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This Newsletter is the publication of the

PHILOSOPHY OF MATHEMATICS EDUCATION NETWORK

ORGANISING GROUP

AIMS OF THE NEWSLETTER

The aims of this newsletter are: to foster awareness of philosophical aspects of mathematics education and mathematics, understood broadly; to disseminate news of events and new thinking in these topics to interested persons; and to encourage informal communication, international co-operation and dialogue between teachers, scholars and others engaged in such research.

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EDITOR OF NEXT ISSUE OF POMENEWS

The editor of POMENEWS8 will be Paul Ernest. Please send any items for inclusion to him. These may include short contributions, discussions, provocations, notices of research groups, conferences, publications, and also books for review.

EDITORIAL

When reacting to an earlier issue of POMENEWS, I suggested to Paul Ernest that it would assist in making the newsletter more international in flavour if at least some of the items could come from sources other than the United Kingdom. That comment from me was enough for Paul to suggest that perhaps I'd be willing to take on the task of editing an issue of the newsletter with this goal in mind. Although I do not claim to have been responsible for the solicitation of all such items, you will see some progress has been made towards this goal with input from writers in China, Canada, and the United States, . Nonetheless, it is a start. Paul and I encourage writers from other parts of the world--and there are lots of them included on the mailing list for POMENEWS--to send their items to Paul for use in future issues.

This issue sees a reorganization of POMENEWS. In the first instance the **Discussion** section is designed to provide a location for in-depth comment around a particular theme or issue. Second, the **Dialogue** section is a place where interaction amongst readers--in response to issues raised in previous editions of POMENEWS, or flowing from the **Discussion** section, or both--can be fostered. Third, a section devoted to brief reports on **International Research Projects** has been added which will, we hope, expand the knowledge each of us have of research efforts around the globe. Sections for news about **Conferences**--past and future--**Publications** of books and journals, and the traditional **Book Reviews** round out this issue of POMENEWS.

I want to particularly thank Anita LoSasso for giving permission for the papers she wrote for a graduate class to be edited for use here. Thanks go as well to the folks in the Print Shop at Simon Fraser University who put the newsletter together on a very tight timeline. Though Paul Ernest did not undertake the editing of this issue, it will be clear to all readers that he contributed much of the material found here.

It has been an enjoyable task putting this newsletter together. Perhaps others will volunteer to take up the challenge in the future, and add the particular flavour of their homeland to the interactions found in these pages.

Sandy Dawson

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DISCUSSION THEME

SOME REMARKS ON THE PHILOSOPHY OF MATHEMATICS EDUCATION

Zheng Yuxin (Y. Zheng)
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What is philosophy of mathematics education? Before answering that question, I believe it would be better to consider first the following similar question: What is philosophy of mathematics? And particularly, can some general philosophical talk of mathematicians, or their conscious philosophical reflections on mathematical works, be regarded as philosophy of mathematics?

Such talk, and especially, the conscious philosophical reflections of great mathematicians on their own works, are no doubt of great significance to the study of philosophy of mathematics (and mathematical research as well). However, as a definite answer to the above questions, I would say that they should not be identified with philosophy of mathematics, that is, they should not be taken as the main part of philosophy of mathematics, because, the latter, just an any other branch of knowledge, also has its special topics

Thus, I think philosophy of mathematics education should also have its special topics, and not be identified with any *arbitrary* philosophical talk or reflections generated in the process of mathematics education (including both research and teaching activities). But, unfortunately, I cannot resist feeling such arbitrariness (more or less) by reading the topic group presentations of ICME 7 and the tentative outline for the book *Mathematics, Education, and Philosophy: An International Perspective.*

Furthermore, just as the recent development of philosophy of mathematics has shown clearly, the nature of mathematics can and should be analyzed from different angles, such as from the social cultural angle; but, philosophy of mathematics is in the final analysis a branch of philosophy rather than other subjects. Similarly, the philosophy of mathematics education also belongs to philosophy, and therefore should not include topics without (explicitly or implicitly) philosophical meaning. For example, I think *gender and race in mathematics education* is just such a topic.

As a direct answer, I think *philosophy of mathematics education* should be centred on the following three questions:

(1) What is mathematics? This is the view of mathematics

(2) Why should we learn (or teach) mathematics? This concerns the value of mathematics and the aim of mathematics education as well.

(3) How should we learn (and teach) mathematics? This is the epistemological study of the nature of mathematics learning.

As this understanding is similar to the explication put forward by Paul Ernest in his presentation on *the philosophy of mathematics education* at ICME 7, I should say my idea was given independently in an article published in 1992 (Zheng, 1992). Nonetheless, the following points seem worthy of emphasis.

Firstly, the shift away from absolute to fallibilist or quasi-empirical views of mathematics is of great importance, because the latter offers a more realistic description of actual mathematical activities. On the other hand, we should not go from one extreme to the other, that is, we should see clearly the duality (dialectical nature) of mathematics. For example, as far as the social constructivist view of mathematics is concerned, we should first confirm its rationality, especially the claim that mathematical knowledge is nothing but the product of human creative acts, which in turn is of the nature of social construction.

We should at the same time also notice the specialness of mathematics from the view of the *products* of mathematical activities, that is, by the analysis of mathematical knowledge. For example, an important aspect of the particularity of mathematics we should confirm is both the *informal* and *formal* nature of mathematics (by the latter we mean the justification of mathematical statements can neither be intuition nor physical experience but *formal* proofs). Otherwise, what we get would not be a real picture of actual mathematical activities. Furthermore, the *formal* construction is also one of the basic ways by which mathematical entities own their objectivity, that is, as an explanation for the objectivity of mathematics. We should see not only that "mathematics is intersubjectively agreed", but also that the formal definitions of mathematics entities (no matter whether they are implicit or explicit) already imply the transformation from "individual creation" to "objective existence" (cf., [5]). Besides, I think it is also a result of analyzing from the aspect of *products* that some mathematicians have proposed that mathematics is the science of patterns ([2], [4]); but, in comparison, I would like to suggest the following *definition*, as it shows the specialty of mathematics from both the aspect of *products* and *process:*

Mathematicians study reality by constructing relatively independent patterns and taking them as the direct objects of their studies.

Secondly, as far as the question why we should learn (or teach) mathematics is concerned, I think the philosophy of mathematics education does not aim to develop some concrete curriculum, neither does it intend to make detailed comments on any extant curriculum, but rather to give some basic criteria for such concrete studies or to offer a basic theoretical framework for them.

To explicate, I think, in order to get an answer to this question, we should first make clear what mathematics (education) *could do*, that is, we should make clear the value of mathematics, which means not only the special significance of mathematics for the development of intellect of individuals, but also its great importance for the development of the whole society. (On this point, I should say the *critical mathematics education* view is very instructive: the influence of mathematics is both liberating and enslaving.)

As a basic criteria, I think we should emphasize the *timeliness* of mathematics education, which is to say, mathematics education should make the following three *accordances*:

(1) an accordance with the development of the society. This means not only that mathematics education should meet the needs of society, that is, to cultivate the people the society needs, but also that it should use fully the material, technical and cultural conditions the society offers. As far as modern time is concerned, it can be described roughly by saying that we should develop mathematics education for the information age.

(2) an accordance with the development of mathematics. This means not only the renewal of content, such as introducing new material, but also that mathematics education should reflect correctly the modern view of mathematics, such as the modern understanding of elementary mathematics.

(3) an accordance with the development of education. This is concerned directly with the question of whether or not mathematics education could be a science. For example, we should pay more attention to the cognitive approach to education.

Thirdly, a deep analysis of the nature of mathematics learning (and teaching) is obviously a main step towards a satisfactory answer to the question how we should learn (and teach) mathematics. Generally speaking, the detailed study of the mental process of mathematical learning forms the subject of psychology of mathematics learning, and is a weak point in the studies of mathematics education today. One reason for this weakness, I think, lies in the following divorce: most psychologists do not know much mathematics so their studies are usually confined to general theory of learning or very elementary mathematical learning. On the other hand, our mathematical educators are not very familiar with new developments in psychology so their work in this area remains at the level of experience.

According to the above analysis, we can see that the basic way to build a scientific psychology of mathematics learning is as follows: we should take the modern results of general psychology as guides, and make further analysis for the specialness of mathematical learning. Modern studies of psychology have shown clearly that cognition is not simply a reflection of the outer world to the mind, but an active process of construction based on former knowledge and experience: the first step to build the psychology of mathematics learning is to shift from the traditional view of passive reception to the constructive view of mathematics learning. Furthermore, a scientific theory of mathematics learning could not be built by simply combining general psychology of learning with mathematics, such as inserting mathematical contents or examples in the theoretical framework of general psychology of learning, but rather relying on direct studies of the special phenomena and problems of mathematics learning with the guidance of the positive results of modern studies of psychology, especially the constructive view of learning. I think the modern study of the process of problem solving has already made a good beginning in this direction (c.f., [3]).

Fourthly, on the whole we should see both the relationship and difference between philosophy of mathematics and philosophy of mathematics education. Obviously, the question *what is mathematics?* belongs to the field of philosophy of mathematics, and what is more, the epistemology of mathematics. The latter is a fundamental area of the philosophy of mathematics, but it also gives rise to the study of the question of *how we should learn (or teach) mathematics*. Therefore, there is some overlapping between these two subjects. However, the aim of a philosophy of mathematics education is to lay a theoretical found for itself, for the actual activities of learning (and teaching) mathematics. A philosophy of mathematics, on the other hand, is usually greatly divorced from actual mathematics activities (including both research and mathematics education). In this regard, a philosophy of mathematics education could serve as a model for a philosophy of mathematics by guiding philosophy of mathematics to actual mathematical activities.

Finally, I would like to emphasize again the great importance of philosophy of mathematics education. It is true to say that the theoretical foundations of the theory of mathematics education include the following subjects: philosophy, mathematics, education, psychology, logic, and computer science. However, given the above discussion, we now know clearly that, as an analysis for the theoretical foundations of mathematics education, we should not limit to list all the relevant subjects, but rather to set up its own theoretical foundation, that is, the philosophy of mathematics education.

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In response to Professor Zheng

Paul Ernest
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Professor Zheng raises some interesting and important issues about the nature of the philosophy of mathematics and the philosophy of mathematics education. In response, I argue below in favour of a broad conception of philosophy of mathematics and the philosophy of mathematics education, and for the centrality of the issues of values and the social milieu in both epistemology and mathematics education. In responding to his remarks, I have not attended to areas of agreement between myself and Prof. Zheng because I know that many of the issues contested by us are of great interest others. We agree on the key importance of the shift from absolutist to fallibilist philosophies of mathematics, and the need for this to adequately account for mathematical methodology and the practices of mathematicians; we agree that the philosophy of mathematics education is not about developing a curriculum, but about a theoretical foundation on which curricula might be developed. We both agree on the novel and important insights that a philosophical perspective can bring to mathematics education.

What is the Philosophy of Mathematics?

First of all, what is the philosophy of mathematics? Does it include a wide range of reasonably well-articulated reflections of persons, such as mathematicians and others with an interest in and knowledge of mathematics, about the nature of mathematics? Clearly the answer will depend on the style and depth of such discussion and reflection. But an ambiguity should be signaled. The academic philosophy of mathematics, is a specialist field of knowledge and research which is today a branch of philosophy, which has its own basic ideas, topics, methods, styles of reasoning, literature, and community of scholars. In addition, there is also a broader domain of questions, interests, literature, and a range of non-specialist but interested scholars, concerned with reflecting on the nature of mathematics. These include mathematicians, historians, anthropologists, sociologists, and educationists specialising in mathematics, and interested in issues concerning the nature of mathematics. It also includes historians of ideas, and interdisciplinary 'European' (e.g. French and German) philosophers who reflect on the nature and role of mathematics in the overall current of human ideas.

One reaction to this distinction is to argue that the former, tighter academic grouping constitutes *real* philosophy of mathematics, and the latter is some more popular conjunction of interests, but not a proper field of study, not the philosophy of mathematics. This claim is defensible, for it is much easier to identify the *problematique* of the former group. It is largely concerned with reflection on issues of epistemology and ontology concerning mathematics, and uses the styles of thought and reasoning acceptable to the philosophical community at large, and publishes in general philosophical journals.

However this claim, in my view, also suffers from a number of overwhelming weaknesses. First of all, the 'professionalized' image of the philosophy of mathematics is a recent construction (Kitcher and Aspray, 1988). Earlier this century the philosophy of mathematics evidenced in the works of Frege, Russell, Hilbert, Brouwer, Heyting, Weyl, Carnap, Curry, Gödel, and others looked more like a branch of mathematics (foundations). Until recently, the foundations of mathematics and the philosophy of mathematics were widely regarded as identical. Modern academic philosophy of mathematics is itself a modern construct. Thus the philosophy of mathematics - even in its narrowest and most professionalized sense - is not static, but continues to develop in scope and focus. Currently there is a growth of interest in the psychology of mathematics within the philosophy of mathematics community, in order to relate reflection on individual knowing in mathematics to the real constraints of psychology and cognition (Detlefson, Kitcher). Thus due to internal forces, a broadening of its scope to include some of the issues from the 'popular domain' is taking place.

Second, there is a multi-disciplinary 'maverick' tradition which has emerged in the academic philosophy of mathematics which has broadened the *problematique* to include especially the issues of the history of mathematics and the methodology of mathematicians in the philosophy of mathematics (Lakatos, Kitcher, Wang, Tymoczko). Thus academic philosophy of mathematics is not a unity, and at least one of the branches has adopted a broad scope incorporating issues of history and practice.

Third, throughout this and the last century there has been yet another a tradition of European philosophers philosophising about mathematics, including Husserl, Enriques, Bachelard, Mannoury, Serres. Such scholars have always related the nature of mathematics either to the conditions of human experience, or to culture and history, or both. But this tradition has been ignored by academic, Anglo-American philosophy of mathematics, until recently.

Fourth, there is no dedicated journal on the philosophy of mathematics narrowly focused in the first sense. Throughout its time of publication the only dedicated journal *Philosophia Mathematica* has included papers from the broader second view of the philosophy of mathematics, and it explicitly intends to continue to do so as the manifesto for the recent revival testifies.

Fifth, there is also a general argument from first principles for a broader view of the philosophy of mathematics. The argument is that the philosophy of mathematics should not be defined retrospectively in terms of what academic philosophers of mathematics happen to have concerned themselves with in the past. The field is better defined in terms of its object of study: namely, systematic reflection on the nature of mathematics and on mathematical knowledge. What constitutes legitimate reflection of this sort is something that needs to be treated explicitly within the philosophy of mathematics, rather than assumed at the outset.

Of course the issue of who decides who is a 'real' philosopher of mathematics' is problematic and risks a vicious circle, in which a professional clique define a field to serve their own interests, and not those of either the broader academic community with an interest in the issue (including mathematicians, educationists, and other scholars) or those in society in the large, who ultimately provide the support for academia. (Likewise, the broader academic community may argue in favour of a broader view of the philosophy of mathematics, as I do, which may be seen as serving to legitimate *its* interest in the field.)

- My position is that the philosophy of mathematics needs to be broadened and reconceptualised to include 'external' factors related to the historical, methodological and wider cultural and social aspects of mathematics, just as has happened in the modern philosophy of science, following the contributions of Kuhn, Feyerabend, Lakatos, Toulmin, Laudan, and others. The breakdown of the foundationist project of Modernism of securing absolutely certain foundations for mathematical knowledge (the logicist, intuitionist and formalist programmes based on the Euclidean paradigm), is well documented (Davis and Hersh, Kline, Tiles). Elsewhere I have argued that the establishment of mathematical knowledge as fallible and quasi-empirical means that mathematics is not hermetically sealed-off and separable from other areas of human knowledge and activity (and values). This means that in mathematics (as in science and other areas of human knowledge), the contexts of discovery and justification interpenetrate. Consequently social and cultural (and ethical) issues cannot legitimately be denied an impact on mathematics and mathematical knowledge and must instead be admitted as playing an essential and constitutive role in the nature of mathematical knowledge.

For the above reasons I have argued (Ernest, 1991, Forthcoming) that the philosophy of mathematics needs to be broadened and reconceptualised to include a broad range of issues as follows:

o epistemology (the nature, genesis and justification of mathematical knowledge, and proof),

o ontology (the nature and origins of mathematical objects and relations with language),

o mathematical theories (constructive and structural, their nature, development, and appraisal),

o the applications of mathematics (and relations with other areas of knowledge and values),

o mathematical practice and methodology, and o the learning of mathematics (and its role in knowledge-transmission and creativity).

The moral to be drawn from the above argument is that the nature of the philosophy of mathematics is an area of contestation, and there are different perspectives of its nature. As a presumably reflexive domain, the issue of what constitutes the nature and scope of the philosophy of mathematics and what is its *problematique*, are issues that need to be debated inside the field, and a multiplicity of viewpoints and positions on these issues acknowledged.

In this conclusion I do not differ too much from Prof. Zheng, who agrees that "the recent development of the philosophy of mathematics has shown clearly, the nature of mathematics can and should be analysed from different angles, such as from the social-cultural angle...".

What is the Philosophy of Mathematics Education?

However, this agreement ceases when the issue of the philosophy of mathematics education arises, for Prof. Zheng argues that "the philosophy of mathematics education also belongs to philosophy, and therefore, should not include topics without (explicitly or implicitly) philosophical meaning. For example, I think GENDER AND RACE IN MATHEMATICS EDUCATION [sic] is just such a topic."

Although it expresses a widespread view associated with traditional positions on the philosophy of mathematics, and indeed of positivistic views of mathematics and scientific knowledge in general, this remark reveals a very different perspective than that held by many associated with the Philosophy of Mathematics Education network. Among the latter the view that mathematics is value-laden, and indeed can be a vehicle of racism and sexism, although undoubtedly remaining controversial, commands widespread acceptance. Thus there is an important question to be asked: Is there philosophical legitimacy in the view that mathematical knowledge is value-laden, and that issues of race and sex have a bearing on matters epistemological?

To strengthen my earlier arguments for an affirmative answer to this question, it is perhaps best to look for support outside of the nascent and incompletely defined area of the philosophy of mathematics education, especially since its nature is currently being contested. As it happens, this view is not solely the province of radical thinkers in education or in the philosophy of education. A number of different philosophical positions share the view that the nature of knowledge, and that of the social group who generate and validate it, are mutually dependent and constitutive. This range of positions includes Feminist epistemology and Standpoint theory (Harding), critical theory (Habermas), some varieties of post-modernist philosophy (Rorty, Lyotard), poststructuralist theory (Foucault), social constructionism in the social studies of science (Restivo), and in social psychology (Gergen, Shotter), social epistemology (Fuller), and the sociology of knowledge (Bloor, Barnes). This is a long list, and even if in response it is said that many of the areas mentioned fall outside of philosophy proper (conceived narrowly), I would wish to argue that at the very least feminist epistemology, critical theory and Rorty's post-modernist philosophy must be admitted to this philosophical core. Thus there is strong philosophical support for the legitimacy of the view that mathematical knowledge is value-laden, and that this is relevant to epistemology.

Values and Mathematics Education

The key issue in the above discussion is that of values, and the relationship between values and knowledge in mathematics (and elsewhere, for example, in other sciences). Both feminist epistemology and critical theory share the view that knowledge at all stages in its production and warranting is not only shot through with the values of the persons and groups involved, but that the very quest for knowledge is preceded and driven by certain social and epistemological interests.

This has a direct bearing on Prof. Zheng's definition and research agenda for the philosophy of mathematics education. He claims it should centre "on the following three questions".

What is mathematics?

Why should we learn (or teach) mathematics? ... the value of mathematics and the aim of mathematics education...

How should we learn (and teach) mathematics? ... the epistemological study of the nature of mathematics learning."

My response to this definition and research agenda is three-fold. First, I agree that each listed element should be included; but second, I think the scope of the philosophy of mathematics education should be broader than this; and third, that each of the above questions, especially the last two necessarily involve values

1. I have argued elsewhere (Ernest, 1991, Forthcoming) that the philosophy of mathematics education should primarily be concerned with philosophical reflection on the:

(i). nature (philosophy) of mathematics,

(ii). nature of the learning of mathematics,

(iii). aims of teaching mathematics and mathematics education,

(iv). nature of the teaching of mathematics

(v). social context of mathematics education, and the interrelationship of all of these factors as a social system.

Each of these five areas of concern gives rise to a characteristic set of philosophical problems and issues pertaining to mathematics education. I have also argued that both the concepts of 'philosophy' and 'relevance' used here need to be understood more broadly than in traditional Anglo-American or linguistic philosophy. Philosophy today includes a whole motley of diverse theoretical perspectives, many of which can be brought to bear on mathematics education issues. Thus, all in all, I welcome and endorse Prof. Zheng's list.

2. But, as my view of the scope of the philosophy of mathematics education shows, I also believe that the social context, structure, institutions and indeed reality of mathematics education cannot be fully comprehended or adequately analysed philosophically unless the essential social nature of the process is included in the *problematique*. (Ernest, 1991, In press). In the context of traditional Western philosophy, it is surely easier to grant the need for a social account of mathematics education than of the philosophy of mathematics; and yet Prof. Zheng has granted the latter but not the former.

One reason for including a social perspective is because in considerations of the epistemology of individual knowing and learning mathematics, there is a widespread shift of opinion towards recognising the essentially social nature of that learning (Cf. Vygotsky, Walkerdine, Bishop, Mellin-Olsen). This is because of many factors, including the social nature of: interpersonal interaction in the teaching and learning of mathematics; the social nature of language, the medium of learning; of institutions and locations for learning; and according to some, the social nature of mind itself. Whether or not one agrees with all of these views, their discussion, and hence *a fortiori*, the need for a consideration of the social, must be a central concern of the philosophy of mathematics education. These positions cannot, in my view, be disallowed from the outset, prior to any consideration of their merits.

3. The issue of values, I claim, is central to the consideration of Prof. Zheng's last two questions for the philosophy of mathematics education. Both include the word 'should', which indicates a normative and hence value-based position. In addition, values and aims are explicitly included within the scope of the philosophy of mathematics education (question 2). But the issue of values and aims immediately raises the question: Whose aims and values? To value something is to make or have made a judgement of worth or goodness, and that judgement has to be made by a person or a group. Once we admit that human values, aims and judgements are involved, then the issues of differing perspectives, of multiple purposes, of vested interests and of contestation arise, once again bringing us into the domain of the social. My claim is that it is philosophically incorrect to consider the aim of mathematics education and disregard the source or ownership of that aim. To isolate and dehumanise aims and claim that they are based on wholly rational grounds is in my view to mislead. What it means is that the values and aims of a particular group or culture are so dominant that they render both their own source and the possibility of alternatives invisible.

Prof. Zheng does consider aims in a social context; namely the needs of the development of society. But once again I would ask: Whose perceptions of those needs? Certainly in modern Britain we have found divergent aims based upon different personal and group beliefs, philosophies and ideologies. So the notion of developing mathematics education to meet the needs of society in the information age need to be unpacked (deconstructed) and the opposing forces revealed (Ernest, 1990, 1991). I consider this insight to apply to all cultures and contexts of mathematics education.

A Branch of Philosophy or of Mathematics Education?

One final issue concerns philosophy of mathematics education: Is it a branch of philosophy or a branch of mathematics education? Prof. Zheng claims that the philosophy of mathematics education belongs to philosophy. However this position is softened when he not only argues that the philosophy of mathematics education and philosophy of mathematics share certain problems (notably, 'What is mathematics?'), but that the former is much closer to mathematics education. Indeed, he criticises traditional philosophy of mathematics for paying too little attention to actual mathematical activity both in research and in education.

My position is that the philosophy of mathematics education is primarily a part of mathematics education. It is one perspective on the problems and issues of mathematics education, but one which draws on and applies the methods and concepts of philosophy. It is one of a number of meta-perspectives on mathematics education, which also include the psychology, sociology, anthropology, and history of mathematics education. I regard each such application as a branch of mathematics education.

A general issue is that of the boundaries between disciplines. One particular boundary that I (and others) have challenged is that between mathematics and empirical science. Of course this is not to deny that there is an important conceptual distinction between these two domains. What I reject is the notion that there is a rigid and watertight boundary between the two, which excludes any matters pertaining to the one area from epistemologically impacting on the other. This follows from a quasi-empiricist philosophy of mathematics (which both Prof. Zheng and I endorse). Similarly I think that philosophy of mathematics and mathematics education might share certain problems and interests, and thus have some overlap. But like some modern philosophers (e.g. Rorty and Lyotard) I reject the notion of philosophy as a privileged, prioritised meta-narrative. Thus there are differences in the overall sets of problems addressed, in the methods employed, in genre and rhetorical styles, differences in the literatures (not to mention the composition of different academic groupings) between different disciplines. But once the privileged position of meta-narratives like philosophy, and rigid subject boundaries are questioned, there is nothing to prevent shifts in discipline boundaries and the sharing or the importation of the problems and methods from one discipline to another.

In Prof. Zheng's final point he lists philosophy, mathematics, education, psychology, logic and computer science as contributions to the theoretical foundations of mathematics education. Of course psychology is the discipline which has contributed the most to mathematics education (not counting mathematics itself). But I would also add sociology, anthropology, ethnomethodology, history, and semiotics to his list, thus emphasising the social sciences and humanities more.

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DIALOGUE

The discussion between Zheng and Ernest in the THEME section is extended in this new DIALOGUE section of POMENEWS7. A number of letters written in response to items in earlier editions of POMENEWS extend and challenge the Zheng/Ernest discussion. LoSasso adds a teacher's perspective as she describes what influenced her own teaching over the years. She then gives further expression to her beliefs about teaching as she critically reviews a book, *Creating Miracles*, which is another teacher's story about his classroom work over a year. I follow on from LoSasso's writings to support my contention that teacher beliefs--and one can call these *teachers' philosophies*--do indeed determine what happens in the classroom. Finally in this section, Phil Davis brings the dialogue back to earth as he talks about his childhood experiences playing marbles, images evoked by Ernest's example in POMENEWS6.

Let's hear first from John Bibby as he exhorts us to demystify what we say!

DEMYSTIFYING THE DEMYSTIFICATION

a letter from John Bibby

Being a "fellow-traveler", I like to keep in touch with what POME is doing, while lacking some of your fervour, much of your vocabulary, and nearly all of the specialised knowledge which POME-groupies display.

POMENEWS6 encouraged me to articulate what I guess must be a common question among such as myself:

If the philosophy of maths education aims to demystify much of what poses as maths education, how can we demystify the demystification?

Underlying this question are various dimensions of unease.

1. We are repeatedly told not to trust absolutists who believe in

o "the Euclidean paradigm of mathematics as an absolute, incorrigible and rigidly hierarchical body of knowledge existing independently of human concerns' (p.2)

o "a body of absolute truths developed from a set of logical axioms through a deductive process" (p.8)

o "the purity of mathematics" (p.14)

o "the prejudice of many people, that mathematics is neutral, carries no ideology, is apolitical' (p.15).

But who *does* believe these things any more? Or rather, who believes that these descriptors of mathematics are in any sense balanced or fair?

Are they not mere rhetorical Aunt Sally's intended to legitimise our voice of protest? To me they are old hat - barely worth bothering about, dead as a parrot, and long buried to boot.

(Is Steve Lerman seriously boasting that he wrote about the social nature of mathematics *as long ago as 1983* (his words, my italics)? Personally I thought it became fairly conventional before the war - God bless the Soviet Union! It's time the debate moved on.)

2. To what extent are relativistic responses to absolutism an umbrella covering a range of other concerns? Paul Ernest's soliloquy on marble playing may be interesting ethnography - kids' games *are* fascinating, aren't they? - and a good starting point for teaching such kids; *but where is the maths?* (For that matter, where is the philosophy?)

3. Extending the above, by emphasising mathematical contents in social situations, we risk focussing on trivial abstractions while throwing away more important things. I suspect Ernest's marble-playing example under this heading. Even more do I suspect Lerman's holocaust (p.17) - "five million has one kind of meaning, whereas six million (means) the holocaust" This particular six million is not a "mathematical particular"; if anything it's a mathematical smoke-screen designed to numb and obscure - "one death is a tragedy; a million deaths is a statistic".

Feeling inside my impatience, I hear the question "So what?" What does all this mean to the practising teacher/learner/person-in-the-street? How does the "philosophy" make our lives more effective?

So I turned with high hopes to Geoffrey Roulet's article *"What does this mean for the children the classroom?"* However, it left me extremely disappointed, and not only because of the narrow view of maths learning portrayed by the title.

Its main aim seems to be the denigration of "tool kit" views of mathematics. But what is maths if it fails to include a set of tools, views, perspectives - call them what you will? The philosopher's focus should be upon what are the *appropriate* tools? how were they created? how can they be used? how can we develop new ones? and how can learners learn to investigate the above questions? Over to you, POME!

Yours sincerely,

John

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PS. My philosophy of maths education?

1. *Of course* it's social - let's get on with it!

2. Anybody can learn *anything* given (a) the motivation (b) enough time.

3. Make it fun!

4. Mathematics is too important to be left to mathematicians.

5. Philosophy? The philosophers are welcome to it!

Was it Goering who said "when you mention philosophy, I reach for my gun"? Personally, I usually reach for my couch - it has that effect on me.

Seriously though, I know I'm wrong - keep up the good work.

[If John Bibby seems to be gently chiding critical fallibilists to defend the proposition that their arguments have practical implications for teaching, then John Threlfall confronts the issue directly as he queries if any of this philosophy *stuff* has any practical import at all! Editor]

ABSOLUTISM OR FALLIBILISM -

What difference does it make to the classroom?

John Threlfall,

University of Leeds

This paper takes up the theme of Geoffrey Roulet's article: "The Philosophy of Mathematics Education - What does this mean for the child in the classroom ?" (Roulet 1992), in which a dichotomy of philosophies of mathematics between absolutism and fallibilism is linked to two distinct styles of teacher practice. I will argue that the relationship is not so clear cut as is presented in Roulet's article and that fallibilism or absolutism has an effect on teacher action only in combination with other attitudes, assumptions and opinions of the individual teacher.

Roulet (1992) distinguishes two views (philosophies) of mathematics, absolutist and fallibilist, and two styles of mathematics teaching, transmission and constructivist, and suggests that they are linked at a fundamental level. This linkage is illustrated by the correspondence in a group of (student) teachers between absolutist views with transmission teaching and between fallibilist views and constructivist teaching: an 'absolutist' view of mathematics as "a body of absolute truths developed from a set of logical axioms" (page 8) is associated with clear and coherent presentation, and *pupil* practice; a 'fallibilist' view of mathematics as "potentially flawed and open to revision" (p. 8) is accompanied by self-discovery and the use of real world examples.

The extent to which this linkage is inevitable or "logical" can be questioned: it seems quite possible to present mathematical ideas clearly and coherently even if they are thought fallible; the praxis of self-discovery can be used in the hope of discovering absolute truths (in the way proposed by Constance Kamii (1986) for example). Why should the skills of mathematics be neglected just because there is an acceptance of the need for further growth in the discipline? Why should real world examples be avoided by those who consider mathematics to contain absolute truths?

Two kinds of "philosophy of mathematics"

Roulet invokes Lerman (1983) in support of a logical connection between philosophies of mathematics and teaching style, but the contrast of philosophies which Lerman describes is of a different kind. Two meanings of the word "philosophy" need to be distinguished - "philosophy" as a belief system which underlies action, and "philosophy" as a consideration of fundamental truth. Lerman's distinction between philosophies of mathematics is between the Euclidean system of proof and deduction, and the quasi-empirical approach championed by Lakatos (1976). These are philosophies of mathematics in the sense of coherent belief systems about what mathematics is and how it should be approached. As Ernest (1991) points out, when Lakatos is disputing Euclidianism he does not address the matter of certainty, truth or the nature of mathematical objects (although he does do so elsewhere). The contrast between the Euclidean and quasi-empirical approaches is fundamentally one of method, rather than metaphysics, of approach rather than belief. They are philosophies in the same sense as one may talk of a "personal philosophy", a set of principles by which one acts. Absolutism and fallibilism, on the other hand, are views on an epistemological question, the issue of absolute and necessary truth in mathematics. It is misleading to refer to them in the same way.

As a consequence of this difference, when considering classroom practice, the contrast between Euclidianism and quasi-empiricism is clearer, simpler and more clear cut, as it is already a contrast of methods, of mathematics as a process, and each method can be treated as a whole: one does it this way, or that. In the contrast between absolutist and fallibilist approaches to classroom practices, on the other hand, there are a range of elaborated positions which can be called absolutist, which are different from one another in many ways, and several fallibilist positions (including that of Lakatos) which are also different from one another in many ways. By listing them to identify the common thread Ernest (1991) also expounds their variety. The contrast between absolutism and fallibilism is not of the whole of one position against the whole of another, but a small, if fundamental, part of a range of positions against the equally small and equally important part of others. So although Lerman (1983) is able to argue that contrasting philosophical positions have direct consequences to the classroom, this may not apply to the distinction between absolutism and its rejection, which is a different kind of philosophical contrast. Philosophy in the sense of consideration of fundamental truth brings views that a person may hold, but not a set of principles for action unless further steps are taken. Any teacher may develop her specific philosophical views into a coherent belief system with action principles by further thinking and reflection. The question is whether these further steps are direct consequences of the view, and would therefore be the same for everyone.

The effect of philosophical beliefs on approaches to teaching and learning

The fact that teachers with a shared view on the nature of mathematical truth seem to be inclined to one way of doing things over another, and that Roulet's student teachers display a correlation between preferred teaching style and epistemological opinion does not show that a teaching approach is unequivocally related to the particular view. Not many individual teachers, student or otherwise, think out their approach from scratch. They acquire beliefs, attitudes and approaches as a package, by acceptance of one of the elaborated belief systems to which they have been exposed, but these have been developed by a relatively small number of individuals or groups. The apparent identification of fallibilism with cognitive models of learning over behaviourist ones, of constructivist models over information processing ones, and so on, has arisen because only a few educators have developed their own views into coherent philosophies of mathematics and mathematics education. The developed beliefs systems are coloured by those individuals' or groups' views on other fundamental issues, such as moral values, the nature of society and political ideology, as well as by their attitudes to teaching and learning. The supposedly 'fallibilist' and 'absolutist' philosophies of mathematics are not simply the corollaries of an epistemological belief, worked out in isolation on logical grounds, but are an expression of the nexus of values that have clustered around opposing positions as a result of the ethical, political, social and educational opinions of some of the influential people who hold or have held each view.

In terms of the particular belief, rather than an extended belief system, the denial of an absolute origin for mathematical truth is perfectly consistent with behaviourism, and the idea that knowledge is constructed in the person rather than acquired from outside can be readily accommodated to an absolutist view of mathematical knowledge. The single epistemological issue has to be considered within a perspective that takes into account other aspects, which may have precedence. For example the 'fallibilist' view is often associated with an emphasis on discussion, and the teacher who said "What is there to discuss? The facts can stand for themselves," might be presumed to be absolutist in his/her view. However, it would be unfair to attribute such an attitude to all those who share one philosophical view. Absolutists within the Piagetian tradition actively value children's discussion in its role of promoting the resolution of 'cognitive conflict' in the "right" direction.

A teacher's view on the nature of mathematical truth could well have an influence on how they teach. However, given the effect of other attitudes and opinions, and the other priorities which sometimes take precedence over philosophy in the classroom, it may be better to consider the influence of epistemological belief in the detail of action and the specific decisions of an individual teacher, rather than in terms of the broad sweep of teaching style or overall decisions about what to do.

For example, the approach a particular teacher makes to continuity and progression might be affected by her views on the nature of mathematical truth. A teacher with an absolutist view may use 'logical' analyses of the connections between elements, and approach the teaching of these in a systematic pre-ordained sequence, whereas a teacher who holds a fallibilist view may seek out empirical studies of 'cognitive maps' and pay greater attention to the actual responses of the children.

Similarly the approach to problem solving might be affected. A teacher with absolutist views may try to build up firm associations between problem situations and the successful procedures to adopt, and promote the establishment of a formula approach to problems, whereas a teacher with a fallibilistic view may be more inclined to try to get the child to make and test hypotheses, to revise them in the light of experience, to try different approaches and to invent their own representational system.

However these contrasts presume many other attitudes and beliefs. There is no intrinsic reason why a teacher with an absolutist view of mathematics should not encourage children to articulate their perceptions or use eliciting language to try to develop the their understanding of the problem, or why a teacher with fallibilistic views should not look for efficient processing solutions.

The association of teaching styles with philosophical beliefs has been exaggerated in Roulet's account. The tendency for certain actions to accompany certain beliefs is interesting and worth pursuing, but should not be considered 'logical'. Equally, "fallibilistic teaching" may appeal as a blanket term to incorporate the quasi-empirical approach proposed by Lerman (1983) to stand in contrast to a Euclidean approach, and the constructivist approach described by Von Glaserfeld (1991) and his followers to stand in contrast to a transmission approach, using the term "fallibilism" because it is a view which the proponents of these approaches seem to have in common, but it would be misleading to suggest that the acceptance or rejection of absolutism in itself suggests a teaching style. The influence of the philosophical opinion on action is matched by psychological, social, political and educational considerations with which it is not directly connected.

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[As noted earlier, the following piece was written initially when Anita was taking a graduate course at Simon Fraser University. Anita has taught for over a quarter of a century in a variety of locations, at a variety of grade levels, with a wide variety of students, and with varying degrees of happiness with the outcomes of her teaching endeavors. But as she says, it is hardly ever dull! Her accounts provide a practical counterpoint to the more philosophical discussions of the previous two articles. Editor]

A VOICE FROM THE CLASSROOM:

Highlights from My Life as a Mathematics Teacher by Anita J. LoSasso

My life as a mathematics teacher has been one long process of trying to find ways with the accepted - indeed, approved - methodological structures to allow students to meet and know mathematics. My definition of the body of work that is mathematics has changed several times, my way of expressing it has varied, but one goal has been consistent: to give my students, if possible, the joyful experience of seeing the patterns and expressing their perception of them in an atmosphere of affirmation. It ain't easy. But then, you know that. It's a given!

There have been times when I was reading Borasi that I wanted to scream. Her candor and her clarity are brilliant. The picture she paints in her last two chapters is one I have tried to produce in my classes many times. Sometimes the success is meager to nonexistent. Very occasionally it has been a resounding success. But never does it meet the goal I have set for myself in terms of an overall program of learning. Facing and dealing with this failure has made this paper and all the papers for this semester very difficult to write. When I have attempted to write out a manifesto or a 'Life of Anita' I have gotten bogged down in contemplation of what I haven't done. Borasi's honesty about this in her experiment heightened the anxiety I felt. The example was set, I could not evade it. Her courage put my reluctance into a perspective of cowardice I could not allow to stand as a self-definition. Those feelings, however, colored everything that follows.

My own high school experience with mathematics classes were a major influence on my teaching. I had an argument with my teacher in grade nine and she slapped my face. The fault was on both sides. As a result, I did not take any math courses in grades ten and eleven. When I got to grade twelve she was gone. I then took the grade twelve course offered, Algebra II and Trig, with the new math teacher. Betty Jaynes felt it was her job to help us do the math we probably already knew how to do. She played a minimal role in our work together, answering our questions sometimes, guiding us to finding them for ourselves. I loved every minute of it.

In college I decided to do a math minor because it had the same number of courses required, whether a major or minor and it had all those men in it. Again I was lucky in my teacher. Dr. James A. LaRue's vision of math was at the same time playful and rigorous. His basic premise was that any and all could learn it; it was, after all, just the product of our own minds. I took every class he taught, auditing when my class load was full and making trips to campus during my long practicum. I even called him long distance from Colorado during my first year of teaching to discuss a problem that arose with an algebraic axiom. I was teaching no math at the time, just sciences, but I had offered to help my students with their homework. I had asked for, but not yet been given, junior math classes to teach. I got them the next year. Since then I have managed to teach or tutor some students in mathematics every year of my life.

My first teaching textbooks were the SMSG courses in their early paperback form. My big mistakes made in teaching that year came from believing that the students could do more than had been expected of them previously. I could have when I was in school, so I was sure they could. They found very dignified and courteous ways to let me know when I was wrong; when I expected too much. I learned from them. I began to listen more carefully.

My second teaching textbooks were the University of Illinois books by Beberman and Vaughn. They were used in a new school of radical design that allowed great flexibility in the delivery of curriculum and the work with pupils. I was struck, while touring Thomas Haney Secondary, by its resemblance in structure and design to that school, Carmody Junior High.

During the time I was at Carmody, I was fortunate in being head of a department of very flexible and professional teachers who could discuss, examine and explore different ways of dealing with the teaching/learning of mathematics in that environment. In addition, I was going around to the feeder elementary schools to teach grade five and six students how to program computers using base two numerals for all commands and calculations. That certainly dates me!

It was during the time I was at Carmody that I had one of those epiphanies that occasionally come in a life of teaching. Although I had believed that any of my students in my classes could learn the mathematics assigned to their level, I was teaching only the academic students or the accelerated students. I was not dealing with the *dumb* classes until I got to Carmody. There I had to deal with the dumb class to the power of three. Not only were they considered dumb at this school, they had been considered dumb in their last three schools. Every year for the last three they had been in a different school as the county tried to deal with a burgeoning population, building at a frantic pace. I got them at Carmody in the fourth year of their trials and tribulations, welded together as a group and as cynical about education as they come. It was with this group that I learned, I had to learn, how to enter into a dialogue with my students. I learned to listen and listen and listen and maybe talk about one day in five. Although the seeds of this had been with me all through my previous four years, it was here they grew to fruition. I learned from this group as from no other that the definitions and descriptions that come with children, through the system, do not truly define them; they define themselves in any new context. They did this so completely that many went back to the academic stream, took more mathematics and at least one of them became a math teacher.

My next major learning experience came in Philadelphia in an elementary school. I went to Philadelphia and Patterson Lower School to be a math specialist in Lore Rasmussen's Learning Centers Project. In my third year there I was the head teacher in a mini-school designed to function on the model of the British Infants' Integrated Day. This experience proved a theory of mine, and God knows how many others, that children can choose those things they need to learn AND LEARN THEM without our telling them which step to take next and when to breathe while they're doing it. We provided the environment. The students provided the learning power, the sustained concentration, the tenacity, the dogged determination to learn what they needed to learn. We learned to watch and listen and work on ourselves to be ready to provide what they needed at the moment they knew they needed it. I never worked so hard in my life - until now as I try to teach a full schedule and do a master's program at the same time.

At this point, with the naiveté of youth and short experience - nine years - I felt it was time to give the world the benefit of my great learning. I applied to become a Faculty Associate at Simon Fraser University, Faculty of Education. (And the rest, as they say, is history.)

I had been using materials written by John Trivett to teach other teachers how to use the Cuisenaire rods to teach/learn mathematics in elementary school. Through a sociometric too complicated to detail, John knew all the folk who had written my references and approved my hiring.

One could not work with John and not learn. Some of the things one learned were about pain and some about pleasure and only in an w/holistic sense could this be interpreted to be about teaching/learning math. But they were. It was John who truly led me to the expression of the idea noted two paragraphs above, that one did not prepare lessons, one prepared one's self. This preparation is a never-ending process. That's what I'm doing now in this program.

After that first experience as a Faculty Associate (and the fourth as a graduate student) I began the experience of teaching in a different country and a different culture than the one in which I was raised and began my work. This has provided some very hard lessons, none of which I would prefer to have learned elsewhere. The context has provided, at times, a very rigid structure in which I careened like a snooker ball, with many bruises to my ego if not my body. But in many ways, it was a very gentle teacher, providing true friends to see me through the struggle. For that I am grateful.

Some of the lessons learned here have been:

Just because we all speak English does not mean we all speak the same language. Even in mathematics, the universal language, we may not be agreed on that of which we speak.

Appeals to authority have a different strength here, than there. They are more defining of personality and all things. There is no dispute, or if there is, it comes with great pain and effort at redefinement.

The bonds of friendship are stronger here, I think. It may be a function of the survival culture that Margaret Atwood discusses in her writings that enhances them or the need for a sheltering relationship in the face of authority.

It may be that I have been here more ready to learn the strength of friendship. Certainly I have learned to respect my students' friendships. They are extremely supportive of each other, especially if they are not in that group of elite known as the 'smart' class, or see themselves as interlopers there.

In the time I have spent here, 24 years, I have done many different jobs in many different venues:

alternate schools, the Vancouver District Office, four traditional secondary schools and the Professional Development Program of this Faculty, both on campus and off. Of late I have felt that my classroom no longer demonstrates any evidence of the many things I have learned from my students. It seems stale and lifeless on many occasions. I am hoping that as a result of my studies I will emerge reinspired and will carry on to inspire new students in my multicultural class groups.

For that is the lesson I am learning in the classes I teach while I learn others in the classes in which I am officially a student - how to communicate about the universal language of mathematics to many whose first language is not my own. I am learning about using the language of English (Ameri-Canadian version) to teach student speakers (or non speakers) of English who have learned that language in Hong Kong, Osaka, Beijing, New Delhi, Saigon or El Salvador. I am not yet sure what I have learned about doing this in any way that I can adequately express in the context of this paper. Perhaps my studies will eventually give me voice to tell of that.

In tribute to John, I am still preparing myself.

[It was also when Anita was taking that graduate course that she wrote a review of Chris Healy's book, *Creating Miracles*. Anita was searching for the voices of other teachers with which her own classroom experience might resonate. What she found in this book speaks to the earlier discussion by Bibby and Threlfall about the use, or lack thereof, of philosophical foundations for teaching. Perhaps in future editions of POMENEWS, other teachers will add their stories to those of LoSasso and Healy. Be that as it may, here is Anita's review of the Healy book. Editor]

A REVIEW OF *CREATING MIRACLES*

by Anita J. LoSasso

Healy, Christopher C. (1993) *Creating Miracles: A story of student discovery.* Berkeley, California: Key Curriculum Press.

This very interesting book was written by a teacher from Los Angeles.. The book is a mixture of fiction and non-fiction. It signals its fiction parts; they are the part that are from the journals of students. Healy wrote them as he imagined the students would have who were in his first No Book geometry class. He says that they are, with one exception, based on particular students. He has had those students read them to see if they matched what the students really thought and felt. The students agreed that the narration pretty much matched what they had felt and experienced in the class. The exception he tried to make a composite. The non-fiction parts are from his journals.

Unlike many other books I have read, the Introduction and the Appendices of this one are worth reading. In the Introduction, the author tells things about his style of teaching and his understanding of geometry that help one to understand how this No Book class could come about. The Appendices give enough information about the process that one could reasonably adapt and do a similar thing in one's own classes.

Creating Miracles was written in a year taken off from teaching to do just that. It is a description, through Healy's journals and his imagined ones of his students, of the process gone through by a geometry class with no text books. They wrote their own.

The No Book situation just sort of developed because the new text books they were to use had not come when school started. Healy had a cupboard full of the old books but was not enthusiastic about giving them out to the class, when some wag on the first day of class said something about supposing they weren't going to have any books. So Healy replied along the lines of how that might be a good idea. He decided on the spot that they would not use a text. This left the class with the impression that the wag was calling the shots. Although Healy attributes this sort of reaction on his part to laziness, partly, and boredom with the textbooks, generally,

I think it takes lot of guts to let it look like someone else is controlling what goes on in one's class.

The class appears to have been a typical one for a public school in the United States, a grade ten class with some accelerated students from grade nine and a few senior students who have discovered late in secondary school that they need another math class for graduation. The aforementioned wag was in grade twelve. His journal is part of the story.

Healy began by putting the class in groups of four, randomly. He changed the groups every two weeks. He always decided the groupings. He tells of one time when a group had not sufficiently developed the idea they were working on and felt they needed more time together. They brought this up in class and told the class why they needed more time. The class voted to allow it.

On the first full day of class, after the students had been assigned their groups, each group was given a sheet of paper with a 'rule' on it. These rules were casual statements of axioms or theorems of Euclidean geometry. Healy gives examples like "Triangles have 180o" and "Parallel lines never meet." For one period the groups were to write any statement that they felt to be true or related to the statement on the page. He read those every evening and took statements from those written on the pages by the students to head the pages the next day. He makes the point that the whole thing could have fallen apart at any time if the students, for any reason, had decided not to work on their statements one day. They never did and neither did any of the classes that followed them.

At some point it became necessary to define terms. Healy took some of the contentious ones, wrote them on the board and told the class that their homework was to write good definitions of them. He took up the homework at the door as the students entered the room. Everyone had done them. He recorded them as done and gave all of them to one group and told that group to come up with class definitions. Every week, on Monday, they reviewed the definitions for the previous week and made any adjustments necessary for the whole class to be satisfied with them.

There was also the matter of what was called Discovered Truths. These were ideas that groups determined to be true and thought were important enough that the whole class should know about them. The groups required that the Truths be put on the board with the definitions for homework. They wanted the Truths voted on and put in the book with the definitions. The group that made the discovery presented the argument for inclusion in the book. This argument for validity often seemed to take a form similar to a proof.

It became apparent that someone was going to have to keep track of all this stuff. Due to his own laziness, Healy tended to depend on the students' records, one in particular, so he asked that student to enter all the work she had recorded on the computer they had in class. The computer was there as part of an experimental program about which Healy did not seem overly enthusiastic. It turned out to be the thing that allowed their book to grow and be published, periodically, for tests and finally for the total work of the term. It had two significant pieces of software: Geometry Supposer and Appleworks.

The students in the No Book class participated in grade-wide tests at the end of the first semester and the academic year. The students made about 5% less, on the average, than the students in the classes with texts. They did this on the first test, the second test and all the tests in subsequent years of classes that came to be called Build-A-Book classes. There was, however, an immense difference in their attitude toward mathematics and to learning altogether. This process truly had the effect of empowering the students. This is what led Healy to write the book. He wanted to see that happen to more students.

In the process of reading the book one learns that there are other teachers also teaching geometry classes at this level and that Healy has others that use the normal textbooks. Because of the success of this chronicled first class, he has maintained at least one class that he calls the Build-A-Book class in every academic year after the first one (1987) until the writing of *Creating Miracles*. The Appendix contains the last one before the writing of the book, the class of 1991.

Healy claims to have written this book to encourage other teachers to let classes do more exploration without the structure of curriculum, for a little while at least if they can't do it for a whole year. He points to his lack of expertise in mathematics (it is his second field of study) and lack of interest in improving that expertise as an indication that anyone can do this kind of thing. He does admit to a fair bit of creativity, though, which I am sure he needed to get through the experience alive. He had to defend it to administrators, after all.

My reaction to the way he chose to tell the story was mixed. I found the journals of the students did not quite ring true. They seemed like an effort on the part of the author to justify what he had done and it wasn't necessary. The Build-A-Book of 1991 that is in the Appendix did that very well. It shows that the students could do real mathematics of quality and rigor.

I am very glad that Healy wrote the book however he chose to do it. I hope a lot of math teachers in the U.S. read it and talk about it and try out similar things in their classrooms before it is too late and public education collapses under its own cumbersome weight. It would be nice if we paid attention to it in B.C. too.

This book makes excellent parallel reading for the last two chapters in Borasi. The format outlined for implementing a Build-A-Book class certainly fits Borasi's outline for implementing a humanistic inquiry. It exploited a real-life problem situation - no text books and a lazy teacher. It focused on traditional topics in a nontraditional way in a lot of uncertainty. Everyone seemed aware of limitations in themselves and their situation, as well as in the mathematics they came up with for their class. The class discovered humanistic elements in mathematics by making it themselves and discovering the power in doing that. Their springboards for enquiry were their own statements. Healy exploited the surprise of working from a student's halfway mocking statement and made a new domain of work for his students. The class certainly presented an alternative to the status quo and they were forced to reflect on the significance of their work, continually. It almost seems as if Healy used Borasi's Chapter 12 for an outline to organize his writing.

Healy's inclusion of the format and time sequence of the class work in the Appendix is most valuable. I'd sure like to try and use it. I have done similar short projects with some grade nine classes too many years ago. I revived an account of those projects and rewrote it for my application to this program. It has many of the characteristics of Healy's classes' work in that it depended on the students' building and defending of theories. The work was also about geometric relations, their explanation and extension. It was not nearly as comprehensive as the building of a whole book, though.

Thanks goes to Key Curriculum Press for putting this book out in the market. I wish it success.

[The next piece supports the point of view evident in LoSasso's reports. It is argued that a teacher's meaning within her classroom is co-determined by the teacher and the environment in which she finds herself. Editor]

MEANING IS CODETERMINED

by A. J. (Sandy) Dawson

As trees grow they add layers of new material to the outside of that which existed before. In doing so, the new layers alter the older ones even as the newer layers are determined by the older ones, all this occurring as the tree grows. Metaphorically and perhaps literally, this captures the enactive view of cognition. With respect to humans and their environments (and here we also consider environments for learning mathematics), Varela et al conclude that :

...living beings and their environments stand in relation to each other through *mutual specification* or *codetermination.* Thus what we describe as environmental regularities are not external features that have been internalized, as representationism and adaptationism both assume. Environmental regularities are the result of a conjoint history, a congruence that enfolds from a long history of codetermination. [VTR, p. 198]

The meaning of any particular situation cannot be prescribed or determined from outside of the learner, because the meaning attributed to a situation is a result of the organization and history of that learner. What any learner does in a particular situation can only be understood in terms of what took place previously as the learner interacted with the environment--the history of structural coupling between learner and environment. Meaning is codetermined by the learner and the environment. Meaning doesn't exist independent of the learner. There aren't external regularities and meanings which learners are attempting to find or discover.

There is not in the enactive view of cognition a straight line path between a learner and the activities into which that learner may have been invited to engage. Rather, the learner chooses to stress certain aspects of the activities and to ignore others. It is not just a one way street, however, with the learner being the predominate force. The environment exerts itself by putting limits on what pathways the learner is able to pursue. Learning occurs at the interstices where the learner meets the environment, stresses particularities within that environment, and generates a response whose viability in the environment is then determined.

The enactive view of cognition contends that feedback is provided by the environment to learners. However, knowledge and/or information is not transmitted to learners from the environment. The transmission model of knowledge acquisition, the metaphor of the *tube of communication*, in the enactive view of cognition, is not tenable. Maturana and Varela contend:

...each person says what he says or hears what he hears according to his own structural determination; saying does not ensure listening. From the perspective of an observer, there is always ambiguity in a communicative interaction. The phenomenon of communication depends on not what is transmitted, but on what happens to the person who receives it. And this is a very different matter from *transmitting information*. [Maturana & Varela, p. 196]

If a very significant part of what takes place in classrooms is negotiation of meaning, and the finding of pathways which are *satisficing*, what then can be taken to be a problematic situation for learners in the mathematics classroom? When is a problem a problem for the learner? The answer, from an enactive point of view, is that a problem is only a problem when the learner and the environment mutually determine that there is something missing in the situation, a something which seems in need of rectification. There is always a *next step* for the system, i.e., the learner within an environment, even if that step is to do nothing! Hence, it is the learner and the environment within which the learner is functioning that together "...poses the problems and specifies those paths that must be tread or laid down for their solution." [VTR, p. 205] This is even valid, according to Cobb, Yackel, and Wood, for children encountering mathematics in the primary school.

The analysis we have presented...illustrates that what counts as a problem and as a conceptual advance has a social aspect even at the most elementary levels of mathematics. [Cobb et al, p. 21]

In the case where the learner takes a positive *next step* (and in a very real sense *all* next steps are positive ones), the notion of finding the optimal step does not operate. Learners try things out and see if they work. If they don't work, then another step may be tried or the activity may be abandoned. If the next step fills the missing link in the situation in a *satisficing* fashion, then again the learner may pass onto another activity, or the learner may, having found one acceptable solution, seek another. In this manner, pathways out of situations are generated. The decision as to whether or not a proposed step is viable is based on the individual's assessment of responses from the environment, including the learner's teacher and peers. The new pathways which are generated play a dynamic role in the structural history of the learner.

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[We close off the **DIALOGUE** section with a light-hearted letter from Phil Davis, who writes in response to a comment by Paul Ernest in a previous POMENEWS. Editor]

AND BACK TO THE PLAYGROUND

a letter from Philip Davis

How wonderful the story about Jane's marbles!

When I was five years old, marbles were a standard part of children's outdoor play as soon as the snow melted, and all the attributes of this play you mentioned are familiar to me.

Alas, I believe marbles have totally disappeared from the children's scene in the USA. Why? TV? A good topic for a social anthropologist.

Although irrelevant now, I believe that a conjectured invasion of marbles by adult/academic mathematics should be avoided. There should be a private sector to life.

When chess was invaded by mathematics/computers, I think it killed chess as far as the average run of the mill player was concerned, and it certainly has altered the game as far as the masters are concerned. Professional baseball, basketball are now deeply mathematized in the USA, and these games were substantially altered by it.

If by ethno-mathematics or ethno-X is meant those enclaves of knowledge, practice, and tradition that operate popularly in the X area, then with some reservations, I think it would be a mistake to academicize them. When X = art, literature, music, religion, etc., this has been going on in the USA under the banner of ethnic peace and mutual respect. The verdict is not yet in on whether the movement has had the desired effect.

Sincerely yours

Phil

Philip J. Davis, Prof Emeritus

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BOOK REVIEWS

by Paul Ernest

Brian Rotman, *AD INFINITUM...THE GHOST IN TURING'S MACHINE: Taking God Out of Mathematics and Putting the Body Back in: An Essay in Corporeal Semiotics*, Stanford University Press (Stanford, California), 1993, xi+203 pages (ISBN paper 0-8047-2127-0)

Since the late 1970s Brian Rotman has been single-handedly developing a semiotics of mathematics, building on the ideas of C. S. Peirce and F. de Saussure, as well as modern structuralist, post-structuralist and post-modernist thinkers. Readers of Valerie Walkerdine's seminal poststructuralist account of elementary mathematics *The Mastery of Reason*, will have seen how central aspects of her analysis of mathematical language are based on Rotman's semiotic theory of mathematics. She cites (and quotes extensively from) an unpublished paper of his from 1980. Ultimately, this was published in revised form as Rotman (1988). It was reviewed in POMENEWS 4/5 where it was judged to be a notable and significant development for the philosophy of mathematics. Before that paper appeared, Rotman published another relevant book, *Signifying Nothing*, which offers a semiotic analysis of zero. In it Rotman boldly advances the analogy between the appearance and acceptance of zero in the Hindu-Arabic number system promoted by the algorists in renaissance Europe; the appearance of the vanishing point in the new art of perspective; and the emergence of paper money. He argues that each is a metalinguistic signifier, absent from the code-system from which it emerges, but signified through that absence, and through the totality of its mother-system. The resulting account is an intellectual *tour de force*, in which what seems initially to be an implausible parallel is made utterly convincing. It is also an *homage* to, or perhaps even a parody of, Foucault's *The Order of Things* in which he develops a similar set of analogies between disparate entities in the history of ideas (language or grammar; scientific taxonomy; economics of exchange value), and which begins with an enquiry into the vanishing point of the gaze and the objects gazed upon in Velasquez's *Las Meninas*.

In *Signifying Nothing* Rotman offers his semiotic analysis of zero, truly one of the special numbers. In *Ad Infinitum* he tackles its reciprocal, that other big number: infinity. The book might be said to fall into two part. In the first part, he offers first a synoptic view and critique of traditional conceptions and philosophies of mathematics. He locates himself firmly in the 'maverick' fallibilist tradition, and argues that mathematics is a primarily linguistic human construction (language is interpreted broadly to include the totality of human practices of writing and ideogram-inscription). He attacks Platonist conceptions of existence in mathematics, arguing that these stem from a mistaken philosophy of language which regards it as representational. Instead of indicating some pre-existent realm of objects, he agues that both signifier and signified reside within mathematical language. He also strongly emphasises the materiality of all mathematical signifiers, and indeed for all signifieds, given the material, embodied nature of persons (and hence their imaginings).

Rotman goes on to elaborate his semiotics of mathematics, which after its many years of development has achieved a finished (and to this reviewer, persuasive) state. In brief this theory posits a formal mathematical Code, in which mathematical knowledge is presented and justified, and a MetaCode, in which both mathematics and the mathematiser can be discussed. He posits three levels of speaker/writer and listener/reader. First, there is the person who has access in the MetaCode to all forms of speech. Second, there is the mathematical Subject, the ideal mathematician, who uses a restricted language which excludes deixis (pronouns and other context-dependent signifiers such as time and place of utterance). The Subject creates in imagination the mathematical domains under discussion in response to inclusive orders (e.g.: define a group of infinite order with the following relations...). Third, there is the Agent, who carries out in imagination the direct imperatives given in the mathematical Code (add this, integrate that, etc.). The Agent is restricted to the realm of signifieds (the SubCode), and has no awareness: it is an automaton that responds to orders. Rotman argues that a mathematical proof is a thought experiment carried out by the Subject, who imagines the Agent carrying out the required actions, realising in imagination the claims of the theorem and its proof, and that ultimately it is designed to convince the Person.

In this overly brief account it is not possible to do justice to the power, originality and import of this new conception. What can be said is that for the first time Rotman's semiotic theory offers an account of mathematics as a human construction which accounts both for the inner and outer nature of mathematics : the role of symbols, constructions, mathematical objects, proofs, on the one hand, and the mathematician and her activity and personhood (and social-situatedness) on the other.

The second part of the book is concerned with infinity, and in particular with the employment of the semiotic framework to define and construct a system of non-Euclidean arithmetic. In effect, Rotman offers a new rationale for constructivist mathematics, for a finitistic interpretation of arithmetic. Arithmetic is based on the notion of successor, iterated at various levels to generate number, addition, multiplication, exponentiation, and so on. Rotman points out that the idealized actions of the Agent are traditionally allowed to stretch out into infinity by an ideal conception of endlessly unchanging iterated operations. This infinitude of activity is allowed or even demanded by classical mathematics. This, Rotman argues, is based on a false and disembodied notion of the Agent, one which forgets that all being is corporeal, that all resources are finite, and which ignores the embodied constraints of the laws of entropy and thermodynamics. All activity must grind to a halt when faced with excessive complexity and work, just as our capacity to understand does. Thus the imagined infinite capacity of the ideal Agent must be replaced by one that is tied to actual performability, at least realisability in principle, when the finiteness of resources and the capability of the body are acknowledged. The endpoint of Rotman's case is that the notion of infinity must be discarded, since it transcends bodily possibility. His claim is that infinity is a deistic idealisation, which must be discarded if we are to reattach arithmetic to the reality of our bodily existence.

My summary hardly provides an adequate representation of Rotman's elaborate intellectual apparatus and argument. What he presents is a striking and novel rationale for finitism in mathematics, stricter in its way than Intuitionism (closer to Yessenin-Volpin's ultra-Intuitionism, as he acknowledges). His account is a fascinating and powerful resource for researchers in finitism in the foundations of mathematics. It may also have unexpected applications in the arithmetic of electronic calculators and computers, where of course the arithmetic is finitistic and limited to the realisable (as he points out himself).

Having said this, I have difficulty in accepting the need to limit the actions of the Agent to what is realisable by the body. I do not think that Rotman's semiotic analysis needs to be linked to these limitations of the body, in view of the central roles of semiosis and thought experiments in his account. For surely in the idealised domain of thought we may imagine some of the concrete characteristics, particularities, and constraints and limitations of corporeal being stripped away. Rotman acknowledges this, and indeed he constructs a rather good analogy between the processes of reduction/truncation in the abstracting passages from Person to Subject to Agent, and the 'forgetful functor' in category theory. Nevertheless, he is driven by the desire to limit the nature of this abstraction to what is bodily-realisable in terms of operations and iteration. My view is that although he constructs and demonstrates the coherence and power of his conception, he does not establish the necessity of his finitistic interpretation. For the Agent operates on the signifiers of mathematics, and a finite sequence of these is all that is needed to signify an imagined infinity of actions or objects. There is no need to limit the actions of the Agent on a finite set of signifiers just because the signifieds they name are conceived as boundless or unrealisable. For me, this is one of the special properties of fantasy: we can conceive what is not and that which cannot be. It is also one of the special features of symbolisation: a symbol which is not an icon need not resemble that which it symbolises, so the finite symbols '...', 'o', '**N**', '***x***' and 'Red' each name infinity or an infinity of instances. Although both the domain and co-domain of the signifier-signified relation are to be found in mathematical language, they maintain their difference. And the signifieds in mathematics are all the while changing from their earlier nature and being reconstituted by changes in signifying and discursive practices in mathematics.

In terming his new system non-Euclidean arithmetic, Rotman is acknowledging, through the analogy with geometry, the multiplicity of legitimate interpretations of number, and offering a new one to add to the others. (We already had uncountable 'non-Euclidean' interpretations of natural number-arithmetic; to them Rotman adds a finite one.) However, he is at best ambivalent about the legitimacy of the classical infinite interpretation of arithmetic, as is evident in his prescriptive use of terms. For his overall message is that the concept of infinity is incoherent and must be repudiated and discarded. Despite this ambivalence, it is clear that Rotman does not expect us to give up classical mathematics. For he combines this scepticism about infinity with the acceptance of multiple perspectives on mathematics and number. This is just as well, for we have heard the clarion call to give up infinity before, albeit argued differently, in the work of Brouwer, Heyting, Bishop and other constructivists in the philosophy of mathematics. I was not persuaded by it then, and I still cannot see why we should be banished from Cantor's 'paradise'.

I can however see the overall power and value of Rotman's semiotic theory of mathematics, and I think it has a great deal of explanatory and philosophical potential for further exploration outside of his program for a finitistic arithmetic. Indeed, in my view, it represents one of the most original and important recent contributions to the philosophy of mathematics (understood in a broad, interdisciplinary sense), reaching far beyond its implications for 'strict finitism'. (Reviewed by Paul Ernest)

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David W. Jardine, SPEAKING WITH A BONELESS TONGUE, Makyo Press, Box 305, Bragg Creek, Alberta, Canada T0L 0KO; 1992, xxxv+256 pages (ISBN 0-88953-161-7).

This is a unique and difficult book to review because it is original both in style and content. The style combines philosophical and pedagogical reflection with elements of poetry and mystical thought. However the result is not something obscure, but an authentic, self-consistent and integrated account. The author's aim is to write in "small interlacing 'bits' or 'chunks' which relate laterally and generatively to all the other bits" (page *vi*). A central organising metaphor or image is "The Jewelled Net of Indra" according to which "the Earth is envisaged as a net, not a two dimensional one, but a system of countless nets interwoven in all directions in a multidimensional space. In each criss-cross of the net is a Jewel which, in each of its infinite facets, reflects all of the other Jewels and all of their faceted relations." (page xiv) The text, with its numerous asides, footnotes and cross-references, draws on this image. But the theme, also, is about an ecological awareness of the connection of all things: ourselves, our knowing, our being in the world, and our relations with students (the author is a teacher educator).

One of the outcomes of the style (which reinforces the content) is that the book has a wholeness, an integrity, an authenticity which vividly and unusually creates the presence of the author and his thought. I found reading this book to be at times a very intense and moving personal experience. In part, this was because I took the time to give myself up to the book over the course of the Summer. I spent a week's holiday in a beautiful part of Cornwall, and each morning I got up before my family and read the book for an hour or sometimes two with my morning tea, before they emerged. This is a special time for me, and the book added a layer of joy to it. Another reason for my positive response is that the unique blend of subject matter suits me, personally, well. It combines post-modernism, philosophy, reflections on pedagogy, mathematics, language, and understanding, with poetry, Eastern mysticism; all subjects that I have a sympathy with. It builds on the metaphors of conversation and body (embodiment) which are currently occupying much of my thought. But I am aware that another reader might not be so well disposed towards the book. As the author is also aware, the style is risky; but a risk which very much paid off for me. More authors should take risks like this! As you will have noticed, I am writing this review in a personal way. This is because the style of book seems to demand it.

The book offers a powerful critique of rationalist modernism (and in part, of Husserl's phenomenology, for its loss of nerve). The critique of the Cartesian cognizing subject and the *cogito*, offers insights and links between this and patriarchy and monovocity (or monologic, as opposed to dialogue). Piaget is also criticised, for falling into this tradition.

As I read the book, I made up a tentative index (the book lacks one), to help me retrieve the ideas I felt were important. The main items in my list are: agent, analogy/metaphor, attunement with Earth, Cartesian A=A, children's conceptions, colonialisation, conversation, critique of constructivism, Descartes vs. Kant, dis/connectedness, dualism, Hermeneutics, incompleteness of knowledge, language, local & particular knowledge, mathematical subject, mathematics, mathematics education, mathematics & semiosis, multiple selves, player/game, power, purity, social responsibility of science, teacher education, unconscious/dream, univocity/monologicality. But my list says as much about me as about David Jardine's book. For some of his big themes which do not show up in this list are feminism, ecological knowing, age/youth, our connectedness with the Earth and with each other, and the necessary wound where that connection is partly severed as we attain independence and maturity (and our resulting grief).

I found it to be a wonderful book. Having said that, I did not read it uncritically. One minor irritation is some of the footnotes. On one page there are 7 references to the same page of a single work, and the references take up half of the page (in the last couple of chapters I counted 22 references to that page!). More significantly, I found some of the works over-cited. In some of the novel ideas towards the end of the book I found an over-reliance on a small number of not-so-well-known works. In fact support could have been found from a broader literature base, which would have strengthened the argument.

Overall, I think this is an important, powerful and brave book. Not only does the author give much more of himself to the reader than is common in academic books, but there are important theoretical insights and syntheses for educators and philosophers alike, including much more than I have indicated. In particular, the cross-disciplinary combination of ecological and feminist philosophy with some of the better known themes of post-modernist epistemology and pedagogy provides a powerful and, unusually, *responsible* overall perspective on knowledge, pedagogy and being. Reading the book reminded me who I am, where I live, and how much the construction of my self, my life and my environment are privileged by my belonging to the 'winners' in the division of wealth and property in the world. (Incidentally, my interpretation of the 'boneless tongue' of the title is a tongue without the rigid hard core of rationalism, univocity, and stiff traditional style; it is a tongue which talks freely and expressively in conversation with others. It is also language and thought made flesh.) (Reviewed by Paul Ernest)

Teun Koetsier, LAKATOS' PHILOSOPHY OF MATHEMATICS: A HISTORICAL APPROACH, North-Holland (Amsterdam), 1991, xii+312 pp. (ISBN 0-444-88944-2) $82.

It is now almost 20 years since his untimely death robbed the world of the man who was potentially the most important living philosopher of mathematics, Imre Lakatos. Although his main work was carried out at about the same time as that of Thomas Kuhn, and was equally radical in historicising his field, unlike Kuhn Lakatos did not succeed in changing the face of his branch of philosophy. His work was never taken up in the same way by philosophers of mathematics. For example, his neglect by traditional philosophers of mathematics is illustrated by the fact that Lakatos' main works are not even cited in the authoritative 20 page bibliography on the philosophy of mathematics in Benecerraf and Putnam (1983) [Shame on you Putnam!]. The reasons for this neglect are manifold: Lakatos challenged the Shibboleth of absolutism in epistemology and the philosophy of mathematics, and he promoted a novel historical approach to mathematical knowledge. He also adopted a non-standard style for his major contribution, and employed a dialogue instead of the traditional, narrowly focused extended analysis and argument. His work was largely exemplary and inexplicit, which also made it easy to subsume to less radical perspectives (concerning the context of discovery, not justification). Apparently the philosophy of mathematics was not ready for his ideas (just as it had not been ready for Wittgenstein's, a decade or two earlier). Lakatos also blurred matters by changing his mind and his focus of interest. Furthermore, his editors shamefully subverted the thesis of his main contribution (Lakatos, 1976)

Perhaps the tide of neglect is turning. This is the first book length treatment of Lakatos' work on the philosophy of mathematics. It is a good book, one which every philosopher of mathematics and every historian of modern mathematics should have on their shelf (subject to price!). But unfortunately it is not the book that will force a re-evaluation of the significance of Lakatos' work, and place him centre stage in the philosophy of mathematics. That book is yet to be written.

Koetsier subtitles his book 'a [sic] historical approach', and this is very much his theme. After all Lakatos is *the* modern anglophone philosopher who brought the history back into the philosophy of mathematics. Some of this book is devoted to applying Lakatosian analyses to further historical developments beyond those considered by Lakatos. These include the works of Archimedes and Riemann; developments in analysis and partial differentiation; as well as Lakatos own case studies.

Koetsier offers some interesting and valuable analyses of Lakatos philosophy of mathematics. Perhaps his main argument concerns the distinction he draws between strong and weak fallibility theses about mathematical knowledge. Strong fallibility implies (S1) that mathematical theories can be refuted as a whole (comparable to the refutation of scientific theories), and (S2) continuity in the development of mathematics and the accumulation of 'truths' could be accidental. Weak fallibility implies (W1) that mathematical theories are revisable but not refutable in their totality, and (W2) the continuity of mathematics and the accumulation of 'truths' are not accidental but due to the nature of mathematics. Koetsier goes on to suggest that Lakatos' work is more in keeping with weak than with strong fallibilism.

Valuable as the distinction at first seems, there are, however, some problems with it. First of all, it is not clear that the two implications of strong (and weak) fallibility are consistent. Secondly, strong fallibility is never clearly enough defined. After all, even in empirical science, the refutation of a whole theory is problematic. Even from the logical empiricist view, according to the Quine-Duhem thesis, in refuting a theory empirically, the refutation can always be directed at ancillary hypotheses, and not the central theory itself. This analysis was adopted by Popper and indeed adapted by Lakatos, in distinguishing between the core and protective belt of a theory. Third Koetsier tries to establish his case by showing with historical cases that mathematics is weakly but not strongly fallible. (My counterexample might be Russell's refutation of Frege's *Grundgesetze der Arithmetik*.) But the issue is what Lakatos thought, and my reading is that at his most radical Lakatos was strongly fallibilist. My final criticism of the book is that no use was made of Lakatos' 1961 PhD Thesis in this book. Since much of his published work was posthumously edited (by revisionist Popperians) a balanced overall assessment of his work must go back to that source.

Overall, this is a valuable and in many respects a bold book. It is most interesting because of the risks it takes: both in terms of novel historical analyses, and philosophical theses about Lakatos philosophy of mathematics. It contains much insight as well as much that can be argued with. It is undoubtedly essential reading for all those interested in Lakatos and the modern movement in the philosophy (and historiography) of mathematics. (Reviewed by Paul Ernest)

Sal Restivo, Jean Paul Van Bendegem and Roland Fischer (Eds.), MATH WORLDS: PHILOSOPHICAL AND SOCIAL STUDIES OF MATHEMATICS AND MATHEMATICS EDUCATION, State University of New York Press (Albany, N. Y.), 1993. (vi+292 pages)

This is an exciting and valuable collection of pieces most of which first appeared in two special issues of *Philosophica* in 1989 devoted to philosophy of mathematics; also in the mathematics education journal ZDM (Zeitschrift fur Didactik der Mathematik), which have also been supplemented by additional papers. In view of their ephemeral and relatively inaccessible sources, the editors have performed a great public service in putting together this volume, which contains many novel and important contribution of central interest to readers of this newsletter.

The contents of the volume are as follows:

S. Restivo, The promethean task of bringing mathematics down to earth.

J. P. Van Bendegem, Foundations of mathematics or mathematical practice: is one force to choose?

M. D. Resnick, A naturalized epistemology for a Platonist mathematical ontology.

T. Tymoczko, Mathematical skepticism: are we brains in a countable vat?

Y. Rav, Philosophical problems of mathematics in the light of evolutionary epistemology.

R. Fischer, Mathematics as a means and as a system.

H. Jungwirth, Reflections on the foundations of research on women and mathematics.

N. Noddings, Politicising the mathematics classroom.

O. Skovsmose, The dialogical nature of reflective knowledge.

P. J. Davis, Applied mathematics as a social contract.

R. Fischer, Mathematics and social change.

H. Mehrtens, The social systems of mathematics and National Socialism: a survey.

S. Restivo, The social life of mathematics.

The book contains important contributions to the philosophy of mathematics, its sociology and mathematics education from a socio-philosophical perspective. As a collection, it is difficult to do more than highlight its central theme, which is a social view of mathematics. Each chapter alone deserves a review; but this would exceed the scope of this newsletter. Overall, it is essential reading, and available as a reasonably priced paperback. (Reviewed by Paul Ernest)

CONFERENCE NEWS

PDME-1, 2, & 3

The Political Dimensions of Mathematics Education: The Third International Conference will be held in Norway, probably July 1995 in Bergen. For further details contact Professor Stieg Mellin-Olsen, Lilletverdtvegen 119, 5050 Nesttun, Norway (Fax: 09 47 5 334701). PDME-1 was held in London, 1990; PDME-2 was held near Johannesburg, South Africa in 1993. What is unique about the PDME conferences is that participants largely share an overt commitment to social justice through mathematics education.

The proceedings of PDME-1 have been published as: R. Noss, A. Brown, P. Drake, P. Dowling, M. Harris, C. Hoyles and S. Mellin-Olsen, Eds.,(1990) *The Political Dimensions of Mathematics Education: Action & Critique*, London: Institute of Education, University of London. The proceedings of PDME-2 have just become available from Maskew Miller Longman, P.O. Box 396, Cape Town 8000, South Africa and is published as: C. Julie, D. Angelis, & Z. Davis (Eds.), *PDME-2: Curriculum Reconstruction for Society in Transition.*

PME XVIII, 1994

The 18th International Psychology of Mathematics Education Conference will be held in Lisbon, Portugal during the period 30 July - 3 August 1994. This conference will include a plenary panel on History of mathematics and mathematics learning (psychological issues), and Hans-Niels Jahnke of IDM (Germany) has been invited as one of the plenary speakers. For further details of the Lisbon PME conference contact João Filipe Matos, PME-18, Department of Education, Faculty of Sciences, Campo Grande - C1 - Piso 2, 1700 Lisbon, Portugal (Fax: 351-1-7573624). For further details of the PME organisation contact your local representative or Dr. Joop van Dormolen, Kapteynlaan 105, 3571 XN Utrecht, The Netherlands.

POME DISCUSSION GROUP, 1993

There was a Philosophy of Mathematics Education discussion group chaired by Paul Ernest at the 17th Annual Meeting of the International Group for the Psychology of Mathematics Education, at the University of Tsukuba, Japan, 18-23 July, 1993. An lively discussion took place, following short lead presentations made by a number of persons including Paul Cobb, Jere Confrey, Paul Ernest, Jeff Evans, and Steve Lerman. The theme discussed was that of the relationship between individual and social views or theories of learning mathematics. Some of the key issues concerned the contrast between the following two clusters of positions, although the following account does not capture the subtlety of the discussion nor the variety of viewpoints expounded.

Constructivist viewpoint

According to this position, learning of mathematics by an individual can be described in terms of an at least partly pre-given individual, who interacts with other persons, and with external representations (including language) and manipulatives. Such an individual makes sense of her surroundings, and tests hypotheses and sense making, by means of her actions, and through responses from others (and the environment). In defence of this position, does it not have to be conceded that a child coming to school is an at least partly formed individual with her own set of constructed conceptions, who then makes sense of her experiences in school on the basis of her previous understandings?

Sociocultural viewpoint

According to this cluster of positions, or at least some of them, an individual does not simply learn mathematics, but learns to operate mathematically in contexts. Thus for many contexts, such as those of school mathematics, a pre-given individual cannot be assumed, for the person's identity in a new context is constructed within that context through the positioning, roles, discourse and already specified practices. Thus learners construct multiple identities according to context, and the issue of the transfer of mathematical knowledge and skill from one context to another becomes quite problematic. This splitting matches the well known discrepancy between school (and formal or academic) mathematics, and street (and informal or ethno-) mathematics.

Steve Lerman and Kathryn Crawford are putting forward a Discussion Group on Vygotskian Approaches to the Learning of Mathematics to continue this and related discussions at PME-18 in Lisbon, 1994.

PSA/HSS/SSSS JOINT CONFERENCE

The Philosophy of Science Association is holding its next (14th) biennial meeting at the Clarion Hotel, New Orleans, USA, 14-16 October 1994. This is being jointly held with the History of Science Society and the Society for Social Studies of Science (of which Sal Restivo is currently chair), and it will therefore be a uniquely interdisciplinary meeting. More information can be obtained via Richard M. Burian, Chair of 1994 PSA Program Committee, Center for Study of Science in Society, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061-0247, USA (E-mail: RMBURIAN@VTVM1.BITNET).

ANNOUNCEMENTS and NEWS

PUBLICATION ANNOUNCEMENTS

Forthcoming book

The first *permanent* fruit of the POME Network (as opposed to the more ephemeral newsletter and conference symposia) is the following book. But for the over-utilisation of the phrases involve, it might have been called *Philosophy of Mathematics Education: The State of the Art*.

Mathematics, Education and Philosophy: An International Perspective, Edited by Paul Ernest, *Falmer Press (London, New York and Philadelphia), 1994.*

This book represents some of the most important aspects of current work in the philosophy of mathematics education, and an indication of some of the more exciting directions for future research. The aim of the book is both to survey the field and to risk predicting future areas of fruitful research. The papers include contributions to the POME Topic Group at the ICME-7 conference in Quebec, 1992; contributions from members of the organising group of POME, and other chapters solicited from significant and forward-looking researchers in related areas.

Below is a selection of the submitted chapters under consideration. This indicates the flavour that the book will have, and the strength of the contributions and contributors. Not all potential contributions are included, and for reasons of space, regretfully it will probably not be possible to include every single one of the chapters listed below. (Hopefully, offers of alternative placements will be made to authors of omitted chapters. It is planned to publish a special issue of the University of Exeter serial publication ***Perspectives*** entitled *Teaching and the Nature of Mathematics*, and to offer a place in it for worthy papers for which room cannot be found in this book.)

Tentative Outline of Contents

The Constructivism Controversy

Ernst von Glasersfeld, A Radical Constructivist View of Basic Mathematical Constructs

Leslie P. Steffe, Interaction and Children's Mathematics

Robert Thomas, Radical Constructive Criticisms of von Glasersfeld's Radical Constructivism

Stephen Lerman, Articulating Theories of Mathematics Learning

Michael Otte, Is Radical Constructivism Coherent?

Cognition, Society and Technology

Brian Rotman, Mathematical Writing, Thinking and Virtual Reality

Katherine Crawford, The Context of Cognition: The Challenge of Technology

Ole Skovsmose, Technology and Critical Mathematics Education

Erick Smith,, Computers and the construction of the other in the mathematics classroom

Psychology, Epistemology and Hermeneutics

David Pimm, Another Psychology of Mathematics Education

Dick Tahta, On Interpretation

Philip Maher, Potential Space and Mathematical Reality

Tony Brown, Towards a Hermeneutical Understanding of Mathematics and Mathematical Learning

Post-modernist and Post-structuralist Approaches

Valerie Walkerdine, Reasoning in a Post-Modern Age

Jeff Evans and Anna Tsatsaroni, Mathematics 'Trauma': Closure or Empiricism?

Paul Dowling, Discursive Saturation and School Mathematics Texts: a strand from a language of description

David Jardine, On the Ecologies of Mathematical Language and the Rhythms of the Earth

Thomas Tymoczko, Structuralism and Post-Modernism in the Philosophy of Mathematics

Humanising Mathematics

David Bloor, What can the Sociologist of Knowledge say about 2+2=4?

Falk Seeger and Heinz Steinbring, The Myth of Mathematics

Paul Ernest, Dialectics in Mathematics: A Historico-Philosophical Account

Philip J. Davis, Mathematics and Art

Hao Wang, Skolem and Gödel

Francisco Speranza, The Significance of History and of Non-Absolutist Philosophies in Mathematics Education

Sal Restivo, The Sociology of Knowledge and Mathematics Education

Gender and Race

Leone Burton, Is there a Feminist Style of Doing Mathematics?

Mairead Dunne and Jayne Johnston, Research in Gender and Mathematics Education: the production of difference

Ubiratan D'Ambrosio, Ethnomathematics, the Nature of Mathematics and Mathematics Education

George Gheverghese Joseph, Different Ways of Knowing: Contrasting styles of argument in Indian and Greek Traditions

Enquiry in Mathematics Education

Stephen I. Brown, The Problem of the Problem and Curriculum Fallacies

John Mason, Enquiry in Mathematics and in Mathematics Education

Marjorie Siegel and Raffaella Borasi, Demystifying Mathematics Education through Inquiry

Charles Desforges and Stephen Bristow, Reading to Learn Mathematics in the Primary Age Range

JOURNAL ANNOUNCEMENTS

CHREODS

CHREODS is a new(ish) journal addressing issues in education, usually in mathematics education. (A 'chreod' is a developmental pathway in space-time, after Waddington.) CHREODS aims to publish papers which stem from contexts of teaching or training, where the practitioners are attempting to work at issues that arise within or impinge on their practice. The intention is to promote a range of enquiry whose unifying theme is a concern to interrogate the realities of teaching or related discourses. The Current issue, CHREODS 6, includes 10 articles by a range of authors including Bill Brookes, Tony Brown, Paul Ernest on such themes as Metaphor, Sustainable Design, Regularity, Meaning, Knowing mathematics, Constructing as a joint activity, Drawing, etc. CHREODS costs £6 for 2 issues. Cheques payable to the "Manchester Metropolitan University" should be sent to Dr Tony Brown, Manchester Metropolitan University, 799 Wilmslow Road, Didsbury, Manchester, M20 8RR, U. K.

PHILOSOPHIA MATHEMATICA: PHILOSOPHY OF MATHEMATICS, ITS LEARNING AND ITS APPLICATION (University of Toronto Press)

Since its rebirth in 1993, Series Three of ***Philosophia Mathematica*** has published two issues. For those seriously interested in the philosophy of mathematics it is essential reading. For philosophically minded mathematics educators, it is also a prestige journal in which to publish. The journal is doing well, but needs broader support. For further details of subscriptions see POMENEWS6 or contact the editor: Robert Thomas, Department of Applied Mathematics, University of Manitoba, Winnipeg, Manitoba, Canada R3T 2N2. Internet: Robert_Thomas@UManitoba.ca Telephone: (204)474-8126 Fax: (204)275-1498

OTHER NEWS

MATHEMATICS EDUCATION E-MAIL DISCUSSION LIST

MATHSED-L is an electronic discussion list to serve the international community of mathematics educators, run from Deakin University, Australia, and also sponsored by the Australian Catholic University. The list aims to encourage the dialogue of mathematics education researchers and theorists. Its aims are in harmony with the research agenda published as an ICMI discussion paper on the nature of research in mathematics education (in JRME and ESM). The underlying philosophy is one of acknowledging and valuing multiple paradigms for such research. The list intends to be a forum where the theories, perspectives and paradigms that will inform future research, to and beyond the year 2000, can be floated. So far there has been discussion of Vygotskian and Constructivist approaches, and an amusing exchange of about a dozen versions of the quatrain 'Roses are red, violets are blue, ...' completed to illustrate different epistemological perspectives in mathematics education. You can join MathsEd-L free of charge by sending an E-mail to: listserv@deakin.OZ.AU

The message should have no subject line and in the body of the message there should be a single line as follows: SUBSCRIBE MATHSED-L <your name> (In place of '<your name>' type your e-mail name-and-address. This may seem obvious, but on my first try I wrongly left the '<' and '>' signs on). To send messages to the list (and all its subscribers) send an e-mail to: mathsed-l@deakin.OZ.AU

Any requests for help should be sent to either of the list moderators:

Andrew Waywood (awaywood@christ.acu.edu.au)

Robyn Zevenbergen (robz@deakin.edu.au)

International Research Projects in Mathematics Education

Djordje Kadijevic, LEARNING, PROBLEM SOLVING AND MATHEMATICS EDUCATION (Report No. 93/3, ISSN 0107-8283)

Published by DIKU, Copenhagen University, Department of Computer Science, Universitetsparken 1, DK-2100 Copenhagen Ø, Denmark. Djordje Kadijevic, Serbian Ministry of Education, Nemanjina 24, YU-11000 Belgrade, Yugoslavia.

This slim but comprehensive volume represents an attempt to situate an approach to problem solving and computing in mathematics education on a sound social and epistemological footing. It represents a novel synthesis of ideas from mathematics education, cognitive science, problem solving research and the philosophy of mathematics education. The contents of the report are as follows:

Part 1 - LEARNING

Chapter 1, Knowledge and Learning. What is knowledge? What is learning?

Chapter 2, Learning Concepts. Learning Strategies. Metacognition. Learning styles, approaches and orientations. Types of learning.

Part 2 - MATHEMATICS EDUCATION

Chapter 3, Mathematics Teachers. Introduction. Teacher education. Types of teachers.

Chapter 4, Mathematics Teaching. Do we fail to teach mathematics? Why do we fail to teach mathematics? How ought we to teach mathematics?

Part 3 - PROBLEM SOLVING IN MATHEMATICS CURRICULUM

Chapter 5, The Art of Problem Solving. Introduction. Polya's approach.

Chapter 6, 30 Years of Research on Problem Solving. Heuristic-centred approach. Control-centred approach. Affect centred approach. Linguistic centred approach. Personal-centred approach. Application-centred approach. Computer-centred approach.

UNA EMPRESA DOCENTE - "A TEACHING ENTERPRISE"

This is a project led by Cristina Gomez and Paola Valero of the University of the Andes, Apartado Aereo 4976, Bogota, Columbia.

Una Empresa Docente has been running since 1988 as a research centre in mathematics education, concerned to raise mathematics education quality in Columbia, to develop mathematics teacher training programs, to develop curriculum designs and to carry out research. The project orientation is based on a forward looking epistemology of mathematics, and on a theory of teacher beliefs and actions, as well as a view of the relationship between mathematics and society. Currently they have published 4 text books and a range of computer software. A new development "Matebasica Matica" (derived from Matematica Basica, the former name of the course) is an explicitly epistemologically-orientated teacher training program in which teachers have the opportunity to reflect on their knowledge, beliefs and attitudes towards mathematics. The overall program and projects currently involve 100 institutions and 400 teachers and researchers. Currently a major focus is on the project "Calculators and pre-calculus: the beliefs of the teacher".