to foster awareness of philosophical aspects of mathematics education and mathematics, understood broadly to include most theoretical reflection; to disseminate news of events and new thinking in these topics to interested persons; and to encourage *informal* communication, dialogue and international co-operation between teachers, scholars and others engaged in such research or thought.

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COMMUNICATIONS

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Please send any items for inclusion to him including a copy on disc or E-mail. These may include short contributions, discussions, provocations, reactions, notices of research groups, conferences, publications, and also books for review.

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FUTURE OF THIS NEWSLETTER

In the future this newsletter will be published via the Internet. See the editor's homepage at: **http://www.ex.ac.uk/~PErnest/**

This carries all editions of the newsletter (minus a few missing bits of early numbers). It is also planned to include a file of reactions, comments, discussion, etc. to be accessed via this homepage so please send anything you would like to say or add to PErnest@ex.ac.uk. Colleagues unable to access PoMEnews via the Internet are requested to write to the editor enclosing self-adhesive (international) address label(s). The subscription for such persons will be UK£5 per issue (Cheque to P Ernest), but free to colleagues in East Europe, ex-USSR, and in developing countries. No mailing list will be maintained and you will need to request each desired issue.

EDITORIAL: CRITICISM AND THE GROWTH OF KNOWLEDGE

The title of this editorial is taken from the eponymous book edited by I. Lakatos and A. Musgrave (CUP, 1970). That book represents the written version of a planned dialogue between Thomas Kuhn and Karl Popper at the Bedford College seminar in 1965, at which Lakatos was secretary. Three other edited volumes (Lakatos editor) preceded this book's final publication in 1970, treating symposia on problems in inductive logic, philosophy of mathematics and the philosophy of science. Each of the volumes embodied the dialogical format of the conference and the dialectical theory of knowledge genesis and justification which is central to Lakatos' philosophy of mathematics (and science).

Conversation or dialogue is increasingly a central metaphor for inquiry, knowledge growth, and even of mind, as the works of Bakhtin, Gergen, Habermas, Harré, Lakatos, Mead, Rorty, Shotter, Volosinov, Vygotsky, Wertsch, Wittgenstein and other thinkers show. But of course philosophical dialogue already reached a near timeless form in the work of Plato. If we adopt dialectical reasoning (understood broadly, not strictly Hegelian) as providing a model of conversation there three moments forming a cycle: Thesis Antithesis Synthesis (the new Thesis). The 'positive' phase of Thesis concerns a knowledge proposal in its first form relative to this cycle, but of course all knowledge claims are embedded in personal and cultural history, and come out of the never-ending conversation of humankind. The 'negative' phase of Antithesis concerns a critical scrutiny of the knowledge proposal; its weaknesses outlined and maybe a counterproposal or progress to a resolution implied. Completing the cycle, a new 'positive' phase of Synthesis concerns a reformed knowledge proposal, which has been strengthened by overcoming and accommodating critique, and which now both embodies new knowledge claims and is partly or wholly warranted by the appropriate knowledge community.

Popper underscored the importance of these phases in his title 'Conjectures and Refutations' (making up his 'Logic of Scientific

Discovery') and Lakatos extended this process to mathematics in his 'Proofs and Refutations' (making up his 'Logic of Mathematical Discovery'). In *Criticism and the Growth of Knowledge* Popper's logicist and internalist 'Thesis' was challenged by Kuhn's relativist and historicist 'Antithesis' and Lakatos offered a 'Synthesis' in his Methodology of Scientific Research Programmes.

What has this to do with the philosophy of mathematics education? I want to make a controversial claim. I believe that research in mathematics education has come to over- focus on the 'Thesis' or proposal stage in knowledge generation, and currently underemphasises 'Antithesis' or criticism. Yet the latter is just as essential as the former, and is necessary to improve and warrant knowledge. Let me elaborate my claim. Three main reasons come to mind.

First of all, there are epistemological and methodological reasons. As a community we have to a large extent shifted from an objectivist epistemology to a subjectivist epistemology. A couple of decades ago knowledge and mathematics were widely conceived in the mathematics education community in absolutist, objectivist terms, although there were, of course, notable exceptions. Now knowledge and learned mathematics are widely conceived in subjectivist terms, in the mathematics education community. Constructivism and the interpretative research paradigm stress the subjectivity of knowledge and of all knowing. But such a focus can over-emphasise the generative act in knowing: conjecturing, theorising, imagining, intuiting, etc. Whilst this is a vital ingredient, it nevertheless represents, in my view, an overemphasis on the 'Thesis' stage, with a neglect of 'Antithesis'. Constructivism does include the accommodation of schema by interaction with the world (the subject's experiential world), but the emphasis is on the subjective knowledge of the individual which progresses by an internal dialectical logic. This helps improve the knowledge, but does not publicly warrant it. My claim is that that mathematics education community has not sufficiently made the shift to a third philosophical position, a social epistemology. Here public criticism and warranting play the role of dual to genesis and formulation of knowledge (public or private). There is increasing lip service paid to interpersonal negotiation, but the subjectivist assumptions remain deeply entrenched.

Second, and related to the above epistemological story, is one of ideology. Mathematics education has typically in the last couple of decades been tied in with progressive reform. We want to respect the child or learner as epistemological agent, emphasising her informal knowledge, discoveries, problem solving and posing, activities, feelings etc. The philosophy of progressive education wants to shield the child from hurtful influences and let her grow and realise her potential creatively from within, like a flower. Now this ideology, for all its strengths, again focuses on the moment of genesis and minimises the moment of criticism. Progressive teachers have said "I don't like to mark children's work wrong" and 'assessment' can be a dirty word from this perspective. Underpinning it is a pervasive ideology of individualism, the individual before the group, which of course is the invisible but all-pervasive ideology of modern consumerist, capitalist society. (For a powerful critique of individualism in mathematics education see the important paper by McBride, 1994).

Third, and related to the above, mathematics educators are in the main nice persons who care about each other and don't want to hurt each others' feelings. This is partly our stock-in-trade. As ex-teachers and teacher educators, in the main, we have to be careful not to crush our charges: to let them grow in confidence and capability. This is in part a reaction to the competitive world of mathematics, where a fierce pecking order exists and error is rooted out harshly. So at mathematics education conferences even at the highest international level, criticism is mostly mild, and harsh critiques are viewed as bad manners. Where criticism is built into the field (and without it, it would immediately fall into disrepute) is in the critical scrutiny of submissions for journal and conferences. This is essential for the maintenance of quality (not according to any fixed notions of quality, but according to the evolving conceptions of active members and leaders in the field). However even here mutterings of discontent have been heard: too much is being accepted for conference presentation that is the product of sloppy thinking and does not add to knowledge. Another reason that we are too nice to each other is that we have banded together in adversity. Education has often been viewed with partial contempt by other university departments, including mathematics.

Having offered three points in support of my controversial claim, I will deviate here to describe a recent and relevant controversy.

Mathematics Versus Mathematics Education: An Epistemological Battle

Recently a dispute erupted in the British national press over problems in mathematics teaching. Mathematicians claimed that standards have fallen and that much of the blame rests with the mathematics education community and its misguided progressivism and mistaken constructivist and relativist epistemology. Epistemology in mathematics education hits the national press? Who would have thought it? On 28 December 1994 The Guardian Newspaper carried the front page banner headline 'School Maths in Crisis'. It also carried an article by T. Barnard and P. Saunders (1984), two mathematicians from King's College, London, from which the following extracts are taken.

There is a very curious situation in mathematics at the moment. There are on the one hand, clear signs of a crisis. There is a strong consensus in the university mathematics world that the mathematical awareness, skills and understanding of pupils completing secondary education have deteriorated rapidly in recent years. Students know less, understand less, have little facility with simple operations, have little idea what is meant by proof, find difficulty in solving any but the shortest of problems.

The problem is not simply the schools being perhaps too complacent and the universities perhaps too demanding. There are fundamental disagreements about what mathematics is, what pupils should learn about mathematics at school, what skills they should learn within mathematics both to be used later on in the subject and elsewhere, what else they should be learning while they are learning these and what the priorities are. They go right to the heart of how pupils learn anything.

Central to the issue lies the epistemological divide between a traditional emphasis on knowledge as an external landscape, and a post-Piagetian emphasis on knowledge as an individually constructed internal picture. The situation is sometimes regarded as if there were two diametrically opposite positions. Firstly, the Absolutists, cultural restorationists who see teaching as presenting a corpus of facts and learning as a passive process of receiving this. Opposed to them are the Relativists, liberal progressives, for whom teaching means presenting learning activities and learning is an active process of building a mental network of relations.

But is this a true picture? In mathematics teaching, the changes are sometimes seen as a jettisoning of content - Euclidean geometry, the arithmetic of fractions, algebra - to allow time for a careful development of process skills: interpreting, communicating, selecting, applying, and so on. But where an instrumental understanding of content is merely replaced by an instrumental understanding of process, there is no improvement. On the other hand, a relational understanding of content carries with it a relational understanding of process. The converse is not true. Pupils cannot genuinely understand processes if they do not understand the content on which these are supposed to be operating, and if you reduce the latter you inevitably limit the former.

The mathematicians' 'thesis' was rapidly answered by an 'antithesis' claiming that the problem was not that mathematics educators had let the side down with their irrelevant theories, but that mathematicians were as usual taking the narrow view and blaming others for the fact that mathematics is a decreasingly popular subject for university study in the modern world. Worse, it is the antiquated, instrumental and content-centred approaches in further and higher education which were the real culprits, and which are driving potential students away . (I must admit to adding my voice to this 'antithesis' group, in Ernest 1995)

It took the calmer voices of Margaret Brown (THES) and Sue Burns (LMS Newsletter) to offer a synthesis, pointing out the fact that although an increasing number of students take 'A' Level exams in the UK at age 18, the number taking 'A' Level Mathematics declines annually in absolute terms, and those who do succeed at it may be opting for medicine, economics, computer science, etc. The number opting for mathematics degree course declines annually. So mathematicians may be having to recruit from the lower strata of a shrinking pool of applicants, and hence observing a decline in competencies.

Conclusion

The point that I have been making is that criticism plays an essential role in the growth of knowledge. This newsletter is dedicated to dialogue on reflective and philosophical aspects of mathematics education. It has tried to counterpose conflicting views, embraced all criticism as part of its conversation, and the present issue can be seen to sit well in this tradition. However not all forms of critique are the same, and it is as well to remember that there is a moral dimension to dialogue. In the Platonic dialogues, Socrates uses the method of *elenchus* or refutation, to lead people to see that their views are perceived by others as wrong. This has been contrasted by feminist philosophers with 'the adversary method', which is "a model of philosphic methodology which accepts a positive view of aggressive behaviour and then uses it as the paradigm of philosophic reasoning. ...The adversary method requires that all beliefs and claims be evaluated only by subjecting them to the strongest, most extreme opposition." (Harding and Hintikka, 1983: xv). Getting the balance right between constructive and destructive criticism is an important issue of judgement. Readers will have to decide if I have got the balance right in this issue.

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DISCUSSION THEME: THE DISCIPLINE OF NOTICING

As in previous issues of POMENEWS the discussion section aims to provide a location for in-depth comment around a particular theme. This issue the theme to be discussed is an emergent research perspective in mathematics education: the 'Discipline of Noticing'. This is primarily due to the work of John Mason (although he would be the first to credit Caleb Gattegno) who is one of the founding members of the Organising Group of the Philosophy Of Mathematics Education Network.

IS 'THE DISCIPLINE OF NOTICING' A NEW PARADIGM FOR RESEARCH IN MATHEMATICS EDUCATION?

Paul Ernest

For at least a decade John Mason has been elaborating what he terms 'The Discipline of Noticing' (in brief: 'DN') (Mason, 1994: 183). This is a perspective or stance with regard to research in mathematics education, and represents John's personal research program in mathematics education over the past decade. This includes a number of strengths (as they strike me) including the following:

- the valuing of individual sense-making, which is so central to the dominant constructivist perspectives in mathematics education,
- a concern to relate this to the actions of the teacher, which is too often missing in constructivist accounts,
- a concern with the authentic nature of lived experience, which strikes a deep chord with many progressive educators,
- an emphasis on an interpretative research methodology, focusing on case study and concrete particulars of life and educational practice.

These strengths make this a seductively attractive approach to educational research, one which is proving popular with a significant sector of those in British mathematics education who might loosely be described as drawn to the teacher-as-researcher movement.

Because of this burgeoning popularity, in this short piece I plan to look critically at DN, rather than to celebrate its strengths. Critique is an important feature of any community of researchers, although it is a feature that is too often neglected in mathematics education, as I discuss in the editorial. I believe it is incumbent on the community of researchers in mathematics education to examine critically any perspective that is proposed or adopted as a novel educational research paradigm, especially when it is used by a growing number of young researchers as the basis for PhD research. Such is the position of DN. Anyway, DN has been widely honoured in many research presentations, including a prestigious plenary talk at the PME conference in Lisbon (Mason 1994). I shall take this most recent exposition of DN as a representative statement of the position. The characteristics of the approach of DN appear to me to be the following. There is:

- A concern with the subjective experience of the learner and teacher (especially with acts of construal, awareness and changes in awareness, the noticing of significance),
- The direction of another's or others' attention to what has been noticed as significant: selecting and emphasising certain events or experiences and directing own or others attention to accounts of them, or trying to enable others to recreate the experience,
- An attempt to develop an authentic voice to describe the researcher's experience (whilst at the same time regarding such accounts as secondary to the experiences they describe),
- The rejection of the epistemology and methodology of the scientific research paradigm and the endorsement of a methodology akin to that of the interpretative research paradigm.

My commentary on DN follows the division of its principles or themes into four aspects. In the words of Mason "The Discipline of Noticing ...[has] four aspects:"

[1] Systematic Reflection; retrospective re-entry through brief-but-vivid accounts of incidents, without judgement or explanation; once a corpus accumulates, it is possible to identify common threads. (Mason 1994: 183-184):

This assumes that it is unproblematic to provide 'brief-but-vivid' accounts of experiences or events without judgemental aspects: that it is feasible to do this. A poet provides 'brief-but-vivid' accounts, but they are not *of* an underlying experience. Whilst an individual may possibly construct expressions which symbolise and help re-evoke a personal experience, the assumptions implicated in claiming so cannot easily be validated. The memory against which the 're-evoked experience' is judged, the very means of judging validity, may be altered by the process of constructing a' brief-but-vivid' account. The assumption that such expressions may serve to give a proxy a matching experience is very problematic. It either assumes that (i) there is an essential meaning in a signifier, in what may indeed be an arbitrary signifier, which is communicable to another (which is refutable); or (ii) that there is no essential meaning in a signifier and the proxy does not have a matching experience, but has some other experience which is deemed valuable in some unspecified way. The problem with case (ii) is that it does not justify the term 're-evoke' for any other than the original experiencing subject. Yet [4], below, claims that "refining task-exercises which permit colleagues to experience (directly or by analogy) something proposed as worthy of noticing" is both possible and feasible. So a problem that arises for me is that if one takes a wholesale subjectivist position on the priority of meaning and experience over expression (as DN does), how is communication of a selected meaning possible?

The second claim is that "once a corpus accumulates, it is possible to identify common threads". This is in part the claim of Grounded Theory (Glaser and Strauss 1968), except that GT is very clear that the researcher's perspective constructs the pattern ('common threads') and that many such patterns are possible. GT then concerns the validation of bottom-up theories through the involvement of others. This comes up in [4] below, but it is not made clear that the aspects of DN make up a cyclic pattern. For *in*validation through others (always a possible outcome) would necessitate 'identifying *other* common threads'. The 'language of identification' used in [1] suggests that there are pre-existing patterns in the experiences. This makes the claim either (i) resemble the inductivism that Popper in his *Logic of Scientific Discovery* critiques decisively (the claim that patterns are in the world and can be induced by finite experiences), or (ii) some phenomenological or rationalist style language of experience (the patterns are derived by *a priori* necessity or some other force *within* the mind).

[2] Recognising and Labelling Choices: locating alternative strategies and gambits for use in particular situations in the future, either from reading, from sharing incidents, or from observing others; labelling typical incidents and corresponding gambits so as to trigger recollection of possible actions spectively [sic]. (Mason 1994: 183-184)

This is an unobjectionable form, suitably elaborated, of the injunction to learn from experience. It ties in through references in Mason (1994) with work on teacher's decision making. However, the research on teacher-decision making is sometimes regarded as having a positivistic or anti-constructivist tinge: based on the assumption that teacher's perceptions, choices and behaviours can be described accurately by an outsider. The language in [2] is that of injunction, not that of knowing, so this criticism can be side-stepped. But there are epistemological issues raised. 'How can I know about my state and position?' is only partially problematic, concerning the difference between knowing and believing. But 'How can I know about another's mental states and processes, and about their potential and actual behaviours?' is more problematic. It is very problematic from the perspective of a subjectivist epistemology. If this is the wrong epistemology to interrogate, what is the appropriate one?

[3] Preparing and Noticing: entering recent experiences post-spectively, then imagining making an alternative choice pro-spectively. It takes considerable effort to move the moment of recognition from the retrospective to the spective. (*ibid*.: 183-84)

Many of the problems raised for me by the injunctions in this section have already been articulated, above. An issue which arises here and above is that of an unspoken set of values and model of good practice. What values underpin the DN, both spoken and unspoken. Is not the emphasis on individual choice an expression of an ideological individualism, which has long dominated Western thought, and is emerging most sharply in the 'free market' or 'market-place' metaphors of recent social policy? McBride (1994) has offered a powerful critique of how individualism is the hidden ideology of much modern school mathematics. She argues how the emphasis on individual choice, and mathematics as underpinning rational choice, permeates school mathematics texts, and represents a perspective whose strategy is to deny or conceal the historical, social, cultural, political nexus in which all knowledge and practices take place. On the cult of the individual: Is the DN a version of the philosophy of Nietzsche's 'Superman', the person whose authentic experience and refined judgement is paramount; either in discerning what is happening, or (more frighteningly) in pointing out to others what matters? This analogy is not completely far-fetched. Consider the tradition of Zen masters which DN parallels: the guru or enlightened one

indicates to the chela or disciple what is of import for the latter to attend to attain wisdom. This is on the one hand a wonderful tradition of mystical teaching, on the other a pure authoritarian power relationship wholly dependent on the integrity (or whim) of the master. The term 'Discipline of Noticing' is explicitly linked to Buddhist thought by Mason, and both 'discipline' and 'noticing' are concepts that occur in Buddhism.

[4] Validating with Others: to guard against solipsism and self-delusion, continued interspective exchange of brief-but-vivid incidents and observations; refining task-exercises which permit colleagues to experience (directly or by analogy) something proposed as worthy of noticing. (Mason 1994: 183-184)

It is central to research in the interpretative paradigm in education that subjectivity is guarded against, and that the equivalent of validity and reliability can be attained for the results of research. This fourth aspect stresses this dimension of research, as an antidote to the possible subjectivity of the insights derived from DN. My criticism here is not that it is unimportant - it is overwhelmingly important - but that it fails to relate to the extensive and growing literature in interpretative and qualitative research in education and the social sciences. The validation of the methodology and results of interpretative and qualitative research has been the major issue of discussion in scores of journals and dozens of books for over a decade, and there is a rich and well-tried literature to be drawn upon. It is largely missing from DN. Is DN reinventing the wheel of interpretative research methodology?

DN as communicated in its four aspects has some interrelated and dominant features. The titles of the four aspects are reformulated injunctions. Injunctions are commands, recommendations, orders or invitations to do something (which relates back to the guru-chela authoritarian power relationship mentioned above). It is therefore my presumption that DN is derived from the mathematical heuristics that Mason *et al*. (1982) articulated and developed from the work of Polya (1945) and others. There is a strong analogy between Polya's (1945) heuristic problem solving cycle and the process of educational research, as is widely noted (e.g. in Ernest 1994). Mason's earlier work on the teacher-as-researcher clearly links the active role of the student of mathematics as problem solver and the active role of the teacher-researcher. My conclusion is that DN is a problem solving-based research methodology; a theory of how to undertake research. Whether my genealogy is justified or not, it is clear that DN is primarily methodological in character. This is also made clear in the text of Mason (1994, 189).

This raises another problem. In my view DN is presented as more than a methodology, it is presented as a fully-fledged educational research paradigm: a new way of looking, being, teaching, and doing research. But an educational research paradigm is usually understood to have three interrelated components: an ontology, an epistemology and following on from the former two, a research methodology, namely a theory of the means to generate knowledge in accordance with the epistemology. DN on the face of it seems to presuppose its ontology and epistemology. They are implicit. It is claimed that it is based on a 'common epistemology' (Mason, 1994, 189), but this seems only to make explicit the fact that it is an implicit, assumed view of knowledge. Only the methodology seems to be given explicitly. Of course one way to address this criticism is to further elaborate the epistemology and ontology of DN. Like any other emergent research program that continues to grow, critique can point to areas that need extension.

Finally, I should return to my initial question 'Is the 'discipline of noticing' a new paradigm for research in mathematics education?' Have I answered it? The answer to this second question must be no. Evidence can be adduced that DN is serving as a paradigm for research in mathematics education, in that it underpins a variety of current research in Britain. But my question was not descriptively factual as this answer suggests: rather it was critical and evaluative. In this latter sense I cannot claim to have offered anything like a decisive critique. I can only claim to have indicated some problem areas, and there is no doubt that DN is continually being elaborated and strengthened.

At this point I invited John to respond (acknowledging that justice cannot be done to any elaborated approach like John's in a few pages).

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RESPONSE TO PAUL'S COMMENTS ON THE DISCIPLINE OF NOTICING

John Mason

One of the fascinating things about seeing other people discuss ideas I have worked on, is the way individuals stress some aspects and ignore others. It provides a useful reminder of Proust's remark, that "every reader is a reader of themselves", which reflects an Amerindian Weltenschaung expressed by Hymenehosts Storm (1985) as "The world is a mirror for the people". I have found myself not just assenting to that adage, but asserting and living by it whenever it comes to mind.

The previous paragraph provides a potted version of noticing. I was struck, when reading Storm, by his adage. It fitted with past experiences and gave me a neat handle (label) to act as focal point for, among other things, a variety of techniques to employ when I catch myself reacting to or being mystified by someone else's reading. Furthermore, as I typed the word *assent*, the word *assert* was triggered into memory. The reason is that *assent-assert* is a framework which Joy Davis and I worked on some years ago as a label for distinguishing between intensities of participation and between kinds of learning. Indeed it finds correspondence in the not-noticing-noticing-marking-recording framework which lies at the heart of the Discipline of Noticing. Labels act as focal points for triggers (usually metonymic), resonances (usually metaphoric), and strategies to choose to enact if it seems appropriate. I desist from inflicting upon readers of PoME any exercises designed to highlight similar experiences in them, but exercises are also a critical part of the methodology of DN.

Brief-but-vivid accounts

I find it useful to locate the phenomenon being analysed, and to distinguish it from the analysis, as far as possible. Paul points out that the notion of brief-but-vivid accounts, which form a core element of DN, is problematic. This is undoubtedly the case, for accounts are based on language, and language is problematic. However, people who have worked on generating brief-but-vivid-accounts, especially with colleagues, know that it is indeed feasible to produce effective BBVs. In fact I maintain that we are all aware of BBVs, for when we launch into a story and we see other people's eyes glaze over, we know we have strayed! It does not take long to discover what is effective and what not. An account of an incident may be brief, may even be vivid, yet not necessarily be brief-but-vivid. That classification is reserved for accounts which others readily recognise. There is not room here to elaborate, but it is one of the key elements of the experience of noticing, as distinct from theories about noticing conducted solely in and through language.

Paul opens his account of DN with a mixture of account-of-like summaries in his list of strengths, and accounting-for, when he uses expressions like *seductively attractive*. It is as if he is warning a younger generation against the immoral diversions of a wicked world. I am intrigued by the multitude of possibilities which could be read into this phrase. DN is at pains to reduce the emotive associations of language in order to focus attention on phenomena and to engage in critique and questioning.

Communication

At the heart of any linguistic enterprise there must surely reside the constant refrain of Paul Cobb: *taken-as-shared*. As long as we seem to be agreeing, we keep talking; when we start conflicting, we have to negotiate or even part company. DN makes no claims to produce certainty or truth in absolute terms, or to study objective events. Indeed I find it informative to think of an event as constituted by the collection of accounts of and for, together with associated psychological states and inarticulable images, not only from direct participants, but also from second and third hand participants. Of course memory is fickle and malleable. But DN does not take events as having an objective ontology to which accounts approximate.

Paul's fundamental question for DN seems to be: If one takes a wholesale subjectivist position on the priority of meaning and

experience over expression, how is communication of selected meaning possible?

DN depends on negotiation of meaning between colleagues, and a constant testing of significance and sensitising to notice in the moment. The social is not denied, and is important, but it is not the totality either. That taken-as-shared communication is possible is self evident; that mathematics educators do not on the whole communicate well could also be concluded from experience of conferences and teacher-training. A disciplined approach to negotiating technical terms, along the lines suggested for the growth of labelled frameworks, would not go amiss in the community!

Paul appears to seek structure, in that he sees a contrast between the cyclicity of Grounded Theory and the apparent lack of structure in DN. DN is not usually presented as a cyclic process simply because as soon as noticing becomes a mechanical sequence of actions, it is no longer noticing. Unfortunately the urge to simplify and routinize has struck other action research methodologies in which cycles have become unnecessarily rigid. All aspects of DN are potentially present at any moment, depending on circumstances. DN is not merely cyclic, but mutually co-evolving, to use Varela, Thompson, & Rosch's (1991) phrase.

I am mystified by Paul's assertion that DN depends on 'pre-existing patterns of experience'. I suspect it is because of our collective uncertainty about how to speak sensibly and fittingly about experience that he resorts to logic, for example by identifying 'pre-existing patterns of experience' with philosophical positions criticised (convincingly for him) by Popper. In the spirit of DN, I invite Paul to work at recounting brief-but-vivid descriptions of incidents, and only then to ask himself whether his ability to relate to the experience of others is due solely to parallel discursive practices or whether there is also a sense in which people in similar situations have similar experiences. How else indeed could apparent communication be possible? Paul asks how he can know about states and processes, about potential and actual behaviours, of others. I submit that within some discourses, this *is* possible. I suggest that it actually happens, quite frequently, otherwise his students would remain distanced and untouched by being in his presence. It is precisely because he can relate, can speak meaningfully to students and colleagues, that I know that he can know about states and processes of others. But this is of course just one discourse among many, an account-of that he may be able to reject by invoking a different discourse.

Ontology and Epistemology

I am disappointed that Paul does not find the epistemology and ontology he seeks, because I have endeavoured to address these. They are essential components of any paradigm. DN addresses directly the problem of descriptions and labels bringing into existence the qualities they purport to describe, by emphasising the ongoing necessity of testing in experience, communicating with a wider community, and using the criterion of future practice being informed. The epistemology of DN is based on the proposal that you cannot do better than that, no matter what methods you employ, which cuts across traditional philosophical approaches to base itself on lived psychological experience.

That some aspects of meaning are due to discursive practice seems to me to be as evident as that individuals construe. But proposing discursive practice as the sole answer to ontological questions concerning meaning does not fit with my experience. It does not assist me to make as much sense of my past or inform my future practice as my own eclectic view that the psyche and the social are interwoven. Notice that I demonstrate my epistemology and criteria for validity within my response: I interrogate my past and test to see how future acts may be informed. I console myself with the assumption that Paul and I need to negotiate what is expected in a discourse on epistemology and ontology as distinct from method. It may be that what constitutes ontology and epistemology may differ between different paradigms.

Paul refers to 'rejection of the epistemology and methodology of the scientific research paradigm'. I am not convinced by the appropriateness of his definite articles, and I certainly do not reject any paradigm out of hand. Rather, I consider fitness-for-task. For the kinds of questions which interest me (how do I develop my sensitivity to notice and appreciate?, what is it like to be ...?, what is the psychology of recognising a possibility?), I have found the DN more useful than any other paradigm. But I draw extensively on any results obtained by any method, if they resonate with or challenge my own experience. The results I ignore are the ones that pass me by, that do not seem relevant to my concerns. In this I suspect we are all rather alike. Indeed this observation is the basis for the epistemology on which the DN is founded.

Links to literature

It is true that writing about DN has not stressed the vast literature on interpretative and qualitative research, nor referenced all related philosophical discussions or occurrences of similar ideas in other domains. I look forward to specific references which can be followed up and which shed light on central issues. I want only to reinforce the difference between what I have read elsewhere and DN. DN is

concerned with the experience of the researcher. It is intraspective in origins and both intraspective and interspective in validation. It draws upon extraspective research where it is informative and relevant. Most accounts of interpretation that I have read immediately move into the extraspective, in which the account, the transcript, and the video, become the phenomenon being studied.

How do you decide what to notice?

The question I am most often asked is "How do you decide what to notice?". In fact, noticing is not initially something most people have control over, and it is precisely in order to experience the wonderful state of freedom, of choice, that the Discipline of Noticing works to make noticing a matter of common intentional experience. I read in various criticisms a concern that if I am led to notice some things, I may overlook or be led away from more important aspects. Yet any stressing has consequent ignoring. As long as our stressings remain implicit and buried we are at the mercy of the habituation and enculturation that has taken place. By bringing our implicit frameworks and theories to the surface we can work at being able to choose to alter them.

Roots and Sources

DN is an elaboration of a way of working carried out over many many years, derived and re-articulated in my context from a few fragments I have encountered in writings from many cultures reporting thousands of years of psychological and sociological investigation. Mevlana Jalaluddin Rumi (the mystic who initiated the whirling Dervishes in the 11th century) said at one point "Don't look at me, take what is in my hand" and I recommend this injunction to anyone who concentrates on the person rather than the seeing.

What matters to me is not pedigree but meaning, not knowing-that but knowing-to. Part of what I have learned from others is that what I can reconstruct for myself is mine; what falls away over time belonged to them. So I have deliberately not emphasised my own sources, but rather dwelt in what I have internalised, made my own, assimilated and accommodated, reconstructed for myself, ... the list of euphemisms is long. It is up to others to do the same for themselves. And that is part of the epistemology of DN: each individual must test assertions in their own experience, past, present, and future. By proceeding in a fairly disciplined fashion, they will develop their own being.

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WORKING WITH 'THE DISCIPLINE OF NOTICING': AN AUTHENTICATING EXPERIENCE?

Tansy Hardy and Dave Wilson

Last year we not only attempted to work ourselves with 'The Discipline of Noticing', but used it as a framework on a Masters Unit that we were teaching. In particular we worked upon the creation of brief-but-vivid accounts. This was, in part, an attempt to test out, by working with others, the conjectures that their creation is both possible and worthwhile. Certainly it required effort and practice and in that sense was not unproblematic. What we observed was that early talk} and writing by the students, and ourselves, was characterised by containing highly generalised anecdotal narratives and sweeping value judgements, both about practice and about pupils. We are suggesting that within these narratives there was nothing to be examined, in the sense that nothing salient, no moment of energy was identifiable. (This energy, we have found, signifies something worth examining further.) The exercise seemed less about giving 'a proxy a matching experience' or communicating a 'meaning' but more one of creating data, getting a sense of what there is to be studied and reflected upon.

The issue of validity was to do with the identification of phenomena which were recognised as part of the participant's practice worth reflecting upon. The telling, and re-telling up until recognition or resonance provided data for reading and re-reading.

In the cultural studies field, posters and films are the artefacts, exterior to ourselves available for reading. Valerie Walkerdine (1988) examined the writings of Hughes and Tizard and offered re-readings of their data which consisted of transcripts of dialogues between children, parents and teachers. A practitioner researcher needs to construct the data from their experience which is exteriorised and held up for examination. Certainly it is possible, at this stage, to question the validity of the data we generated about our teaching and the 'truth' of our readings and rereadings. There is a debate going on within psychoanalysis at the moment Reference to on the psychoanalysis mailing list (at psychoanalysis@netcom.com) about the validity of the case studies produced by influential practitioners, including Freud. Stekel, when taken to task by Freud for revealing the names of his clients at a conference is supposed to have replied that he had not only made up the names, but also the stories. Kohut's case studies are thought to have been based upon his own self-analysis.

How much does this matter? We want to suggest, that from the viewpoint of a reflective practitioner, the issue of validity is much more one of whether the retold anecdotes are recognisable by other practitioners and are so discussable than of whether they are objectively true or not. The anecdotes come with their own truth, in a way similar to that which Flaco Jimenez 'brings his own authenticity on stage with him', as Ry Cooder said (on the BBC 2 Arena transmission on Ry Cooder's music).

The awareness that we were not dealing with a cyclical procedure was reinforced by our experience of working with teachers on this Masters Unit. Within their enquiries the teachers were involved in creating data, generating and identifying a focus of enquiry (the strands they identify as running through their experience) and interpreting and validating those strands. However these elements were not engaged with in a linear order, cyclical or not.

There is a sense in which these elements of the enquiry process are inextricably caught up with each other, that in identifying relevant data we cannot help but simultaneously reflect on that data and consider its relation to other data. This data is compared to existing strands in our experience, and we notice jarring and resonance with previous interpretations. New interpretations start to form.}

We have attempted to create a structural form to represent the simultaneity and symbiosis inherent in this process and capture this diagrammatically. We have written further on this, including our diagrammatic models (Hardy and Wilson 1995). This image has no asserted beginning, no end. We are always in the middle, between things, interbeing, intermezzo, within Deleuze and Guattari's (1988) definition of a rhizome.

This allows us to suggest that the identification of focus is necessarily in a symbiotic relationship with the creation and role played by data, and its validation. However, the 'Noticing' process is not undisciplined.}

Of course, we come to the enquiry process with existing interests and concerns, and there is always the question of whether we are merely reinforcing current concerns (or even fetishes) 'the obsessive' seeing 7s everywhere but we find that we can insert a gap and hold these at a distance and conduct a valid enquiry.

In looking for ways to work with our Masters students and also to support our own research we found that other available frameworks seemed neither to give explicit recognition of our personal experience as teachers, nor to offer us an authentic description of how we had come to our professional knowledge. We chose to engage with the Noticings framework as it supported this experience, whilst making our reflections more systematic.}

One of the things which we noticed whilst reading Paul's critique was that we are now positioned as John Mason's chelas. Through our choice to explore Noticings as a possible tool for our enquiries Paul positioned us. We were effectively silenced, in advance of responding, by Paul's metaphor of the guru and his chelas. As disciples we are bound to reiterate the words of the master. Here we have tried to speak validly with a voice from our own practice. We hope this can be heard.

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THE PRINCIPLE OF ECONOMY IN THE LEARNING AND TEACHING OF MATHEMATICS, BY DAVID P. HEWITT

Published by Dave Hewitt, c/o The Sciences Academic Office, School of Education, University of Birmingham, Edgbaston, Birmingham, B15 2TT, U. K. Price £10.

Discussion/Review by Paul Ernest

This book comprises the author's PhD thesis submitted to the Open University and awarded in 1994. The relevance of this review in the present context is that it reports a research project undertaken within the Discipline of Noticing, as the following author's *Abstract* makes clear.

This thesis looks at the learning and teaching of mathematics through the issue of economy. Here, economy is concerned with the personal time and effort given by a learner to achieve some desired learning. The study sets out to establish that the principle of economy informs the learning and teaching of mathematics, and to establish a list of principles which can assist an economic approach to the teaching of mathematics.

The study is carried out within the Discipline of Noticing and is based on the development of theory from significant events building on the work carried out by Caleb Gattegno on the subordination of teaching to learning. An *account of* these events are given, followed by *accounting for* them, and linking the generality contained within these isolated events with everyday learning experiences. At times, the reader is asked to carry out simple tasks which assist in drawing their attention, through a personal experience, to the points being developed.

The learning process which turns something newly met into something which can be done with little conscious attention, is analysed and called *functionalisation*. The analysis of this process produces the idea of *practice through progress*, where the learner's attention is placed in a task which requires the desired learning to be subordinated to it. Particular attention is given to the learning of young children before entering school, since this is impressive in terms of economy. This study identifies powers children use in their early learning, and how these link in with root notions in mathematics called *mathematical essences*. A list of principles of economy are developed which provide guide-lines for approaches to teaching to make use of children's powers and utilise mathematical essences. A computer program, *GRID Algebra*, is developed to demonstrate how the principles of economy can be incorporated into a resource. (Hewitt 1994: v)

One of the disarming strengths of this account is that it is a lucid account of an action research enquiry by the author, who honestly positions himself as a teacher-as-researcher. There are a number of striking strengths in this thesis, which I detail in the 'sympathetic review' below.

The book addresses a number of central themes: the Discipline of Noticing , drawing primarily on Mason and Gattegno; learning, first in general, then in terms of number, and finally in terms of algebra; curriculum development: the development of GRID Algebra, a teaching package; theory, reflection and the 'Principle(s) of Economy', critical reflection on some existing curriculum material (secondary school mathematics text pages); the [intellectual] powers of children.

Looking at the text, I feel myself torn in two directions and adopting two roles: that of empathy and sympathetic reader and that of critic. So I offer two complementary reviews reflecting this ambivalence.

Sympathetic Review

First, the empathetic role. What I see here is an authentic account of the teacher as teacher-as-researcher. There is an in-depth intellectual biography, describing Dave Hewitt's education, career as a teacher and curriculum developer. This indicates intellectual influences, insights and investigations. It speaks to the interiority of his personal development: the ideas that resonated and seemed to be fruitful in his experience. The whole account is very much an 'investigation' (i.e. a open-ended problem solving project) writ large

and transposed from mathematics to mathematics education. This is precisely what the teacher-as-researcher movement endorses (as I do). Namely that all individuals can be epistemologically empowered, and that the knowledge creation of the child, student, student-teacher, teacher and researcher are all of a piece, differing in degree and kind (and in the discursive practice in which they seek to validate their knowledge) but identical in principle. For example, a report of a mathematical investigation typically posits a problem domain, generates examples, looks for pattern, provides conjectured generalities capturing the pattern, offers independent tests of the conjecture and other forms of justification, and includes additional supporting material to describe the overall processes of investigation. Now this description fits very well here. The conjectures are 15 Principles of Economy derived in 3 chapters as the distilled fruits of reflection on practice. But these are anticipated by the identification of important ideas earlier, which finally emerge in relation to each other as the Principles of Economy.

Many of these principles are eminently sensible or wise proposals for teaching in a meaningful and inspiring way. E.g.

- View mathematical content through the eyes of a pedagogue rather than a mathematician. This means considering children's powers, mathematical essences, and awarenesses.
- Use imagery and movement so that awarenesses can be gained within a context of generality.

The account fuses personal development with emerging 'theory' or at least generality of description. It is a confident, articulate, reflective and honest account. It insightfully describes illuminating teaching episodes. and ends with the development of a rather good sounding curriculum pack (GRID Algebra).

Critical Review

Secondly, I find myself also looking at the study in a detached critical light. Then my criticisms concern undefined terms, weak methodology, weak theoretical and literature base, atheoretical nature of discussion and claimed outcomes, and a weak justificatory structure, as I detail in the following paragraphs.

There are key terms used which are undefined or inadequately defined and used problematically including 'children's powers', 'mathematical essences', 'awareness', and even 'economy'. How can any claims for knowledge be mounted until terms are clarified?

The so-called methodology chapter includes reflections on objectivity/subjectivity, discussions of ontological assumptions, a constructivist position on learning; action research, teacher-as-researcher and the Discipline of Noticing and validity, in 13 pages. In effect the reader is told that the researcher has reflected a on these issues and the data, and that the researcher derived the data by what felt right, and that the reader can validate the data by empathy or resonance with her own experience. I don't find this adequate or acceptable. There is an established interpretative research tradition in education which has devoted a great deal of careful thought to these issues which cannot be treated so cavalierly.

The theoretical foundations and the literature on which the study rests are primarily the Discipline of Noticing. There are 90 references in all, and 28 might be said to be professional rather than educational research publications. Of the total,15 are on Discipline of Noticing by Mason, Davis and Gattegno; 8 are by the researcher himself; 6 are school or college mathematics texts, 5 are on the teacher-as-researcher and one more is an introductory text on research methodology in education. Such counting settles little in itself, perhaps, but is indicative. It can be said that the study lacks conventionally strong foundations in either the mathematics education or social sciences research literature. The key case study described is of the development of an algebraic computer package, but the literature on the learning of algebra, or on the use of computers is thin. That on the testing and evaluation of curriculum developments is non-existent.

Possibly in consequence of this, the discussion and the claimed outcomes are almost wholly atheoretical. The 'Principles of Economy' are injunctions to practice, induced from a few examples. Their meanings are not made unambiguous and no attempt is made to test them systematically. The concepts and propositions put forward are not clarified by, based on or validated in terms of a systematic research literature (other than the Discipline of Noticing).

There is a 2 page section on validity and an acknowledgement of the import of validity and reliability, but there is no notion of scientific or critical testing of the claims. The central quotation in the section on validity is as follows: "The essence of validation is two fold: selecting and honing descriptions which others instantly recognise; refining tasks which highlight fruitful skills or sensitivities." (Mason with Nevile, 1992: 25) Knowledge *qua* knowledge needs warranting, some means of justifying the claims as knowledge. What is offered here is in my view a weak justificatory structure, one which fails to engage with the powerful methodology of the interpretative

research paradigm.

So I am torn between two views and reviews. Or rather I find myself in both positions simultaneously. The first is a sympathetic view of an epistemologically empowered teacher-as-researcher generating knowledge within the Discipline of Noticing. The second is a rigorous critique of weaknesses of theory, methodology and validation in the account. In my view, both reviews have something important to say, and it is not a matter of interpretative vs. positivistic educational research paradigm positions. The intrigued reader will have to read it and make up her own mind!

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RESPONSE TO PAUL'S COMMENTS ON THE PRINCIPLE OF ECONOMY IN THE LEARNING AND TEACHING OF MATHS

Dave Hewitt University of Birmingham

He [sic] [the 'ideal mathematician'] can think of no condemnation more damning than to say of a student, "He doesn't even know what a proof is." Yet he is able to give no coherent explanation of what is meant by rigor, or what is required to make a proof rigorous. In his own work, the line between complete and incomplete proof is always somewhat fuzzy, and often controversial. (Davis and Hersh, 1981, p34)

I welcome the opportunity to discuss issues relating to my thesis, and the methods used and why they are not outside a rigorous paradigm but are part of an alternative and equally 'rigorous' one. My thesis is written in a style which is consistent with a different methodology - that of the Discipline of Noticing, consequently it differs in style to that of most PhD theses. The methodology is also consistent with, and appropriate for, the particular questions and challenges of my research. What I have done is to present a personal development of theory relating to the economic use of personal time and energy in the learning and teaching of mathematics, and introduce several new notions which inform the teaching of mathematics. Some of these are briefly mentioned below (and in my abstract), and form the substantive content of my thesis - something which Paul does not discuss.

Interpretation - the Individual

Sense-making goes on all the time but is often hidden within some methodologies where definitions, procedures and validations are assumed to be unproblematic. The so-called 'rigour' can hide the individual and local event of sense-making. What I have done within the thesis, and what the Discipline of Noticing offers, is to acknowledge that communicating understanding, and validating, involve personal sense-making for both the writer and the reader of a thesis, or any written work.

All methods involve a researcher interpreting events, and someone else observing the 'same' event will have different personal interpretations. Within my thesis, the culmination of my particular interpretations of many events build into a theoretical perspective which can inform the dynamics of teaching and learning. The events which are described and interpreted only matter in so much as it was through them that I developed notions (such as the placement of attention, subordination, functionalisation, practice through progress, neutral zone, teacher-as-amplifier and teacher-as-editor) which proved significant to my developing theory. What is important is not whether my interpretations are 'correct'. The importance is in the notions which are developed from my interpretations, and that these notions are not particular to me, but are phenomena which are recognised by others as well.

Communication and validation - the community

Communication is problematic. I am reminded of von Glasersfeld's (1987) comment '... it appears that knowledge is not a transferable commodity and communication is not a conveyance.' (p16). There are many differences between each of the individuals within any community. This is acknowledged in the acceptance that other people may not notice the same things as I notice within an event. However, there are also things which we all share. One technique I use is to relate a particular event to other events, so that the

particular event becomes an example of more general situations. I deliberately choose events from our everyday learning (something we share, such as walking, writing, speaking,...) to increase the likelihood of the reader being able to examine what I am saying from their own experience. In addition I use the technique of asking the reader to carry out simple activities. These are carefully designed so that the reader can find, through their own noticing of what is happening to them, examples of the points I am making at the time. The creation of activities to assist with this form of communication is not always easy. Such simple but effective activities, which provide examples of the notion being discussed, demand careful construction. Likewise, it is not a trivial exercise to find examples of a notion within 'everyday' learning. Thus, this form of communication and validation is demanding for the writer (to devise such activities and examples) and demanding for the reader (actively to engage in the activities and seek what is being discussed from their own experience). I offer the following as a simple example:

A situation which involves the rule 2(x+3), for example, is sometimes worked towards by considering particular cases, such as when x=7. However, children's attention is often on the particularity of '7', rather than seeing '7' as an example of any number. Withdrawing attention from the particularity of '7' can be difficult. For example, I invite the reader to carry out the following activity:

For the next five minutes, do not think about eggs

When I have asked people to carry out this task, most reported thinking of eggs straight away. Those that were able not to think of eggs, did so by putting their attention into something else rather than trying to suppress thoughts of eggs. To withdraw attention from the particularity of '7', there is a need to shift attention onto something else, rather than asking children questions such as "what if it was another number?", etc., which keep attention on particular numbers. Within the thesis, I discuss techniques to do this, and how an 'x' can be introduced relatively unproblematically. How ever, here I offer the above exercise as a way for you to become aware, from your own experience, that in order to withdraw attention from one thing, attention needs to be actively placed somewhere else, and that directing attention is a significant aspect of a teacher's role. In this case, there is the unusual step of directing attention away from an area where learning is taking place (i.e. away from the shift from '7' to 'x').

I have only had space here to offer a small indication of what is involved. However, such activities offer a strong form of validation and a radically different one to those in other methodologies. The question is whether the research community is going to broaden their image of what is 'acceptable' validity, since comparing the Discipline of Noticing to others is not a way of getting to grips with what is offered through it. I believe this to be one of the most powerful notions within the Discipline of Noticing - that others can notice from their experience what I have noticed from mine. In effect, Paul is inviting people to do what I am saying - check it out for yourself. People will come to their decisions through their own experience of reading the thesis and not through reading the opinions of either Paul or myself.

These methods are different and offer alternative ways to research, communicate and validate mathematics education. What is required is a broadening of view as to what constitutes research, how it is done and what is the 'data' on which generalisations are built. There is also a need to accept that 'rigour' is problematic within all methodologies.

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DIALOGUE

PHILOSOPHY OF MATHEMATICS EDUCATION: POM(E), PO(ME) OR POE(M) ?

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As readers of this newsletter know, the field of philosophy of mathematics has been infused with a human spirit on numerous occasions within the past couple of decades. Such infusion has had the dual effect of transforming the questions asked about the nature of mathematics and of inviting a more robust range of human drama to define the scope of its answers.

At the same time, philosophy of mathematics education has emerged as a field of inquiry. In defining that field, we have borrowed heavily from philosophy of mathematics -- both in our reaction to the old 'isms' schools focused on de-personalized foundational epistemological issues and in our embracing the new one(s) focused upon the social context of knowledge-acquisition, modification and dissemination.

Though there is an awareness by authors that philosophy of mathematics is only one of several domains from which mathematics educators seek wisdom, it is also the case that the magnetism of that perspective frequently tends to overpower others.

This brief essay begins with a recollection of two recent ones in the POME newsletter. My purpose is twofold: (1) to show in a concrete manner what I mean when I claim that the connections between philosophy of mathematics and philosophy of mathematics education have a tendency to be inappropriately (at very least prematurely) drawn; (2) to raise some general questions that suggest how we may be unduly limiting the scope of the field of philosophy of mathematics education as a consequence of our effort to strive for such connections.

Two Recent Essays

In a recent issue of the POME Newsletter, Roulet (1992) reports on the linkages drawn by prospective teachers -- indicating that while most connect an absolutist view of mathematics with transmission of facts, a small number connect a fallibilist view of mathematics with a constructivist view of pedagogy. His plea is that as teacher educators, we need to "provide [prospective teachers] with experiences that enrich their image of the discipline" if we want them to teach in a constructivist mode.

In a response, Threlfall (1994) tells us that Roulet may not have attended carefully enough to the distinction between philosophy as a set of (perhaps inappropriate or illogical) beliefs vs. a 'consideration of fundamental truth'. I interpret that description to refer to a well reasoned, persuasive argument. His point is that teaching practice is influenced by many factors other than legitimate deduction based upon the philosophy of mathematics that one adopts. He points to such categories as "moral values, the nature of society and political ideology, as well as their attitudes towards teaching and learning" (p. 12). He argues that one should therefore not conclude that Philosophy A (absolutist) alone logically implies teaching strategy X (transmission view), while Philosophy B alone (fallibilist) implies strategy Y (constructivist).

While it appears that Threlfall has inappropriately concluded that Roulet was intending to make a conceptual rather than an empirical claim based upon a loose notion of philosophical beliefs held, it is the case that if Roulet is interested primarily in having his prospective teachers adopt a constructivist orientation towards teaching, he has other options than the one he depicts. The logical point that Roulet does not appreciate is the following: If belief in Philosophy A results or is linked empirically with belief in teaching strategy X, and if belief in Philosophy B is linked empirically with teaching strategy Y, then we cannot conclude that it is *only* belief in Philosophies A and B that creates those linkages. Put suggestively but inaccurately in terms of logic: If A implies X and B implies Y, then that does not preclude the possibility that C also implies X and D also implies Y where C and D are neither philosophies nor necessarily equivalent to A and B respectively.

One might have predicted that Roulet (1992) would have seen an alternative to that of explicitly teaching a fallibilist view of the discipline as a way of establishing a constructivist attitude towards teaching. The reason is that his empirical inquiry with prospective teachers is based upon his use of Schwab's "commonplaces of teaching: the subject (mathematics), the student of mathematics, the teacher of mathematics, and the relation of mathematics learning to society in general" (p. 8).

I return then to my earlier observation that even when we see philosophy of mathematics as just one option that directs our attention to philosophy of mathematics education, in what might be an overzealous effort to overthrow the 'isms' of yesteryear, we tend to oversubscribe to the influence of philosophy of mathematics.

Towards A Broadening of the Field of Philosophy of Mathematics Education

Related to that observation, I have a number of questions and suggestions for exploration in order to influence dialogue in the domain of philosophy of mathematics education. In this short space (and limited wisdom) I cannot be as explicit as I would like to be in framing

these questions / observations / suggestions, but here are a few:

1. Below is a sketch of two competing views of philosophy of mathematics and two competing views of the way in which knowledge is acquired by the student. The Roulet article suggests that A and D are the only (most?) viable boxes. Threlfall suggests otherwise. What are arguments of a theoretical nature that would justify the inclusion of boxes C and B as possibilities? In what ways would we have to attend to the other three Schwabian 'commonplaces' in creating such linkages?

		The Student:	
		learning as knowledge	
		received	constructed
Philosophy of Math	math as absolute	**A**	**C**
	math as fallible	**B**	**D**

---

2. There is an interesting sort of ellipsis or equivocation that takes place when we conceive of the emerging field of philosophy of mathematics education. That is, many of us identify something akin to the four commonplaces of Schwab as elements that contribute to its conception. Yet notice what we do when we speak of subject matter, the first commonplace. We end up speaking not of mathematics but of *the philosophy* of mathematics. What is particularly interesting is that we do not so qualify the other three commonplaces: the student, the teacher and the relation of mathematics to society. Why is that? It seems to be a peculiar sort of qualification. Where in the equation below do we imply *philosophy of* on the bottom line? There is obviously much to say about

Philosophy of Math Education =			
Philosophy of (?)			
Math	Learning	Teaching	Society

learning that is not philosophical in nature. But for that matter there is much to say about mathematics education that is also not philosophical in nature.

3. While there was good historical reason for identifying the emerging school(s) of philosophy of mathematics in reaction to the essentially epistemological and foundational ones of the 'isms' schools, we may be unnecessarily identifying that discipline -- philosophy of mathematics -- with those two orientations alone. More precisely, what are some important questions in philosophy of mathematics education that might be informed by a philosophy of mathematics orientation that either transcends or that minimizes the differences between these two schools? Here are some questions that we appear not to ask because we are reluctant to focus on a philosophy of mathematics that levels such differences:

1. What concept(s) do we as educators convey when we speak of 'relating math to the real world'? We seem frequently to have in mind that either mathematics is to be used as a model for a 'real world' phenomenon or vice versa. Regardless of the view of philosophy of mathematics that we adopt, however, there are other options. What are they? Is there a sense in which mathematics *is* the real world? Could it be the case that the language we use for fundamental mathematical ideas (function, transformation, identity) is not separate from the real world but rather captures what it is that we view as essential qualities of that world? (See Keyser, 1916).
2. Regardless of what view of philosophy of mathematics we adopt, the concept of reason-giving and reason-seeking is central to

mathematical thought. We as educators often assume that the reasons we should seek are those that justify movement from one step to the next. But this is a limited view of rationality. What is the span of reasons that we either do or might engage in as we try to understand mathematics?

3. To the extent that rationality is something we are interested in inculcating and something we view as positive (realizing that there are factors that do and ought to temper our soul focus on rationality), how do we think about the mathematical experience in such a way that we are not only fostering *the ability* to be rational but the *desire* to be so as well?
4. To what extent and in what ways can the concept of 'problem' in mathematics influences one's view of problem in other contexts (See Brown, 1994).

I have probably overstated the case for ignoring distinctions among different philosophical orientations in raising these as legitimate questions in the field of philosophy of mathematics education. My point, however is that if we keep our eyes glued too closely to the distinctions among the schools, we may very well end up not asking some interesting philosophical questions that bear on 'mathematics' without qualification.

4. By virtue of our having distinguished these different orientations to philosophy of mathematics, we tend to identify all of mathematics with one or the other of them. Is it possible that we might find elements of different sub-fields within different schools? Or is it possible that as the history of a field evolves it tends to be better associated with one school than another?

5. The general field of philosophy of education has advanced significantly in the past four decades by virtue of its focus on educational problems making use of tools of philosophical analysis. Concepts such as teaching, learning, indoctrination, creativity, critical thinking, brainwashing, liberal education, curriculum, problem, understanding, rational emotions, morality have been explored both independently and as part of the nexus that composes the field of education. Appreciating that not all concepts refer to something in the world, but that we can gain insight into the meaning and confusion associated with them by virtue of the way in which they are used in language, the field has made significant headway in understanding such ideas and in connecting them with 'near relatives'. With regard to critical thinking and to morality for example, philosophical exploration has unearthed the sense in which these concepts cannot be 'add-ons' in an educational enterprise, but rather are built into the very concept of education. That is, given an understanding of what it means to be educated, one cannot achieve that state in the absence of critical thinking and devoid of moral manner. It is one thing to make such assertions; it is another to back them up with philosophical evidence that illuminates each of the components in the analysis beyond our understanding of their non-relational meaning. What is as valuable as the analyses of these concepts is the subsequent debate that has made us aware of the extent to which some groups have been marginalized by insensitivity to unarticulated political agendas inherent in some of the analysis. Part of the success of the field is a consequence of its disinclination to adopt a model of scholasticism in which one seeks 'first principles' from which everything of importance supposedly flows purely by an act of deduction. A relatively small amount of work in philosophy of mathematics education however seems to have harnessed this work. So many of the above concepts are integrally connected with what the field of mathematics education is about that one would have predicted that both the consequences of that research and the style of analysis would have been adopted. By relating many of these analyses to mathematics *per se* we would expand considerably our conception of philosophy of mathematics as well as philosophy of mathematics education.

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A REJOINDER TO OTTE BY ERNST VON GLASERSFELD

From the perspective of the old the new is always wrong. E. Weichselbaum

Reading Michael Otte's contribution to Ernest (1994) I realized once again how difficult it is to prevent philosophically trained readers from grafting their interpretation of constructivism upon the prevailing philosophical presuppositions. Otte's title "Is Radical Constructivism Coherent?" heralds an examination of the internal consistency or non-contradictoriness of the constructivist proposal. However, Otte proceeds to criticize it for not being consistent with conventional epistemology. He brings to bear a number of erudite arguments that would clinch his dismissal of radical constructivism if - and only if - the constructivist orientation were based on and required the metaphysical assumptions of traditional theories of knowledge.

As Otte refers to me several times in his article, I feel justified in trying once more to clear up the misunderstanding which, by the way, has led other authors to make similar but sometimes far less civilized comments. In fact, it was a pleasure to read Otte's critique, because he approaches the subject from a fund of knowledge and with a clarity that no other critic I have so far come across could muster. Nevertheless I contend that his analysis misses the fundamental element that sets radical constructivism apart from the epistemological tradition. This becomes patently obvious in the concluding paragraph of his paper.

Epistemology is not independent of metaphysics, because if we insist on identifying the object with the definition theory gives of it, we also. perhaps unwittingly, tend to define the human subject. This is in opposition to the idea that the essence of man is existential freedom. Mathematics may in part construct its own reality but always in face of the continuum of yet undefined real possibility. Otherwise such a construction loses its subject becoming instead a quasi-mechanistic process, as in the case of radical constructivism. (p.61)

The initial statement in this quotation formulates the very dogma that, as I have reiterated many times, prompted constructivism to break with tradition and to devise a theory of cognition that does without the metaphysical assumptions inherent in any ontology. Constructivism is intended to provide a model of how we may come to *know* the things we call 'objects' and is in no way concerned with what their ontological source might be. At best it may define objects we know. It has no ambitions concerning objects as they might *be* independent of a knowing subject. Nor does it want to define the knowing subject as an ontological entity, The subject enters merely as the active agent of construction, characterized by the repeatable operations it seems to be carrying out. As Otte says of mathematics, the subject "constructs its own reality", and its construction, too, faces a continuum of undefined possibilities (and impossibilities) that arise from the constructs with which it started. These possibilities and impossibilities are 'real' in the world of knowing, not ontologically real in the world of being.

In Otte's piece, the misunderstanding is launched at the beginning by his reference to Parmenides, who first spoke of an "identity between thinking and being" (p.50). I would suggest that the Parmenidean dictum is ambiguous: if "being" is taken in the ontological sense, the statement is that of a mystic who believes that our thinking and knowing is divinely preordained to coincide with the world as it is; but I prefer to take it in Berkeley's sense, namely that all that 'exists' for us is what *we* perceive and conceive. If one accepts this second interpretation, one is free to adopt any metaphysical belief one chooses, including Berkeley's that the existence of the real world is vouchsafed by the fact that God perceives it. Hence I do not think that constructivism demeans "the dignity of the individual human subject" (p.51). On the contrary, it enhances it by strengthening an attitude of tolerance and freedom, since it never resorts to claims of ontological truth as a weapon against others.

The confusion surfaces at other points. When Otte argues against Luhmann and says "We talk with other subjects about objects ..." (p.52), he implies that these objects are 'things in themselves' and not, as Luhmann and I would say, constructs arising from a subject's distinctions in his or her experiential world.

Being a meticulous thinker, Otte raises a point that goes to the heart of the matter. "If we represent an idea is there something beyond the individual symbolic representation? Are numbers different from numerals?" (p.55). In *Children's Counting Types* (Steffe *et al*., 1983), we presented the constructivist answer to that question. Numerals "represent" numbers to any subject who has acquired the mental operations necessary to construct numbers and is able to *re-present* them to him or herself. For a constructivist, this is analogous to the functioning of non-mathematical language. Words do not 'refer' to things in themselves, but they have the power to call forth *re-presentations* of experiences the subject has associated with them. Constructivism is simply a model that suggests how what we know and re-present to ourselves might be constructed in the first place. But for metaphysical realists it is as difficult to give up the traditional theory of objective reference as to relinquish the age old belief that knowledge somehow must reflect onto-logy.

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THE PEIRCEAN INTERPRETATION OF MATHEMATICS BY CHRISTOPHER ORMELL

Attitudes to mathematics tend to begin, for most of us, in the classroom. Some mathematics teachers instil in us an early awe for the subject, partly, no doubt, as a result of mechanisms they use ---consciously or unconsciously--- to hide their own mathephobia. Others, more confident mathematics teachers, try to bring-out a modified 'fun' attitude to mathematics, as a sort of *game* which can be played with symbols, and which, incidentally, reaps a pretty good harvest of social brownie points.

Thus there comes about a spectrum of attitudes towards mathematics, ranging from extreme awe (usually expressed as some form of high 'Platonism') through moderate awe ('quasi-empiricism') and slight awe ('ethnocultural mathematics') to fun ('constructivism' in various forms). The points on this spectrum represent attitudes towards mathematics, which, everyone agrees, looks at first sight like an otherwise pointless and/or mysterious 'cultural' ritual.

Since the early 1980s the status-quo defined by this spectrum has been challenged by a totally different view of mathematics - which sees mathematics as a useful aid or *tool* in science, technology and industry. According to this approach, striking attitudes towards mathematics misses the point. To the 'applicabilists', as we may call them, mathematics has a purpose: namely to help the scientist, technologist or industrialist when he or she gets in a tight corner where he or she needs some numbers. It is a point of view which gets a lot of support from politicians and industrialists, who know very little mathematics, but it leaves genuine mathematicians shaking their heads. They (the genuine mathematicians) know that only a tiny fraction, perhaps less than 1%, of mathematics has this role as a 'tool' in science, technology and industry. So the applicabilists may get a lot of political and industrial backing for their stance, but it is fundamentally flawed as a total view of mathematics. No cogent theory or interpretation of an activity can reasonably claim to apply to only 1% of that activity. In practice applicabilists tend to mix their approach with a conventional attitude towards mathematics, coloured by whatever point on the spectrum of attitudes happens currently to be in favour.

And because the total view of mathematics projected by the applicabilists is flawed, there tends to be an awkward reflection of this ambiguity in the classroom, where the kind of motivation unleashed by simple applications is likely to lead pupils astray in trying to understand 'mathematics' as a whole.

How, then, have the applicabilists got away with this conjuring trick for more than a decade? The answer is that there is a second and much more credible account of the applicability of mathematics, based on a 100% cogent view of mathematics as a subject, hovering in the background. The second non-spectrum account of mathematics is the neo-Peircean interpretation of mathematics. It has been around now for more than a quarter of a century in the form, first, of the 'Mathematics Applicable' project 1969-78 which was a conscious attempt to express it in sixth-form classroom materials, and, second, in the form of the Mathematics Applicable Group (MAG), which has carried the approach to all levels of the school mathematics curriculum. The MAG, though, is only a small group of enthusiasts, and bits of its materials have only been tried in a few hundred schools in the UK and Australia. Although the Mathematics Applicable project never wielded any power over the main UK mathematics curriculum during its ten years of operation, it did train hundreds of teachers, mostly senior teachers, in the approach. This groundwork, I'm afraid to say, served in the event only to provide a subliminal background, which made the applicabilist's case in the early 1980s much more credible than it would otherwise have been.

In effect the new exciting approaches to mathematics pioneered by the Mathematics Applicable project throughout the 1970s were partially hi-jacked, and turned, to appear to lend support to a different, older and plainly flawed 'simple applicabilist' attitude towards school mathematics.

Peirce and Russell

Peirce was already in late middle age when Russell began his logistic crusade in the 1900s to show that mathematics could be based on, and derived from, logic. Peirce's response was an essay of great perceptivity and penetration 'The Essence of Mathematics' in

which he states roundly that formal logic is not a science which helps the mathematician very much. Formal logic can be regarded, Peirce thought, more as a branch of mathematics than vice-versa. This was an area Peirce knew backwards, because he, Charles Peirce, had argued (as a logician) for many years with his father Benjamin Peirce, a leading US mathematician and Harvard professor. Russell's attempt to reduce mathematics to logic, Peirce saw, was a quest, however misguided, for philosophic respectability. To counter it, he Peirce, needed to give a properly 'philosophical' account of mathematics, an account which would place it vis-à-vis other human activities. This he did with his doctrine that mathematics was the 'science of hypothesis'.

This sounded at first innocuously obvious. After all, Russell too had said that pure mathematics is the study of propositions of the kind 'if p then q' (p and q containing only variables and logical constants).

But Peirce meant much more than that. He meant that mathematics is the activity which turns the human mind's natural tendency to entertain hypotheses into a disciplined and telling affair. It was a brilliant theory, because it gave mathematics simultaneously a fundamental human purpose (taking it off the spectrum of mere attitudes to obscurely ritualistic schoolwork) and one of great generality and intellectuality.

The rest I'm afraid is history. The West was going through a cultural crisis with end-of-century blues, and Russell's logistic philosophy of mathematics had the advantage of providing a powerful new frame for envaluing Cantor's transfinite calculus. It would be impossible to over-estimate the initial cultural appeal of Cantor's transfinite calculus, with its unique heady blend of infinity, mysticism and mathematics. It was, in effect, a religion substitute. There was a major act of faith involved: because mathematicians who swallowed it had to disown the clear perceptions of Descartes, Newton, Leibniz, Gauss, etc. that infinity was not any kind of 'number'.

Peirce, by this time, was a little-known figure, and there was no way in which, from the New World, he could mount a cultural offensive to try to wrest the initiative from the young, glamorous logicistic philosophers of Europe. After Peirce died, his work was, we know, virtually forgotten until re-discovered in the 1930s.

The position was such that, although Russell and Whitehead had to stoop to fudges to maintain nominally that mathematics reduced to logic, their overall view of mathematics as a science with high platonic authority suited the age. It was what a great many intelligent people desperately *wanted* to believe.

In my opinion the time is now ripe at last ---a century late, but better late than never--- for the Peircean interpretation of mathematics to be adopted in schools. It is an account of mathematics which links mathematics closely to the exercise of constructive imagination, because it offers the consummation of this imagination in telling answers. Unfortunately few philosophers of mathematics have sufficient imagination, themselves, as individuals, to see that this role of mathematics - to consummate the constructive imagination - can transform school mathematics into a wholly new kind of exciting subject, and one incidentally much more 'educative' than ever before. Only when a large corpus of exciting Peircean-type materials has been produced is the penny likely to drop.

The Peircean interpretation applies to the whole of mathematics, not merely to the 1% of it commonly labelled 'applicable' and used in simple applications. The applications are all to 'ifs': the hypotheses the human mind comes naturally to entertain in areas where vision is obscure. The main answers give you a valid long-term *overview* of some area of science, technology, society and even mathematics itself, not simply some 'numbers' you need (now) to continue doing what you are already doing. And on the higher levels, mathematics itself provides a uniquely fertile source of puzzling 'hypotheses', and hence for the Peircean application of mathematics.

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ON CONFORMITY - A SEARCH FOR A POWER BASE? UNA HANLEY AND OLWEN MCNAMARA

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The pattern of conformity in society exists in a fundamental tension between the desire to assert individual identity and the urge to accrue power by conforming to group norms. The latter, while offering a power base requires a surrendering of individual characteristics in order to maintain the solidarity of the collective. Whilst the collective characteristics are stressed the individual features are ignored. Recent history shows that length of hair has often been used as an indication of membership of particular groups. The long haired hippy cult of the 1960s can be seen as a reaction against the staidness of the 'short back and sides' favoured by their

parents. The subsequent arrival of the punk look served as an assertive and aggressive counterpoint to the laissez-fair manner of the hippies. It was difficult to conceive of an alternative form, it was easier simply to invert the norm.

In the context of the world of mathematics negotiating entry into the power base can be much more problematic than adopting a different hair style. Entry qualifications are more sophisticated, requirements include an affective and cognitive dimension specific to mathematics. Many students, although willing, feel themselves to be left on the periphery hampered by their perceived lack of skill or bound by the limitations of their experience and negative self-image. For most students of mathematics the rules of engagement are promoted in the mathematics lesson. The external pressure and perceived need to conform is paramount for those who choose to enter. When the task is closed the student must attempt with the rest of the group to converge on the 'right answer'. This is the mathematics he or she has come to know and expect. When the task at hand is open ended and creative, however, rules of engagement dictate what she must attempt to be different. Is this a viable option? Is creativity in mathematics simply an illusion? In a mathematics lesson concerned with creating patterns out of tiles, there is a fundamental dilemma. On the one hand the student is trying to assert her individuality through presenting a geometric pattern, on the other hand, she is influenced by the patterns she cannot avoid noticing around her and these are clearly restrictive. Her work is patterned by the properties of the geometric shapes available to her in the exercise and she is restricted by her knowledge of shape and pattern from within the society in which she lives. It is difficult for her to conceive of an alternative form. The search for something different yields as ever, versions of existing types, rather than completely different forms.

The pressure to assert identity by attending to similarities and differences is part of the negotiations involved in entering into the power base of mathematics. In attempting to be the same, the student's own thought must appear to identify very closely with the existing constructions in mathematics which many find difficult. In attempting to be different, the student finds that she can only offer variations of a kind. Creativity in mathematics may well be an illusion!

In mathematics as in other social groups, entry to the power base requires that new members have the potential to conform to existing norms. This involves an almost total surrender of individual characteristics, it is very difficult to assert difference other than by the relocation of existing structure. With so little room for manoeuvre at what cost is that entry made?

THESE BOOTS WERE MADE FOR TALKING TONY BROWN

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In his book, 'Post-Modernism, or the Cultural Logic of Late Capitalism', Jameson (1991) makes an interesting distinction between Modernism and Post-Modernism. He associates hermeneutical depth interpretation with the former and the textual superficiality of post-structuralism with the latter. He illustrates this by contrasting two paintings; 'A Pair of Boots' by Van Gogh and 'Diamond Dust Shoes' by Warhol. He suggests Van Gogh's painting of a pair of peasants boots gives rise to the possibility of various interpretations. He offers the magnificence of the bucolic landscapes we might normally associate with the paintings of 'ol' One-Ear' or alternatively, the stark peasant lifestyle that might be associated with such clothing. Either view, he suggests, can be developed as a fairly full account of what the painting might be seen as evoking. Warhol's effort, however, a dark, sparse, shadowy affair that may have been produced with the help of an x-ray machine seems to defy any such generation of stories. It seems to be all in the surface - it begins and ends with the painting.

With this as our analogy in mind how might we distinguish between Modernist and Post-Modernist production of mathematics? Interpretation of mathematics might be seen as a relatively new idea, unless you are talking about statistics or mechanics. There needs to be an emphasis on the 'activity' of mathematics before interpretation seems tenable. It is only when we consider people choosing to use some piece of mathematics or other that alternatives present themselves. This choosing however, has only recently been re-introduced into the vocabulary of school mathematics. The emphasis has generally been on mathematics where the choices have already been made. The students have typically been delivered to the text and told to work with it.

In another reflection on Post-Modernism Zizek (1989) explores the Coca Cola advert which declares 'This is it'. What is 'it'? he asks. He suggests nothing other than America itself and the associated glossy lifestyle of which Coke is supposedly part. I suggest 'mathematics' has pulled off a similar coup d'etat. Mathematics is ordered, it is logical, obeys strict methods, is fully decidable... or so they say. After carrying out mathematics for years we have looked back on it and claimed certain features as being 'it'. These features however, are completely devoid of the humans that brought them into existence... or so they say. A suggestion, such as John Mason's

(1988), that mathematics is received and assimilated as 'fragments' seems revolutionary, un-mathematical, wrong even! Mathematics which has been distilled from the activity that has given rise to it, is, it might be claimed, all in the surface.

Am I moving to suggest that mathematics has always, in a sense, been Post-Modernist? I think not! To be a real Post-Modernist you need to reject over arching accounts of the sort favoured by Plato and quite a few renowned mathematicians. Another, feature Jameson sees as distinguishing Modernism from Post-Modernism is that styles of the former have become icons of the latter- for example, a process of iteration becomes a button on a calculator. Things shrink as the field they are in expands into ever greater complexity. Interpretation comes firmly into play as we are forced to choose between an ever increasing number of things. The activity of choosing at all levels forces a permanent oscillation between interpretable mathematical activity (a Modernist bias?) and making statements as if free from the situation which gave rise to them (Post-Modernist?). It may be that this manifestation of the hermeneutic circle (Brown 1991) asserts the dynamics which prevent mathematics standing still long enough to be defined in either way.

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BOOK REVIEWS

ASPECTS OF PROOF: SPECIAL ISSUE OF EDUCATIONAL STUDIES IN MATHEMATICS

Volume **24 (1993)**. Edited by Gila Hanna and H. Neils Jahnke.

Review: **Dennis Almeida**, University of Exeter.

It can be uncomfortable for the mathematician to come to terms with the social and temporal nature of proof standards or, for that matter, the undecidability of formal logical statements: it does, after all, affect the notion of *absolute* mathematical truth. However she need not be overtly concerned for mathematics literature enjoys a very high level of reliability not shared by any of the physical sciences. In physics, for example, knowledge the half-life of the literature can be worryingly short: "The primary literature often becomes so irrelevant that it is abandoned wholesale" say physicists Jaffe and Quinn.

How is it that, despite the various vicissitudes brought about by factors including attacks on the 'unmathematical' practices of the analysts of the 18th and early 19th century, the implications of Gödel's theorem, the bickering between the various philosophies about what constitutes an acceptable proof, and transient notions of rigour, mathematics has retained a very high degree of reliability? There is ample evidence to suggest that this is due to, what Hilbert called, the 'self-correcting mechanism' in mathematics. The wife of a mathematician has put it more succinctly "I know what you guys are doing. First you shoot the arrow and then you draw the bullseye." Even Bourbaki acknowledge this: Dieudonné has said of the self-correcting mechanism that "we consider it today as one of the great victories of the human spirit.".

At the present time the stability enjoyed by mathematics via the role of mathematical proof is being subjected to two important influences at different levels. One is due to the increasing mathematization of the sciences. Here the concept of proof is being re-examined - is a computer proof surveyable and reliable? Is a heuristic proof in applied mathematics always unacceptable? The other is, of course, the relatively recent reforms in mathematics teaching which place emphases on learner engagement in the construction and acquisition of knowledge. This has influenced the concept and role of proof in the teaching and learning of mathematics. At the most fundamental level an argument may be seen as a proof if the learner is thus convinced about the truth of the relevant proposition. This contrasts with the higher level where an argument is viewed as a proof if an expert (the *Ideal Mathematician*!) is thus convinced about the truth of the relevant proposition.

The Special Edition of Educational Studies in Mathematics: *Aspects of Proof* is a response to the recent developments in the role and meaning of mathematical proof. *Aspects of Proof* contain papers by Philip Davis (on visual theorems), Man-Keung Siu (on proof practices in ancient China), Daniel Chazan (on geometrical reasoning by students), Reuben Hersh (on the nature and role of proof), Jean Dhombres (on a many proof method in mathematics), and Gila Hanna and H. Niels Jahnke (on proof and application).

I have found *Aspects of Proof* to be informative and valuable. Firstly, as one involved in mathematics education, I was enlightened by the papers which dealt with proof practices in the classroom (Chazan, Hanna and Jahnke, and, latterly, Hersh). Secondly, as one who has been concerned by the variation in proof standards between mathematics and mathematical physics, I was deeply interested in Davis' paper which boldly discusses redressing imbalance of credit which currently favours the prover rather than the discoverer: the rationale is 'yes we need logical proof, but logic rarely leads to any discovery'. Thirdly, from a historical and cultural viewpoint, it was important to see and read Man-Keung Siu's paper on proof in ancient China: the picture that emerges is that proof is seen an 'explanatory note'. Finally, as one convinced that different perspectives in mathematics teaching and learning can aid understanding, I empathised with the paper by Jean Dhombres which studies the 'many proof' practice of the seventeenth century mathematician Gregory of Saint-Vincent.

The paper by Chazan stood out as it highlights an interesting concern which I share. Chazan says "as students become more sceptical about measurement of examples might they also become sceptical about the power of deductive proofs?" Put another way: it is *natural* to be convinced by a carefully conducted experiment in the real world; if suspicion is cast on this *natural* method of convincing then the same shadow of doubt may fall on the *abstract* method (this may have some significance in the mathematics/science debate). Parallel research by Finlow-Bates, Lerman, and Morgan has indicates why this could be so: they found that students in their study viewed empirical verification as 'proof'(it was more convincing) and logical argument as 'explanation' (it explains the examples). The issue must surely be given further attention by researchers.

Coincidentally, *Aspects of Proof* was published at around the same time as the Bulletin of the American Mathematical Society published papers reflecting an ongoing debate in the mathematical community about, amongst other things, rigour, proof, and variations of standards of proof in applied mathematics. Given that *Aspects of Proof* is a response to the recent developments in the role and meaning of mathematical proof it is not very surprising to find close or identical statements being made in the BAMS debate. It is interesting to note the parallel expressions in *Aspects of Proof* and the debate in BAMS (B) - for it gives additional credence, if any were needed, to the fact that the developments in the role and meaning of mathematical proof that *Aspects of Proof* address are real and relevant:

Hersh describes the phenomena of variation in standards of proof and rigour as does Armand Borel (B). Hersh also talks about the (un)surveyability of proofs and computer assisted proofs, something that Morris Hirsch (B) also discusses at some length (he is doubtful about computer assisted proofs). The mathematician Thurston (B) and Man-Keung Siu both argue that proofs should primarily aim to increase *human understanding* rather than aim to be rigorous - indeed Rene Thom (B) proposes that rigorous mathematics should be identified by a tombstone to indicate "graveyard mathematics". The paper by Chazan which deals with the interaction between empirical and deductive reasoning in high school students have some links with the paper by Jaffe and Quinn (B) which compare proofs in mathematics with experiments in physics. Hersh and Davis discuss the critical relationship between intuition and logic, as do Atiyah and Zeeman (B).

There is an important feature in the paper by Man-Keung Siu: "If the only role of a proof were verification, nothing would be gained by giving different proofs of the same theorem. But different proofs serve not merely to convince but also to enlighten". In the face of computer assisted 'proofs' mathematicians are beginning to *recall* the necessity of alternative proofs which will reflect, what Thurston (B) calls, " a continuing desire for *human understanding* of a proof, in addition to knowledge that the theorem is true". Hersh also describes this phenomenon and quotes Halmos' view of the Appel-Haken proof of the Four Colour Theorem: "The present proof relies in effect on an Oracle, and I say down with Oracles! They are not mathematics."

The paper on *Visual Theorems* by Davis also stands out because it deals with the validation of knowledge in the physical sciences. Davis says that acceptance of the visual aspects of mathematical intuition and reasoning "would allow the mathematical education establishment to come to terms with those aspects of mathematics that are required by physicists, engineers, etc. and of the criteria by which these related professions validate their work". If the mathematics education establishment does attend to these aspects it will have to relinquish what Davis describes as "(the assertion) that mathematical proof as currently practised is the only certification of mathematical truth". Benoit Mandelbrot (B) uses stronger language: "For its own good and that of the sciences, it is critical that mathematics should belong to no self appointed group; no one has, or should pretend to, the authority of ruling its use".

Formal mathematics with its reliance on mathematical proof (as currently practised) is detrimental to the practice of discovery: in mathematics there is the rule that the prover gets the lions share of the credit, the inventor or discoverer is given little. Davis' proposal is aimed at restoring the balance.

In their important paper, Hanna and Jahnke, outline concepts which relate the formal aspects of mathematical proof to their pragmatic dimensions: justification and the basis for justification are intrinsically related issues in the classroom. They also, relevantly, discuss the implications of this proposal for teaching. In proposing a *pragmatic basis for proof* in the classroom they rightly, in my view, state that "mathematics education has a genuine scientific and philosophical task to perform". The same could also be said in relation to the proposal by Davis.

In the current era where mathematics is attempting to come to terms with the global reforms in the mathematics curriculum, the use of computing technology, and the increasing mathematization of the sciences, the issues discussed in *Aspects of Proof* are highly relevant to all creators, users, and philosophers of mathematics.

EXPLORATIONS IN ETHNOMATHEMATICS AND SIPATSI, PAULUS GERDES ET AL.

Explorations in Ethnomathematics and Ethnoscience in Mozambique, Paulus Gerdes (Ed.), 1994, 77 pp, and **SIPATSI: Technology, Art and Geometry in Inhambane,** Paulus Gerdes & Gildo Bulafo, Eds, 1994, Instituto Superior Pedagógico, Maputo, Mozambique, 103 pp

Ethnomathematics (resp. science) is the holistic and evolutionary relationship between culture and mathematics (resp. science) pioneered by Ubiratan d'Ambrosio and Paulus Gerdes.

One of the themes of the Ethnomathematics Research Project from which these two books originate is that ethnomathematics and ethnoscience is relevant for nation building in the former colonies. At the time of independence, the ex-colonies had the choice of continuing with development and educational systems imposed by the ex-imperial powers or, as argued by Julius Nyerere in Tanzania, to reform these taking their own cultures into account. The latter perspective has not always gained currency in other ex-colonies; in India, Gandhi who shared Nyerere's view had to give way to Nehru who had a vision of an European-industrialised subcontinent. In **Explorations** Gerdes argues that the penalty for such a misguided perception is "indifference, alienation and social discord" .... a description that could apply to present day India

Ethnomathematics and ethnoscience is also relevant for the school curriculum in the first world. An aculutural procedural presentation of mathematics and science is likely to evoke negative responses from both black and white school pupils. For all pupils these include a lack of understanding of the concepts. For black students a poor self-image may result. These words by Gerdes about the situation in southern Africa are equally applicable in the first world with its multicultural aspects: "Unadapted and elitist, the existing educational systems feed the crisis by producing economically and socially unadapted people, and by being heedless of entire sections of the active population."

Explorations contains a numbers of papers and reports of on-going research in ethnomathematics and ethnoscience by workers in the Instituto Superior Pedagógico. The paper by Jan Draisma on the theorem '8 + 5 = 13' compares the responses of illiterate women, street-children and school children to the problem '8 + 5 = ?'. Draisma poses the question that all mathematics educators and student teachers ought to address: *If the illiterate street-children and school-children of the same age have the same arithmetic ability then what is the point of schooling?* Daniel Soares and Abdulcarimo Ismael present an interesting and delightful account of some popular counting methods in Mozambique. **Explorations** also contains a number of papers on symmetry in cultural artefacts; these possess many resources to enrich the mathematical experiences of all school children. **SIPATSI** is a detailed account of the frieze symmetries found in *sipatsi* or woven handbags made in Inhambane province. Using the parallelogram frame that the weavers use, Gerdes classifies sipatsi patterns using a generic classification system using the dimensions of the unit parallelogram frame and then establishes a correspondence between these and the seven conventional frieze types. Gerdes also includes reflective sections on the geometry of sipatsi. For both its cultural aspects and excellent mathematics, **SIPATSI** ought to read by all mathematics educators and student teachers.

Finally, on a personal note, I was pleasantly surprised by the papers on ethnoscience in **Explorations**. Not least because they

reminded me of my own experiences in school in India where I also learnt about roots and twigs for cleaning teeth, fruits that could substitute for soap, and traditional interpretations about nature. It was during these school experiences that I learnt about the Hindi words *dus* (10) and *mul* (residue) which when combined together is the root of the English word *decimal*. Only recently I realised why, for a long period after reaching secondary school age, I believed that the word *decimal* originated in Europe.

PUBLICATION ANNOUNCEMENTS

One outcome of the contribution of the Philosophy of Mathematics Education Group to ICME-6 in Quebec, 1992, was a book building on the contributions of speakers. This was ultimately realised in the following two books, the first permanent fruits of the PoME Network.

MATHEMATICS, EDUCATION AND PHILOSOPHY: AN INTERNATIONAL PERSPECTIVE

Edited by Paul Ernest

Studies in Mathematics Education Vol. 3, The Falmer Press, London, 1994. £35. ISBN 0 7507 0290 7.

This book addresses the central problem of the philosophy of mathematics education: the impact of conceptions of mathematics on educational practice. It also embodies a far-reaching interdisciplinary enquiry into philosophical and reflective aspects of mathematics and mathematics education. It combines fallibilist and social philosophies of mathematics with exciting new analyses from post-structuralist and post-modernist theorists, offering both reconceptualisations and critiques of mathematics and mathematics education. The outcome is a set of new perspectives which bring out the human face of mathematics, as well as acknowledging its social responsibility.

Reconceptualising the Philosophy of Mathematics

Reuben Hersh, Fresh Winds in the Philosophy of Mathematics

David Bloor, What can the Sociologist of Knowledge say about 2+2=4?

Paul Ernest, The Dialogical Nature of Mathematics

Thomas Tymoczko, Structuralism and Post-Modernism in the Philosophy of Mathematics

Post-Modernist and Post-Structuralist Approaches

Valerie Walkerdine, Reasoning in a Post-Modern Age

Brian Rotman, Mathematical Writing, Thinking and Virtual Reality

Anna Tsatsaroni and Jeff Evans, Mathematics: The Problematical Notion of Closure

David Jardine, On the Ecologies of Mathematical Language and the Rhythms of the Earth

Paul Dowling, Discursive Saturation and School Mathematics Texts: a strand from a language of description

Jeff Vass, The Dominance of Structure in 'Post-Structural' Critiques of Mathematics Education

Tony Brown, Describing the Mathematics you are a part of: A Post-structuralist account of Mathematical Learning

The Human Face of Mathematics

Philip J. Davis, Mathematics and Art

Hao Wang, Skolem and Gödel

George Gheverghese Joseph, Different Ways of Knowing: Contrasting styles of argument in Indian and Greek Traditions

The Social Context of Mathematics and Education

Sal Restivo, The Social Life of Mathematics

Mairéad Dunne and Jayne Johnston, Research in Gender and Mathematics Education: the Production of Difference

Ubiratan D'Ambrosio, Ethnomathematics, the Nature of Mathematics and Mathematics Education

CONSTRUCTING MATHEMATICAL KNOWLEDGE: EPISTEMOLOGY AND MATHEMATICS EDUCATION

Edited by Paul Ernest

Studies in Mathematics Education Vol. 4, The Falmer Press, London, 1994. £35. ISBN 0 7507 0354 7

This book provides a panorama of complementary and forward looking perspectives on the learning of mathematics and epistemology. It explores constructivist and social theories of learning, and discusses the role of the computer in the light of these theories. It brings new analyses from psychoanalysis, Hermeneutics and other perspectives to bear on the issues of mathematics and learning. It enquires into the nature of enquiry itself, and an important emergent theme is the role of language. Finally it relates the history of mathematics to its teaching and learning. The book both surveys current research and indicates orientations for fruitful work in the future.

Constructivism and the Learning of Mathematics

Ernst Von Glasersfeld, A Radical Constructivist View of Basic Mathematical Constructs

Leslie P. Steffe and Ron Tzur, Interaction and Children's Mathematics

Robert Thomas, Radical Constructive Criticisms of Von Glasersfeld's Radical Constructivism

Stephen Lerman, Articulating Theories of Mathematics Learning

Michael Otte, Is Radical Constructivism Coherent?

Paul Ernest, Social Constructivism and the Psychology of Mathematics Education

Erick Smith, Computers and the construction of the other in the mathematics classroom

Kathryn Crawford, The Context of Cognition: The Challenge of Technology

Psychology, Epistemology and Hermeneutics

David Pimm, Another Psychology of Mathematics Education

Dick Tahta, On Interpretation

Philip Maher, Potential Space and Mathematical Reality

Tony Brown, Towards a Hermeneutical Understanding of Mathematics and Mathematical Learning

Falk Seeger and Heinz Steinbring, The Myth of Mathematics

Enquiry in Mathematics Education

Stephen I. Brown, The Problem of the Problem and Curriculum Fallacies

John Mason, Enquiry in Mathematics and in Mathematics Education

Marjorie Siegel and Raffaella Borasi, Demystifying Mathematics Education through Inquiry

Charles Desforges and Stephen Bristow, Reading to Learn Mathematics in the Primary Age Range

History, Mathematics and Education

Francesco Speranza, The Idea of 'Revolution' as an Instrument for the Study of the Development of Mathematics and its Applications to Education

Anna Sfard, Mathematical Practices, Anomalies, and Classroom Communication Problems.

STUDIES IN MATHEMATICS EDUCATION SERIES

Falmer Press, Series Editor: Paul Ernest

This series is founded on the philosophy that theory is the practitioner's most powerful tool in understanding and changing practice. If you write to Falmer they will put you on their mailing list for news of future volumes. Nos. 2-6 were published in hardback only but Falmer have agreed to publish *all* of the books in paperback after a year.

CURRENT VOLUMES

1. The Philosophy of Mathematics Education, Paul Ernest, 1991.

2. Understanding in Mathematics, Anna Sierpinska, 1994.

5. Investigating Mathematics Teaching: A Constructivist Enquiry, Barbara Jaworski, 1994.

6. Radical Constructivism: A Way of Knowing and Learning, Ernst von Glasersfeld, 1995.

PHILOSOPHIA MATHEMATICA

This unique journal is devoted to the philosophy of mathematics and but treats broader subjects of interest to mathematics educators. For details contact Robert Thomas, Applied Mathematics, University of Manitoba, Winnipeg, Manitoba, Canada R3T 2N2. E-mail Robert_Thomas@UManitoba.ca Fax: (204)275-1498

IN MEMORIAM STIEG MELLIN-OLSEN

Stieg, who died earlier this year, was a very special person. He was a practitioner, researcher and theorist in mathematics education who lived his beliefs with total integrity and eschewed any egotistical activity. He will be remembered as a fine human being; for his total commitment to social justice, and for his important work such as creator of the relational-instrumental distinction, his ground-breaking book *The Politics of Mathematics Education (Kluwer, 1987),* and for founding with London colleagues the Political Dimensions of Mathematics Education conferences. PDME-3 will be held

this Summer in Bergen and it will be infused with good feelings and good memories of Stieg, as well as with the inevitable sadness that follows such a great loss.

MATHS EDUCATION ON THE INTERNET

E-Mail Discussion Lists

To join MathsEd-L send an E-mail message to: listserv@deakin.OZ.AU. It should have no subject line and in the body a single line:

SUBSCRIBE MATHSED-L <your name>

Discussion group associated with Chreods: Email mailbase@mailbase.ac.uk with in the body of the message (without the brackets!): subscribe chreods <first name> <last name>

History and Philosophy of Science and Science Teaching List: HPSST-L@QUCDN.QueensU.CA

To join - maths-education@nottingham.ac.uk subscribe similarly to: owner-maths-education@nottingham.ac.uk

Maths WWW Pages

Paul Ernest's homepage & PoME can be found at **http://www.ex.ac.uk/~PErnest/** This also carries all previous editions of the newsletter. It will also include a file of reactions, comments, discussion on PoMEnews. Send your contribution to **PErnest@ex.ac.uk**

Mathematics archives - URL: **http://archives.math.utk.edu**

Leeds maths www server - **http://WWW.amsta.leeds.ac.uk/**

MAA - http://www.maa.org/

TERC - http://hub.terc.edu/

http://s13a.math.aca.mmu.ac.uk

Teachers as Researchers: **http://www.cs.uop.edu/edulink/CAR.html**

Nottingham pages about Maths Education **http://acorn.educ.nottingham.ac.uk//SchEd/pages/gates/**

Dave Wilson's pages at Manchester and links to CHREODS - **http://s13a.math.aca.mmu.ac.uk/**

Freudenthal Institute - **http://www.fi.ruu.nl/**

Nottingham Shell Centre pages - **http://acorn.educ.nottingham.ac.uk/ShellCent/welcome.html**

ADULTS LEARNING MATHS: A RESEARCH FORUM (ALM)

Dr Diana Coben, *ALM Chair, Goldsmiths College, University of London, Lewisham Way, New Cross, London SE14 6NW, UK. Fax: 0171 919 7313 Email: aea01dcc@gold.ac.uk*

ALM is a new international research forum bringing together researchers and practitioners in adult mathematics/numeracy teaching and learning to share ideas, information and research findings in order to promote the learning of mathematics by adults. We welcome members who are involved in philosophical research with adults and are keen to network with POME and other organisations which

share our interest in research on adults learning mathematics. Details of ALM are in the ALM Newsletter, on the Internet on <numeracy@world.std.com>; copies of the ALM Newsletter and Proceedings of the Inaugural Conference are available from Diana Coben.

ALM-2, the second ALM conference, will be held July 7-9 1995 at the University of Exeter. For details contact: Anne Chammings, CET Division, DCAE, University of Exeter, Cotley, Streatham Rise, Exeter, Devon EX4 4PE. tel. 01392-411906 fax. 01392-436082.