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This Newsletter is the publication of the

PHILOSOPHY OF MATHEMATICS EDUCATION NETWORK

AIMS OF THE NEWSLETTER

The aims of this newsletter are: to foster awareness of philosophical aspects of mathematics education and mathematics, understood broadly; to disseminate news of events and new thinking in these topics to interested persons; and to encourage international cooperation and dialogue between scholars engaged in such research.

SUBSCRIPTIONS

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UK£ 5 to Paul Ernest (address below).

US$ 10 to Stephen I. Brown, Graduate School of Education, SUNY at Buffalo, Buffalo, NY 14260, USA.

AUS$ 10 to Kathryn Crawford, Faculty of Education, University of Sydney, NSW 2006, Australia.

CAN$ 10 to A. J. Dawson (address below).

No cheques made out to POME Group, or credit card numbers, please, as we cannot cash them. Colleagues in Africa, Central & South America, Eastern Europe, Mid and Far East with difficulties in obtaining these currencies are currently welcome to it free.

EDITOR OF NEXT ISSUE OF POMENEWS

The editor of POMEnews 7 will be **A. J. (Sandy) Dawson**, Faculty of Education, Simon Fraser University, Vancouver, British Columbia V7A 1S6, Canada. Tel. 604-291-4326, Fax 613-478-7621, E-mail: userdaws@sfu.bitnet Send any items for inclusion to him.

PHILOSOPHY OF MATHEMATICS EDUCATION TOPIC GROUP AT ICME

The major event of the year for the POME Group was the Philosophy of Mathematics Education Topic Group at the 7th International Congress of Mathematical Education, Quebec, August 16-23, 1992. The following is the official report for the conference proceedings.

All mathematical pedagogy, even if scarcely coherent, rests on a philosophy of mathematics. **René Thom** (at ICME2)

Epistemological and philosophical issues are central to mathematics education research and practice. Topics illustrating this include constructivism, problem solving and investigational pedagogies, curriculum theories, teacher beliefs, applications of the *Perry Theory* and *Women's Ways of Knowing*, ideologies of mathematics education, ethnomathematics, multicultural and anti-racist mathematics, work on gender, values and mathematics, and the epistemological foundations of research paradigms.

Behind this is a 'Kuhnian revolution' in the philosophy of mathematics. The Euclidean paradigm of mathematics as an absolute, incorrigible and rigidly hierarchical body of knowledge existing independently of human concerns is being challenged. Instead, many seek a philosophy that accounts more fully for mathematics, including the practices of mathematicians, its history and applications, its place in human culture, including issues of values, education and learning. Davis and Hersh (1980), Kitcher, Lakatos and Tymoczko (1985) propose new quasi-empiricist paradigms for the philosophy of mathematics. Parallel developments in the sociology of knowledge, e.g., Bloor and Restivo (1985), mathematics education, e.g., Ernest (1991), and poststructuralist and postmodernist thought, propose social constructivist accounts of mathematics. All have important implications for mathematics education.

THE TOPIC GROUP

The sessions aimed to offer the opportunity for interested persons to engage with current developments, theories and controversies.

The speakers were: **Stephen I. Brown** (*USA*), **Kathryn Crawford** (*Australia*) **Paul Ernest** (*UK*), **Ernst von Glasersfeld** (*USA*)**, David Henderson** (*USA*), **Reuben Hersh** (*USA*)**, Christine Keitel** (Germany), **Sal Restivo** (*USA*)**, Anna Sfard** (*Israel*)**, Ole Skovsmose** (*Denmark*), **Thomas Tymoczko** (*USA*).

KEY ISSUES RAISED

What is the Philosophy of Mathematics Education? It concerns the subject (mathematics), the teacher and teaching, the learner and learning, and the social context. Each gives rise to a characteristic set of philosophical problems and issues, including the philosophy of mathematics, aims and theories of teaching, the use of mediating cultural artifacts (text-books, computers), theories of learning, and the relationships between these as realized in the social milieu.

How is the philosophy of mathematics relevant to education? In practice it influences the mathematics classroom especially via assessment. The use of ticks and crosses, and the impression that every question has a single correct answer (which the teacher possesses), communicates an absolutist epistemology to the learner.

Teacher beliefs (personal philosophies of mathematics) profoundly affect their classroom practices. Research results suggest that preservice teachers hold views of mathematics interpretable as absolutist or fallibilist, which intimately relate to their views of teaching and learning (e.g., transmission or constructivist, respectively).

Metaphors for mathematics include: rigidly structured Eiffel Tower, near-eternal mountain, or continually growing, self-renewing forest? What do these metaphors presuppose and entail?

Mathematical objects may be real to mathematicians, but are accessed through decidedly human psychological means (imagery and intuition). The question therefore is not 'Do the objects of mathematics exist?', but 'How can we best offer students entry into 'math worlds'?'

Mathematical symbolism enables the mathematician to manipulate the objects of mathematics. However, too often learners mimic their movements but are in fact empty-handed. Symbolic mathematics is meaningless unless conceptually underpinned.

Radical Constructivism argues that symbols and texts have no inherent meaning other than that constructed for them by the reader. 'Sharing of meaning' is not possible. Improved meshing of meanings is all that is possible. This view of interpretation has far-reaching consequences, e.g., for the notion of proof, and for the teaching of arithmetic.

The fallibilist branch of **Hermeneutics** recognizes that the interpretative cycle never arrives at ultimate meanings, for it can never leave the knower's own realm of cognitive constructs (paralleling Radical Constructivism).

A radical sociology of mathematics sees mathematical forms, objects, symbols and theorems as collective objects and worldviews, embodying 'math worlds'. Their social histories reveal them to be materials and resources constructed around social interests. But can this social account dispense with the experiential world of the individual subject, constructed by its own intellectual endeavours?

Critical mathematics education: Mathematics education justifies and reproduces mathematics, a central shaping force in modern society. Mathematical thought abstractions generate realized abstractions embodied in artifacts and social patterns including supermarket check-outs, test scores, voting systems, assembly lines, bureaucratic structures, social institutions, etc. Our mathematized society leads not only to economic power and freedom but also to tight constraints on people's lives.

We need a **Critical Perspective of the mathematics curriculum** as conditioned by the social function of mathematics in society, with mathematics reforms interrogated to see whose interests they serve, and their relations with power, social structure, ideology and values. We need alternative concepts for mathematics education based on the social function of maths, and ways of acting together in the light of this knowledge.

The aims of mathematics education are to empower learners to understand and act on their environment, to be able to make competent and autonomous judgments through mathematics. How does this square with its social role?

Ideologies of mathematics education: Is there a parallel between dichotomies such as fallibilist versus absolutist philosophies, constructivist versus passive reception theories of learning; facilitating, problem-orientated versus authoritarian theories of teaching? How do ideologies of mathematics education mediate power and serve the reproduction of inequalities?

Problems: Is the concept of problem inseparable from that of solutions? What about the problem as project for the future, which involves uncertainty and risk? Problem posing offers ownership of problems and empowerment of learners.

What implicit assumptions do different theories and philosophies contain? What assumed model of a human being? Radical Constructivism assumes the knowing subject has consciousness, memory, and a desire for order in its experiential world. The philosophy of mathematics education is about examining the underlying assumptions of the field.

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DETAILS OF THE TOPIC GROUP PRESENTATIONS AND DISCUSSION

Space limitations (1000 word limit) in the above report did not allow the detailed account that follows to be included. This gives more detail of the individual speakers' (and the audience's) contributions.

SESSION 1: PHILOSOPHY OF MATHEMATICS AND ITS EDUCATIONAL IMPLICATIONS

Reuben Hersh spoke on *Recent Thinking in the Philosophy of Mathematics*, and emphasized the shift away from absolutist to fallibilist or quasi-empirical views. He drew a metaphorical diagram showing traditional and other philosophies of mathematics sitting in the heavens and raining down their influence via teachers beliefs about mathematics to profoundly affect their classroom practices. A semi-humorous response from a member of the audience was that the influence travels upwards, from theory to practice, thus inverting the implied priority.

Thomas Tymoczko addressed some *Issues in Philosophy, Mathematics and Education.* He stressed the importance of a socially orientated philosophy of mathematics and drew upon his image of mathematics as a mountain; something which can be experienced differently by different climbers. He also stressed how mathematical objects really do seem to exist, like the mountain. Ultimately, his emphasis was on a humanistic view of mathematics and its importance for education.

Sal Restivo, *The Sociology of Knowledge and Math Education, P*resented a radical sociology of mathematics beginning from the assumption that talk, persons and intellect are constitutively social. Consequently mathematical forms or objects are seen as sensibilities, collective objects and worldviews, and embody 'math worlds'. To explain the content of mathematics is therefore to unpack the social histories and social worlds embodied in objects such as theorems. Notations and symbols are tools, materials and resources that are constructed around social interests and orientated towards social goals. Consequently, the mathematics curriculum is conditioned by the social function of mathematics in society, and mathematics reforms must always be examined closely to see their relations with broader issues of power, social structure and values, and to see whose interests they serve.

Ernst von Glasersfeld questioned the possibility of a wholly social account of mathematics and raised the problem of access to the social world Did it not presuppose the experiential world of the individual subject, constructed by its own intellectual endeavours?

A general question addressed to the speakers was: Who should develop the mathematics curriculum? One answer was that curriculum development should be entrusted largely to the experts, not mathematicians or mathematics educators, but teachers of mathematics.

Kathryn Crawford emphasized the importance of the cultural context of mathematics education, and argued that this is where the significance of a social philosophy of mathematics arises. Mathematics does indeed rely on cultural and intellectual tools, from symbol systems to computers, and it is their subtle and embedded social roles that gives them their power.

David Henderson, spoke about how the practicing mathematician draws on imagery and intuition, and how the absolutist or formalistic image of mathematics may be a caricature. The objects of mathematics are real to the mathematician, but are accessed through decidedly human psychological means. The question therefore is not Do the objects of mathematics exist? but How can we best offer students entry into 'math worlds'?

One of the questions that recurred was 'What do the philosophies of mathematics and mathematics education mean for the child in the classroom?' One answer is that teachers' views of mathematics are intimately related to their views of teaching and learning (transmission or constructivist, respectively). Thus powerful classroom implications might flow from the teacher's beliefs and subsequent actions and practices.

SESSION 2: PHILOSOPHICAL ISSUES IN MATHEMATICS EDUCATION

Anna Sfard, drew upon the mountain climbing metaphor introduced in the previous session. She said the teacher is positioned at the top, with a wide ranging vista of the mathematical landscape, whilst the learner struggles upward without the benefit of this overview. With such a perspective the mathematician can manipulate the objects of mathematics, but too often, the learners whilst mimicking their movements are in fact empty-handed.

Ernst von Glasersfeld, *Radical Constructivism and the Philosophy of Mathematics,* argued that symbols must be linked to concepts by a thinking subject. Mathematics depends on concepts of unity and plurality, which we arbitrarily abstract from the field of our experiences. Plurality cannot be derived from sense impressions alone, but by reflective abstraction from individually constructed mental abstractions. Educationally, the key point is that symbolic mathematics is meaningless unless underpinned by conceptual structures. Symbols and texts have no inherent meaning other than that attributed to them by the reader. Thus the 'sharing' of meaning is not actually possible. All that we can strive for is improved compatibility of meanings. This view of the processes of interpretation has far-reaching consequences for the notion of proof, and, more importantly, for the teaching of arithmetic

Nicholas Herscovics asked how this account of the meaning of symbols squared with Hermeneutics.

Ernst von Glasersfeld replied that of the two schools in Hermeneutics one is absolutist, seeking ultimate meanings, whereas the other is fallibilist, recognizing that the Hermeneutic cycle never arrives at ultimate meanings, for it can never leave the knower's own realm of cognitive constructs. This latter view fits well with the Radical Constructivist account.

Stephen I. Brown, *The Problem of the Problem and Curriculum Fallacies.* A fundamental curriculum fallacy is to believe that if we get the philosophy of mathematics and the concept of problem straight in our heads, then we could solve the problems of mathematics curriculum and pedagogy. But there are no ultimate answers, and routinizing problem solving by reducing the problem to an imposed task loses something essential. Indeed a more fruitful approach is to ask what would still be left out if problem solving was centrally situated in the maths curriculum? One answer is that problem *posing* would be left out. Problem posing leads into problem solving, and offers both ownership and understanding of problems. It offers the inside view Dewey said is needed for understanding. We act as if problem and solution are integrally connected; but is the concept of problem inseparable from that of its solution? What about the problem as project for the future, involving uncertainty, risk? Issues of personhood are not adequately connected with problem.

Nicholas Herscovics asked whether the fact that he did not personally engage with Fermat's Last 'Theorem' meant that it ceased to be a problem for him?

Stephen Brown replied that posing problems should not just lead to solutions, but can be revelatory about the poser's mind-set, emotional state, view of the world, etc. Problems are more than invitations for solutions.

Christine Keitel, spoke on *Philosophical Aspects of a Critical Mathematics Education.* She asked What is there to be critical about, when mathematics education seems to be ideology-free like pure mathematics, having purity, indubitability, unity, and universality? The answer is that mathematics education serves to justify both explicit and implicit mathematics, which together comprise central shaping forces in Western society. We want to empower learners to understand their environment, to be able to make competent and autonomous judgments through mathematical powers and abstract thinking. But we must be aware that thinking abstractions, such as worlds and images, have given rise to realized abstractions in artifacts and social patterns Realized abstractions today include supermarket check-outs, test scores, voting systems, assembly lines, bureaucratic structures, social institutions, the military and big business. Our implicitly mathematized society leads not only to economic power and freedom but also to tight constraints on peoples lives. This is where a critical perspective is needed, for the influence of mathematics is both liberating and enslaving. We need alternative concepts for mathematics education; an awareness of this aspect of social function of maths, usually neglected, and more positively, ways of acting together in the light of this knowledge.

Paul Ernest, offered a synoptic vision of what *The Philosophy of Mathematics Education* might be, and distinguished four initial areas of its concern: the subject (mathematics), the teacher and teaching, the learner and learning, and the social context. Each of these gives rise to a characteristic set of philosophical problems and issues, including the philosophy of mathematics, aims and theories of teaching, the use of mediating cultural artifacts (text-books, computers), theories of learning, and the relationships between these as realized in the social milieu. He offered a provocative parallel between the two endpoints of a continuum of positions in each of these realms. A fallibilist, socially constructed philosophy of mathematics (as opposed to an absolutist, objectivist philosophy) parallels a constructivist (as opposed to a passive reception) theory of learning, and a facilitating and problem-orientated (as opposed to authoritarian and basic skills orientated) theory of teaching mathematics. He concluded with a sketch of work in progress: a social constructivist philosophy of mathematics which links the learning and warranting of mathematical knowledge through human interactions and conversation.

It was asked: How does the philosophy of mathematics actually influence classroom practice? It was answered that one rich area to consider is assessment. The use of ticks and crosses communicates an absolutist epistemology to the learner, and the idea that every question has a single correct answer (of which the teacher is in possession.)

Avery Solomon argued that many of the philosophies discussed contain inexplicit assumptions. Radical Constructivism seems to assume the outside-inside distinction, and that our learning is based on sensory information. He also raised the issue of the assumed nature of a human being.

Ernst von Glasersfeld agreed that Radical Constructivism, like any position, rests on a number of assumptions. He identified three. The knowing subject must be assumed to have consciousness, memory, and a desire for regularity or order in its experiential world. Ironically, although the Behaviourists claimed to eliminate values, they had to use hungry rats for experiments, whose cognitive endeavours were underpinned by desire.

Another answer to the question of assumptions was that, above all, the philosophy of mathematics education is about examining the underlying assumptions of the field.

Ole Skovsmose, drew together the themes of the group, asking if the philosophy of mathematics and the philosophy of mathematics education were not concerns very distant from the study and practice of mathematics education, as some questioners had implied. He concluded that despite initial appearances, the philosophy of mathematics education was an overarching field of inquiry that asked questions central and essential to research and practice in mathematics education.

As **Jim Neyland** pointed out afterwards, the presentations, controversies and subsequent discussions succeeded in raising many of the most pressing issues that a research agenda for the Philosophy of Mathematics Education should include.

COMMENT ON TG 16

The Philosophy of Mathematics Education: "What does this mean for the child in the classroom?"

Geoffrey Roulet, Queen's University, Kingston, Ontario.

In the final minutes of Topic Group 16's first session (August 18) one participant, challenging the ongoing debate, posed the question appearing in the title above. This issue was raised again by Ole Skovsmose when he concluded his reaction to the second session's deliberations by asking, "What is the connection between philosophy of mathematics, philosophy of mathematics education, and mathematics education itself?" One answer to such questions lies in the parallels presented to us by Paul Ernest. This scheme and its more expanded version appearing on pages 138-139 of The Philosophy of Mathematics Education (Ernest, 1991) suggest reasons why we, in some jurisdictions, are experiencing difficulties generating significant change in mathematics instruction.

Two views (philosophies) of mathematics, absolutist and fallibilist, are set out. The former takes mathematics to be a body of absolute truths developed from a set of logical axioms through a deductive process while the second regards the discipline as potentially flawed and open to revision and further growth. Similarly, styles of mathematics teaching may be described and grouped under the titles, transmission and constructivism. The parallel between views of mathematics and teacher practice is argued by Lerman (1983): "The fundamental issue from which mathematics teachers cannot escape is that a commitment to a theory of mathematical knowledge logically implies a particular choice of syllabus content and teaching style" (p. 65). My research with secondary school mathematics teacher candidates enrolled in a one-year pre-service program following completion of an undergraduate degree illustrates this link.

The initial exercise of a course in mathematics curriculum and instruction required that student-teachers write position papers giving their images of Schwab's (1978) four "commonplaces of teaching": the subject (mathematics), the student of mathematics, the teacher of mathematics, and the relation of mathematics learning to society in general. These papers were not graded but the ideas expressed were analyzed and grouped under common themes. students holding an absolutist view of mathematics, describing the discipline as a "set of rules" with "no grey areas or exceptions", echoed this position in their illustrations of "good" teaching. In their eyes teaching involves transmission of the mathematical facts through clear and coherent presentation of concepts and procedures followed by supervised pupil practice. These teacher candidates expressed the view that school mathematics must be made relevant to students and suggested that this be done "by solving practical problems using concepts" after they have been taught and practised. This "toolkit" image of mathematics and its teaching and learning was held by the majority of this class of future mathematics teachers.

A smaller group of class members took a fallibilist position on mathematics, noting the existence of contradictions and the possibility that one may "achieve multiple answers based on various assumptions". Mathematics was seen as a "language" for discovering and expressing "patterns and relationships". The descriptions of good lessons as those that "allow the students to 'own', through self-discovery, ... the concepts being taught", captured a sense of the constructivist position. Subject relevance was also an issue for this group of students but here the suggested instructional sequence was reversed with concepts introduced and developed by "using key examples chosen from the real world".

In general the students' images (philosophies) of mathematics clearly generated corresponding views of teaching. Moreover the dominant, toolkit view leads to teaching practice that is unlikely to capture pupils' imagination and set them on a path of problem posing and solving. Given the link between views of mathematical knowledge and styles of instruction, teacher candidates and practising teachers are unlikely to adopt classroom practice reflecting any sense of constructivism unless they are provided with experiences that enrich their image of the discipline. The question for those of us involved in teacher education is "What features in a teacher preparation program, both in mathematics content courses and those focusing on instructional methods, could help generate for 'our' students an image of mathematics as a social construction with adventures still to be followed?"

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PHILOSOPHY OF MATHEMATICS EDUCATION DISCUSSION GROUP (PME)

There was a Philosophy of Mathematics Education discussion group at the 16th Annual Meeting of the International Group for the Psychology of Mathematics Education, at the University of New Hampshire, 6-11 August 1992. An exciting *genuine discussion* took place, following short lead presentations made by a number of persons including Marcelo Borba, Jere Confrey, Kathy Crawford, Clive Kanes and Steve Lerman. Participants put forward a list of questions and issues for the philosophy of mathematics education:

What are the different senses of evidence and justification in developing mathematical knowledge?

The role and function of quasi-empirical methods in mathematics and math education

Why does there seem to be a strong emotional component to philosophical commitments, and irreconcilable differences between absolutism/fallibilism?

Why teach mathematics? What is the role of teaching and learning mathematics in society?

The ideology and educational expectations that are emerging from information technology and the collective unconscious

Identity as it evolves in math learning

Alternatives to abstraction as a way to relate and integrate different views of math (i.e. fields of math, ethnomathematics, etc.)

Teacher and meta-teacher -- who is who?

How to help[teachers become aware of and reflect on their personal philosophies of mathematics?

Philosophies of mathematics and their impact on teaching (through teachers hidden beliefs, etc.)

Is the emancipation/liberation of teachers/learners educational? What is educable?

Perspectives of knowledge.

Critique of psychological theories, cf. their application to the problems of mathematics education.

Theories of learning and mathematics education; Vygotskyan theories of learning and teaching; sociocultural aspects of learning mathematics

Following a heated exchange on the significance of absolutism versus fallibilism in the philosophy of mathematics and education, Gerry Goldin and Paul Ernest spent an evening eating lobster in Maine and discussing these differences. Gerry summarized the position reached.

"Sources of 'uncertainty' in mathematics (in the sense of bases for questioning the 'warrant' in asserting a mathematical statement as 'true'): We take the warrant to consist of (a) axioms and 'true' by convention, (b) statements obtained from axioms through agreed-upon inference procedures; i.e., theorems are 'true'; so this warrant is really [i.e. means] 'provable'. Now sources [of uncertainty] include:

(a) variability in the meanings and uses of terms and/or symbols over time, or over possible worlds (meanings of terms are social).

(b) the possibility of undetected human error' in proving a conjecture (always a non-zero probability); this may actually be of practical importance in the case of completed proofs.

(c) Gödel's results concerning the non-provability of consistency of axioms when they are sufficiently complex to include the natural numbers [Peano Arithmetic]. This does not apply to less complex systems.

We agreed on these sources of uncertainty or 'fallibility'. However we seem to have major differences on how these sources of uncertainty influence philosophy of education in mathematics. My position: not very much, Paul's position; profoundly. We have yet to explore this [difference]."

CALL FOR CONTRIBUTIONS TO BOOK:

MATHEMATICS, PHILOSOPHY AND EDUCATION: PAPERS FROM THE POME GROUP

Edited by Paul Ernest

Tentative Publisher: SUNY Press (Science Technology and Society Series, Editor, Sal Restivo)

This book will collect together papers representing current work in the philosophy of mathematics education, that is contributions to the philosophy of mathematics with an import for education, and contributions to mathematics education with a philosophical or theoretical flavour. In recognition of the nascent character of this field, and because of a deep commitment to conversation, multi-authored dialogues are actively encouraged. The collection will include the papers given at the Philosophy of Mathematics Education Topic Group at ICME-7, 1992, and will build on contributions published in this newsletter.

Contributions are also sought more widely from readers of POMEnews, both in the form of dialogically linked pieces by multiple authors, and single paper contributions to one of the following sections, or other issues pertinent to the philosophy of mathematics education. Offers to read one or more thematically linked contribution(s) and then add further comments are particularly welcome. (Authors will have the opportunity to see any comments on their work, and to reply) Please send proposals and draft contributions before the end of February 1992. The final deadline is currently planned for March/April 1992.

TENTATIVE OUTLINE AND POSSIBLE CONTRIBUTORS

The following outline is tentative, and the headings will be further worked, void ones discarded, and categories reconceptualized to fit the contributions as received. Not all of those named are yet committed to writing a contribution (italics name pieces received or firmly proposed).

1. Introduction What is the philosophy of mathematics education? An Agenda for research in the philosophy of mathematics education

2. Aims and Values in Mathematics Education Aims, rationale and values. Curriculum reflection and critique. Ideologies of Social Groups in Mathematics Education. (Jim Neyland) Philosophies of mathematics in education: personal and curriculum philosophies. Problems in Mathematics Education: their philosophical basis. *The problem of the problem and Curriculum fallacies*. (Stephen I. Brown)

3. Radical Constructivism Discussion of the philosophical foundations of radical constructivism. *A radical constructivist view of basic mathematical concepts* (E. von Glasersfeld). *Radical constructive criticisms of von Glasersfeld's radical constructivism* (Robert S. Thomas) Incorporating the Social in Constructivism

4. Epistemology, Psychology and Theories of Learning Mathematics Psychological Theories of Epistemological Development. (Anna Sfard) Social Constructivism. Activity Theory.

5. Epistemology of Research in Mathematics Education Epistemology of Research. Qualitative Research Paradigms in Mathematics Education: Their Epistemological Basis. The Teacher-as-researcher in Mathematics Education

6. Developments in the Philosophy of Mathematics Recent Developments in the Philosophy of Mathematics. Reflections on their impact on pedagogy. (Reuben Hersh, Philip J. Davis, David Henderson, Thomas Tymoczko)

7. Philosophy of Mathematics and Mathematics Education. *How does the Philosophy of Mathematics impact on education?* (Eduard Glas) *The role of philosophy and history in mathematics education* (Francesco Speranza), (Michael Otte)

8. The Image of Mathematics The import of Teacher Beliefs and Personal Philosophies of Mathematics. Cultural images, social images, and mathematics in the media. *Dialogue on the popular image of mathematics (POMEnews 4/5). What does the philosophy of mathematics education mean for the child in the classroom? (*Geoffrey Roulet)

9. Critical Mathematics Education: Critical Mathematics Education: Its Philosophical Basis; empowerment, critical mathematics literacy, and democracy. *Technology and critical mathematics education* (Ole Skovsmose), *Towards a Critical Mathematics Education* (Christine Keitel). *The ideology of certainty in mathematics* (Marcelo Borba)

10. Gender and Race in Mathematics Education *Is there a female mathematics or a feminist style of doing mathematics?* (Leone Burton) Cognition, Society and Technology. (Kathryn Crawford)

11. Ethnomathematics Ethnomathematics, the nature of Mathematics and Mathematics education. (Marcelo Borba. Ubi D'Ambrosio) *Dialogue on Ethnomathematics and Mathematics (POMEnews 6)*

12. History, Philosophy and Pedagogy of Mathematics *Dialogue on Revolutions in the history of mathematics (POMEnews 4/5)*. History in Mathematics Education. *Historical style: an historical and cross cultural variable* (David Wells) *The influence of some mathematical revolutions over philosophical and didactical paradigms* (Francesco Speranza)

13. Social views of Mathematics *Sociology of Knowledge and Mathematics Education* (Sal Restivo)

14. Postmodernism, Post-structuralism, Mathematics and education *Dialogue on Postmodernism, Mathematics and Mathematics Education (POMEnews 3)*. Post-structuralism, Semiotics and Mathematics Education. *Dialogue on Language, Psychoanalysis and Mathematics.* (David Pimm and Dick Tahta). Hermeneutics and Mathematics Education

15. Philosophy of Mathematics Education: Prospects Prospects for mathematics education practice, and for philosophical and theoretical research in mathematics education in the third Millennium (CE.)

COMMENT ON A CONTRIBUTION TO POMENEWS 4&5

On Manifestly (or, at least, apparently) Timeless Objectivity

Ernest von Glasersfeld, SRRI, University of Massachusetts, Amherst, MA 01002, USA.

I receive several newsletters. They pile up beside my bed because I find them a harmless soporific. Not so the last issue of POME (Nos. 4 & 5, 1992). It kept me reading long past midnight up to page 23, and then one sentence launched me into a state of untimely wakefulness.

"...we can talk about anatomy as a study without danger of our words being misapplied to a passing elephant."

Where are we, I wondered - and my mind went back half a century to a happy elephant wallowing in the muddy waters of a river in Ceylon (now Sri Lanka), where I had spent two enchanting days.

Unfamiliar with the author's name, I looked to the end of the article to find out where he lived: Canterbury, Kent, U.K. So far I had associated Canterbury with a cathedral, a bishop, and Chaucer. Now my thinking was switched to another track. What topic, I asked myself, would present the danger "of our words being misapplied to passing elephants" at Canterbury? - An answer grew out of what I went on to read.

"This elephant is walking past quite independently of any logical or observational reasons that I may have for thinking that it is walking past. An omniscient God..." (p.24) Yes of course, I thought, we are talking about God's reality, and she can have elephants can have elephants walking about anywhere, even in Canterbury. Vico (whom I love to quote) made it quite clear: "God is the artificer of Nature, man the god of artefacts." And he also said, as Kant did some decades later, that human reason can <u>know</u> only what human reason has made, and this does not comprise any Nature that God might have cooked up. Just as Ray Godfrey says, we haven't "any logical or observational reasons" for knowing it. Hence, for all I can know, the elephants God has walking about Canterbury might even be white.

Consequently when I came to the end of the article and read the expression "timeless objectivity", I thought that this referred to God's ontology. But then I asked myself, how could any part of God's reality be 'manifest' or even 'apparent' to us? - I realized that the author had arithmetical facts in mind, like the wrongness of 4 × 5 = 30 or the rightness of 2 + 2 = 4.

I do not and, as Vico said, cannot know whether God is in the habit of generating counting units the way I do. Nor could I know whether He/She uses the standard number-word sequence that we had to learn in order to become viable members of our human community. Personally, I think it's unlikely - but my opinion about the matter is obviously irrelevant. In contrast, I consider it extremely relevant to observe that arithmetical facts are, as the word says, things that are, as the word says, things we make (*facere*). And we construct them in accordance with the habits of generating units and counting that we have found to be viable not in God's reality but in our own experiential world. From this perspective, such constructs may become 'manifest' whenever we construct them, but to call them 'timeless' and 'objective' is a little misleading, because it tends to be interpreted as *existing irrespective of one's construction* rather than the result of intersubjectively compatible metal constructions.

I concluded that there is still justification for believing that there is no harm and perhaps some good in convincing teachers to admit to their students that mathematics is not a God-given dogma and to switch from indoctrination to the fostering of individual building. And I went to sleep as a happy radical constructivist.

DISCUSSION THEME:

ETHNOMATHEMATICS: ITS EDUCATIONAL, POLITICAL AND PHILOSOPHICAL SIGNIFICANCE

Ethnomathematics: the voice of social-cultural groups in mathematics education

Marcelo C. Borba

In a recent article about ethics in school, Giroux asked: "Whose history, story, and experience prevails in the school setting? In other words, who speaks for whom, under what conditions, and for what purpose?" (1991, 306). These questions imply that an education for critical citizenship needs to have plural voices. Questions such as these may seem foreign to mathematics education. Due to a belief in the purity of mathematics, mathematics education is often used to reinforce myths about the neutrality and apolitical nature of education. (Sal Restivo, 1990, has made a nice critique of the notion that mathematics is pure.)

At least one new development in mathematics education has brought political issues into the mathematics education debate: the notion of ethnomathematics. In emphasizing the plurality of mathematics, this notion challenges the idea that "true" mathematics is a uniform, objective and mono-cultural phenomenon. Ethno-mathematics can be seen as a field of knowledge intrinsically linked to a cultural group and to its interest, being in this way tightly linked to its reality, and being expressed by a language, usually different from the ones used by mathematics seen as science. This language is umbilically connected to its culture, to its ethnos (Borba, 1990). "Ethno" and "mathematics" should be understood in a broader sense: "ethno" as referring to socio-cultural groups, and "mathematics" to activities such as ciphering, measuring, classifying, ordering, inferring, and modeling (D'Ambrosio, 1985). Examples of socio-cultural groups include the different peoples studied in Africa by Gerdes (1988) and Ascher (1991) and in Brazil by Ferreira (1990); as well as groups such as South African carpenters studied by Millroy, Brazilian children in a slum (Borba) or US grocery shoppers (Lave, 1988). The very power of ethnomathematics and of the work done in this area is to challenge the notion that mathematics is only produced by mathematicians.

In an ethnomathematical view, academic mathematics is just one among other mathematics. Mathematics produced in academia is also "ethno" because it is also produced in a setting--academia--with its own values, rituals and special codes in the same way as other [ethno] mathematics. Even so, academic mathematics was and is being produced by many different historical cultures and by the "live" cultural diversity of the present.

When applied to mathematics education, the notion of ethnomathematics may be even more powerful. Studies have already been done in both formal and informal educational settings. Nobre, Gerdes (1988) and Frankenstein (1989) (in Brazil, Mozambique and the USA respectively) have all done classroom-based studies in which the experiences of the socio- cultural group is taken into account. Ascher (1984) has used the mathematical ideas developed by different cultural traditions to teach a college-level course.

In a Brazilian study, Borba used ethnographic research techniques to map some of the mathematical ideas and activities of children in a slum that evolved into an interdisciplinary pedagogical project in an informal educational setting. This study found that problems based in the children's culture, such as planting a vegetable garden and selling its produce, were more meaningful for them and therefore could better enhance the development of mathematical concepts. One group of children developed the notion of scale, foreign to them in that context, by developing a chart to organize the garden. Other members of the group learned arithmetic skills and concepts so they could do the bookkeeping correctly.

These experiences with ethnomathematics are groundbreaking and much work remains to be done. The notion of ethnomathematics (Frankenstein and Powell, in press; Borba, 1990) may shed some light on a critical approach to mathematics education--an approach that might address Giroux's (1991) concern with education as a means of achieving critical citizenship. Therefore, an important task would be to look for ways in which we can understand students' cultural ways of producing and expressing their mathematics.

Ascher, M. 1991. *Ethnomathematics: A Multicultural View of Mathematical Ideas*. Belmont, CA: Brooks/Cole.

Borba, M. C. 1990. Ethnomathematics and Education, *For the Learning of Mathematics*, 10, 1.

D'Ambrosio, U. 1985 Ethnomathematics and Its Place in the History of Pedagogy of Mathematics, *For the Learning of Mathematics*, 5, 1.

Ferreira, E. S. 1990. The Teaching of Mathematics in Brazilian Native Communities. *International Journal of Mathematical Education in Science and Technology*, 21, 4.

Frankenstein, M. 1989. *Relearning Mathematics*. London: Free Association Books.

Frankenstein, M. and Powell, A., in press. Toward Anti-Domination Mathematics: Paulo Freire's Epistemology and Ethnomathematics, in *Paulo Freire: a critical encounter*, edited by P. McLaren and P. Leonard. NY: Routledge.

Gerdes, P. 1988. On possible uses of traditional Angolan sand drawings in the mathematics classroom. *Educational Studies in Mathematics*, 19 (1): 3-22.

Giroux, H. 1991. "Beyond The Ethics Of Flag Waving: Schooling and Citizenship for a Critical Democracy," *The Clearing House*, 64 (5).

Lave, J. 1988. *Cognition in Practice.* Cambridge University Press.

Zaslavsky, C. 1973. *Africa Counts*. NY: Lawrence Hills.

Restivo, S. (1990) "The Social Roots of Pure Mathematics", in Cozzens, S. E. and Gieryn, T. F. Eds. *Theories of Science in Society*, Bloomington, Indiana University Press, 1990.

RESPONSE TO MARCELO BORBA ON ETHNOMATHEMATICS: Ubiratan D'Ambrosio

By citing H. Giroux in the beginning of the paper, Marcelo Borba places himself in the growing group of Critical Educators, and its subgroup of critical mathematics educators. And thus invites the reader to ask 'What does it mean to be a critical mathematics educator?'. Borba himself points to the prejudice of many people, that mathematics is neutral, carries no ideology, is apolitical. Indeed this opinion reflects a major historical and philosophical falsification of the nature of mathematics. Mathematics is a socio-cultural construction of human mind. And as such it reflects both the actual and ideological context in which it is generated, transmitted, institutionalized and diffused. Clearly both actual and ideological, or in broader terms, socio-cultural contexts obviously generate different modes of explanation, of understanding and distinct ways of coping with the environment. Mathematics is nothing but a way human beings developed in order to cope with the environment, to explain and understand it. But since these environments are distinct, we expect diverse forms of mathematics to be generated. The program ethnomathematics, presented by Marcelo Borba, aims at looking into these different techniques ('tics') of explanation, understanding and coping ('mathema') with distinct socio-cultural and natural environments ('ethno'). Himself one of the proposers of this broader way of looking into mathematics ('ethno-mathematics'), Marcelo Borba refers to his own works and to works of other scholars in this new area of research and pedagogical practice.

ETHNOMATHEMATICS AND MATHEMATICAL DIVERSITY: A RADICAL CONSTRUCTIVIST PERSPECTIVE Erick Smith, USA

Borba's use of ethnomathematics as a way of seeing diversity in our ways of doing mathematics complements the work of others in that area and forms the framework for understanding the important role of culture and context in mathematics teaching. The perspective of this work is, as Borba states, closely related to the creation of a viable form of critical mathematics. I do have three concerns with the short article which I would like to mention. These are not necessarily criticisms of his position, rather extensions of the perspective. I offer them as one way that I see a complementarity between radical constructivism and the work being carried out in ethnomathematics.

First I think it is important not to give the impression that mathematics is a well-defined area of human activity that can be neatly divided up into areas of well-defined ethnomathematics. The kinds of activities that Borba describes as mathematics, "ciphering, measuring, classifying, ordering, inferring, and modeling" are themselves cultural constructs (as is the very notion of mathematics itself). Thus they do not define mathematical activity as much as point at general kinds of activities that are more or less mathematical from the perspective of any particular observer, or, in a more social sense, through a process of agreeing to agree by a number of observers. Mathematical activity should not be seen as clearly separable from other kinds of human activity, rather as smoothly blending with those activities.

Second, given this rather amorphous notion of certain kinds of activities that humans engage in which we want to call mathematics, we should be aware that there are many perspectives we might take that allow us to see commonalities in the mathematics of particular groups. Those working from an ethnomathematics perspective have chosen to 'slice up the pie' using social and cultural criteria, thus have described the mathematics of Mozambique sand painters, Brazilian slum children, South African carpenters, and so on. It is because they have identified commonalities in the mathematical activities within these groups that Borba can describe ethnomathematics "as a field of knowledge intrinsically linked to a cultural group and to its interest, being in this way tightly linked to its reality." However, given a different way of seeing commonalities, one might see more in common between the mathematics of academic geometers and Mozambique sand painters than between academic geometers and academic algebraists. Likewise one might find more in common between Brazilian slum children gardeners and English gardeners than between Brazilian slum children gardeners and Brazilian slum children non-gardeners. To paraphrase Maturana: 'Every commonality is made by an observer.'

Finally, and perhaps most importantly, I would argue that we should not allow an emphasis on the diversity of the mathematics of individual groups to dilute our concern for the diversity of mathematics among individuals within any group. As I have argued elsewhere (Smith, 92), it is the richness of the diversity of the mathematics of individuals within a group (or community) that gives rise to the richness of the identifiable mathematics of the community, just as it is the richness of the diverse mathematics of separate communities that provides the robustness of the mathematics of the larger society.

As stated earlier, I do not believe any of these concerns contradict Borba's main thesis and might possibly have been included in a longer description of his perspective. I do think that they are important, however, if we are to maintain a critical perspective that allows us to see mathematics as a celebration of human activity and not as a prescribed kind of activity belonging to any particular group (or groups).

Borba, Marcelo (1992) Teaching Mathematics: Ethnomathematics, the voice of sociocultural groups. The Clearing House, 65 (3). pp. 134-135

Smith, Erick (1992) Constructivist mathematics as an ethical process. A paper presented at the Second International Conference on the History and Philosophy of Science in Science Education. Kingston, Ontario, May 11-15, 1992.

REPLY TO MARCELO BORBA: Steve Lerman, UK

Marcelo Borba spells out quite clearly and powerfully the potential of the 'ethnomathematics' perspective on school mathematics. So much research, to which he refers, indicates that to ignore the varieties of mathematical activity and thinking of different social groups is to diminish the learning experience of children in school. It also has a longer term implications politically, in that young people come to see schools as part of an establishment in which they have no voice and in which their life experiences are given no value or worth. The anthropological view of knowledge of which it is a part is a powerful contribution to the deconstruction of the myth of absolutist mathematics.

I want to take issue with Marcelo, though, in three aspects of his paper, that are inter-related.

1. Although we as mathematicians, may identify certain social activities as 'mathematics' (which can be thought of as an abstract name for a whole family of social activities), they are different things within those contexts. They are identified by their functions and purposes and uses. Money, although written in figures and with a decimal point, is associated with need, power, presence of or absence of, value, work, profit, etc. When I buy something in a supermarket and decide what is the best 'value' to me, that includes taste, the shape of the container, the absence of packaging if I am concerned with conservation, no animal products, and perhaps economy of size, but that depends on how much I can afford this week. Am I doing mathematics? Another illustration: is the Navajo carpet-weaver doing (second-rate in academic terms) geometry or (first-rate in craft terms) carpet weaving? Is it empowering if I tell the Navajo person that she is *really* doing geometry (see P. Dowling in Harris, 1991)?

2. People have *voices* rather than one voice. My mathematics has one kind of meaning for me when I mix with mathematics educators, quite another when I mix with 'real' mathematicians, and another one again when I mix with people for whom mathematics means dull, dry, difficult, meaningless stuff. Mathematical particulars have different meanings for me too: five million has one kind of meaning, whereas six million has quite another (to do with the collective cultural trauma of the Holocaust).

3. One <u>voice</u> that is essential to develop in children, essential in terms of enabling people to have access to power, is the school-academic-mathematics voice. We shouldn't deny that, we should find ways of helping children to participate in that. You can say "my half is bigger than your half" so long as you *also* realize that in the mathematics classroom "half" has a specific connotation. Having both is fine, in fact essential, and this is a point that Marcelo misses out. (I am not ascribing any transcendental existence to 'school-academic-mathematics', I wrote as long ago as 1983 about the social nature of mathematical ideas and objects.)

The major item on the agenda for ethnomathematics, in my view, is to find ways of valuing those activities in themselves, drawing them into the classroom in some respectful form (which will alter them of course) but also to enable the development of other voices, including the game of succeeding at school mathematics. When my daughter says to me "I can see how to do this trigonometry but what is it for, will I ever use it, what meaning does it have for my life?" part of my answer is "Suspend disbelief, think of it as a game to be learned, because your GCSE [school examination] certificate at the end is pretty useful".

Harris, M. Ed. (1991) *School, Mathematics and Work*, Falmer, London.

REFLECTIONS ON THE ETHNOMATHEMATICS DISCUSSION; Paul Ernest, UK

Marcelo Borba's paper, and the discussion it initiates, raises for me many of the key issues concerned with ethnomathematics. I want to elaborate on a couple of points that emerge in the above discussion. There is the important pedagogical and cultural issue, clearly present above, that ethnomathematics is not about the exotic conceptions of 'primitive peoples'. The unique and universal characteristic of human beings is that they (we) all have and make cultures, and every culture includes elements we might choose to label as 'mathematics'. I want first to give an example of ethnomathematics from my own 'backyard'.

Ethnomathematics 'at home'

When my daughter Jane was 9-10 years old, she joined in with the 'marbles culture' at her school in Exeter. This involved collecting and buying a variety of marbles to enjoy owning, to show and exchange, and to use in playing marbles in the playground. The game and culture of marbles, as understood at that school (refracted through Jane's understandings) involved a complex system of exchange value and, equivalently, the number of 'goes' or turns for each marble in play. These were based on two major variables: marble type or variety, and marble size (other important factors were marble condition, 'prettiness', and colour). The names of the marble types used included: 'cat's eye', 'Dutch oily' (also called 'rainbow cleary'), 'squid', 'Frenchy', 'pixie', 'spotty dick', 'bare lady', 'jinx', 'misty', etc. The marble sizes were termed (in ascending order): 'mini' (or 'small'), 'ordinary', 'middley', 'bonker', 'granny', 'king-size', again showing the creative use of language in that culture.

At the school there were two playground locations for playing marbles, each operating with a different unit of value. At the 'Little end' an 'ordinary' marble was used as the unit of value. At the 'Big end' it was a 'cat's eye' marble. A selection of the 'values' of marbles at the two 'ends' are listed in the following table.

Marble Type	LITTLE end	Big end
Cat's eye	5	1
Dutch oily	6	3
Dutch oily bonker	8	5
Dutch oily	12	7
Frenchy	3	not valid
Ordinary	1	not valid
Ordinary bonker	2	not valid
Ordinary granny	6	3
Granny squid	16	8

A mathematician might note inconsistencies in these ratios (as they were reported to me). But it must be remembered that any exchange or match required the willing participation of children, and if they did not like the proposed exchange, due to an unfavourably perceived ratio, or due to special features of the marbles, or, crucially, due to their own desires, they did not accept the contract. So the issue .of inconsistency does not arise. Any exchanges or plays are context-bound, and an abstracted, mathematical perspective or critique is irrelevant

Alan Bishop has offered a parallel example from Papua New Guinea, in which the ethnomathematical custom of calculating land area for tribal purposes required the length and breadth in paces to be added. This seems to lead to mathematically inconsistent results, for 10×10 and 18×2 fields are assigned the same 'area'. When asked which of two such fields they would prefer, local trainee teachers declared the question unanswerable. You cannot judge which is better without seeing where it is, how it slopes, what grows on it, what the soil is like, etc. (The practices of 'academic mathematics' enculturate us into the view that we can compare or equate what are in fact unique particulars.) Furthermore, the students themselves perceived no inconsistency with the school-based method of length times breadth, which they happily applied in decontextualised 'school maths'.

I offer the story of Jane's marbles as an instance of a local ethnomathematical practice that I came across as a parent (and recorded as a mathematics educator). It was part of her schoolmates spontaneous playground culture (although doubtless it had its own history). The involvement of children in it offered an opportunity for a teacher to develop some classroom mathematical activities based on marbles, to build upon the children's informal knowledge and meanings. But to my knowledge, this was not done. (Would this, done 'respectfully' be a good thing to do, as I am inclined to think, in agreement with Steve Lerman, or would it be the teacher appropriating, colonizing and perhaps spoiling the children's own cultural pursuits?)

Ethnomathematics Versus 'Academic Mathematics'

Another issue already hinted at is that of the relationship between culturally-embedded ethnomathematics and the modern academic discipline 'mathematics'. The traditional, absolutist (and purist) view of mathematics is that it is specialist knowledge 'owned' by mathematicians, which is applied consciously (to real world or other problems) by applied mathematicians and scientists, and applied unwittingly in informal cultural contexts (i.e. ethnomathematics). The mathematicians, according to this view, own the pure 'essence' of mathematical knowledge, and 'dilute' versions are used by others.

In contrast, an ethnomathematical or cultural view of mathematics might argue that (ethno)mathematics is an intrinsic part of most people's cultural activities, and that academic mathematicians have appropriated, decontextualised, elaborated and concentrated that mathematics, reified it even, until it seems to have a life of its own. (Certainly it *has* a social existence, if not life, of its own.) Since a typical analytical philosophy and mathematical ploy is to factor out the origins of knowledge and only consider its final rationalized form, mathematical knowledge is seen to be a pure substance that reflects the structure of a superhuman and timeless realm (Plato's World of Forms, Cantor's paradise), thus denying its ethnomathematical origins. As D'Ambrosio points out, this is a major historical and philosophical falsification.

A critical mathematics educator perspective might argue that to see mathematics in cultural activities is only possible for those enculturated into the mathematical worldview; that it is a form of intellectual-cultural imperialism. (Erick Smith makes this point, albeit in a less politically-loaded form). Exactly half a millennium ago, Columbus set foot on Jamaica, where not a single Native American (Carib) person survives. He opened the way for the rape of the Americas, the appropriation of the land, the cultural artifacts, the agro-technology, and the Native American people's own bodies. Are not now mathematical anthropologists and educators exploiting what remains of that culture, and claiming ownership of, and building careers on the exploitation of the products of intellectual and manual labour under the banner of ethnomathematics? When were royalties ever paid on basket and rug designs, sand and body patterns, quipu, etc.?

A poststructuralist perspective might be that academic mathematics, school mathematics, and ethnomathematics represent collections of quite distinct discursive practices in which the languages, knowledges, artifacts and other resources, and the positionings of the participants vary according to the social contexts and purposes, power and economic relations, etc. The archeology of mathematical knowledge reveals historical relationships between some of these discursive practices, and the discursive formations that some of them give rise to. For example, Jens Høyrup (1980) argues that ethnomathematics first became organized into the discipline of mathematics in Sumer (around 5000 years ago, shortly after the crucial development of mathematical notation) when systematic instruction in mathematics began, and mathematical recording, calculation techniques, and problems were systematically embodied in a novel information technology (cuneiform clay tablets) as part of institutionalized schooling. Halfway between then and now, a rupture in mathematical thought (a revolution in the history of mathematics? see POMEnews 4&5) seems to have been brought about by the Hellenic civilization, with their particular emphasis on pure abstract knowledge and proof for its own sake. George Joseph (1991) has helped us reconceptualize some of the archeology of knowledge that brings mathematics closer to the present day.

Of course, apart from the purist, absolutist view, none of the other perspectives I have sketched (caricatured?) above need be inconsistent, and a blending of them fits with most of what the preceding contributors say. What this overall discussion does show, albeit briefly, is the central importance of ethnomathematics for the history, philosophy and pedagogy of mathematics.

Høyrup, J. (1980) in Fauvel, J. and Gray, J. (1987) *The History of Mathematics: A Reader*, Macmillan, Basingstoke, 43-45.

Joseph, G. G. (1991) *The Crest of the Peacock: Non-European Roots of Mathematics*, Tauris & Co., London and New York. (Paperback publication by Penguin books).

NEW JOURNAL ANNOUNCEMENT

Beginning in 1993, Series Three of **PHILOSOPHIA MATHEMATICA: PHILOSOPHY OF MATHEMATICS, ITS LEARNING AND ITS APPLICATION** (University of Toronto Press)

Prof. J. Fang founded ***Philosophia Mathematica*** in 1964, and published the last volume of Series Two in 1991-92. The journal has now been completely reorganized with organizational sponsorship, new editor, new editorial board, and new publisher. The editor, to whom all editorial correspondence, including submissions for publication should be sent, is Prof. Robert Thomas, Dept. of Applied Mathematics, University of Manitoba, Winnipeg, Manitoba, Canada R3T 2N2. Internet: Robert_Thomas@UManitoba.ca Telephone: (204)474-8126 Fax: (204)275-1498

The editorial board includes: Jon Barwise, Ubiratan D'Ambrosio, Michael Detlefsen, Paul Ernest, Nicholas Goodman, Donald Gillies, Reuben Hersh, Philip Kitcher, Saunders MacLane, Charles D. Parsons, Michael D. Resnick, Gian-Carlo Rota, Stuart G. Shankar, Mark Steiner, Thomas Tymoczko, and Jean Paul Van Bendegem.

The aim of the journal is scholarly interchange. It is the only journal specifically devoted to philosophy of mathematics. To encourage the scholarly discussion that is so badly needed at the present time PM will publish new work in philosophy of mathematics, including what can be learned from the study of mathematics, whether under instruction or by research, and including the application of mathematics, including computation.

What is exciting about this new journal is that it acknowledges the Lakatosian revolution in the philosophy of mathematics, and it aims to break down the barriers surrounding the traditional concerns of academic philosophy of mathematics, and will admit other types of theorizing about mathematics, including that coming from education, sociology, and other pertinent realms.

In addition to main articles, sometimes grouped on a single theme, there will be shorter discussion notes, letters, book reviews, etc. It is intended that each issue of the journal have contents of interest to both mathematicians and philosophers and also to those concerned with the teaching of mathematics at every level. Subscriptions are sought from individuals and institutions. The institutional price (no discount to subscription agencies) is $60 (US.). The individual price is $29 (US.), Subscriptions should be sent to Journals Division, University of Toronto Press, 5201 Dufferin Street, Downsview, Ontario, Canada M3H 5T8.

HOW MATHEMATICIANS WORK. NEWSLETTER No. 1. JULY 1992.

Organising Group: H. Hearnshaw, P. Maher, P. Allan Muir, Samuels, J. Steed and D. Wells.

Communications and requests for copies (currently free) to Allan Muir, Department of Mathematics, City University, London EC1V 0HB, U.K.

"The How Mathematicians Work project has been started by a group of individuals who wish to work together to research such problems as how mathematicians communicate and collaborate; how mathematicians vary in their behaviour, personality and experiences; how Mathematics is created; the role of aesthetics and intuition in Mathematics; the impact of computers and other tools on the way mathematicians work."

The following is a wonderfully vivid extract from an article in the newsletter by Philip Maher: *"How I came to see the Spectrum of a Normal Partial Isometry and its Associated Retraction"*

The Problem: For a long time (all of 1980, some of '81, '82, and '83) I had been working on what seemed a hard problem (which incidentally, has its origins in quantum chemistry): how close can a variable partial isometry be to a (fixed) positive operator? ... [Technical details omitted - Ed.]

The Solution*:* The day itself (early Summer 1983) we went to the seaside, to Worthing I think (because we had missed the train to Brighton: the first of many contretemps that were to happen that day). The most interesting one occurred on the way back home on the bus. My wife and I got involve in a fracas with a young woman who was taunting a disabled man; next, a gleaming flick-knife was held in front of my face; I had the presence of mind to (pretend to) keep calm and say "Cool it"; no dire injuries were inflicted.

I did not think about partial isometries on our trip and -- given the events I have just described -- they must have been maximally far from my mind as we returned to the house where I spent some time bemoaning urban violence.

Then, looking at the wall of the staircase, I saw -- almost as if it were looking at me -- an apparition bathed in a curious light: the picture of two concentric circles, radii ½ and 1, together with the origin at their centre; and superimposed on them, the dynamic of a mapping taking points inside the smaller circle to the origin and outside it to the outside circle.

Although the apparition I saw -- like that of an image in a dream -- did not seem to exist in time, I cannot have experienced seeing it for more than a few seconds. Nevertheless, it was instantly clear to me that the outer circle was the spectrum of a normal partial isometry, and the mapping I saw was a retraction of it. My problem was solved.

Extract from *How Mathematicians Work Newsletter* 1, 1992, 3-7.

REVIEWS (by the editor)

David Bloor, KNOWLEDGE AND SOCIAL IMAGERY, second edition, University of Chicago Press (Chicago and London), 1991 (paper 0-226-06097-7).

Back in 1976, the first edition of this book played a leading role in launching the 'strong programme' in the sociology of knowledge. As is well known, Bloor challenged the objectivity and absolute truth of mathematical knowledge, and argued instead for its cultural relativism. There is no doubt that its publication was a milestone in the sociology of knowledge and in the philosophy of mathematics.

The main addition to the second edition is a 23 page afterword. Here Bloor restates his earlier claim that alternative mathematics is not only conceivable but, at least in part, exists. Critics have dismissed the examples he cites as falling outside of mathematics proper. But, he argues, this is circular, because mathematics proper is defined to exclude examples that challenge the absolutist myth of mathematics. Another criticism is that although mathematical definitions may be negotiated, contrary to Bloor's claim, mathematical truths or proofs cannot. This argument is supported by reference to the formal deduction account of proof. Bloor argues that some forms of inferences (e.g. *modus ponens*) are possibly innate, and that anyway, when they are contradicted we reject the falsification. (The counterexample he gives is the 'sorites paradox' concerning the fate of a heap of sand following the iterated removal of a grain.) A further criticism that he rebuts claims that since there is universal agreement about mathematics, a relativist sociological account is refuted. He points out that even if it were granted that there is universal agreement, this is still only a consensus, and does not of itself establish the absolute nature of mathematical knowledge. It still needs to be shown that such an opinion is not only widespread but correct.

The book as a whole offers a way of seeing mathematics that explicitly makes room for ethnomathematics.

This is an important book, and the afterword complements and extends the case made in it. As the back cover comments rightly say, it is a very influential book, yet widely neglected in the philosophy of science; even more so in the philosophy of mathematics. Anyone who wishes to maintain an absolutist or objectivist account of mathematics still needs to address Bloor's arguments.

Mary Tiles, MATHEMATICS AND THE IMAGE OF REASON, London and New York: Routledge, 1991 (0-415-03318-7)

This book begins as a welcome critique of mathematics from a postmodern perspective. Modernism is characterized by the masterplan of reason, and mathematics and/or logic supplies the core of rationality and reason. With this perspective in mind, Mary Tiles sets the scene with Descartes' and Arnauld's views of reason, then recounts the logicist and formalist programs in detail, focusing on the work of Frege, Russell, and Hilbert. There is plenty of insight here, and the book is rich with detailed mathematical reflection on the work and achievements of these seminal thinkers, and the developments that followed.

In a tantalizingly short final section, Mary Tiles brings the critical tools of Bachelard and Derrida into play, and she makes the important point of the open-ended nature of mathematics in practice. She extends the postmodern insight that texts are not closed to all but a single reading to mathematics, whose proofs, theories and texts are provably polysemous. She shows how the claims for logic to offer closed accounts of reason never describes the practices of mathematicians, which all the time treat greater areas of uncertainty. However I looked in vain for the delivery of the final promise of the coverflap: "Mary Tiles demonstrates that mathematics and mathematically enformed reasoning are in fact far from irrelevant to the power structures of a technological culture." Unlike an unkind tyro who reviewed the book in the *Times Higher Educational Supplement* (17 January 1992: 27), I am left not wanting more of the latest foundational details, but a more extensive argument about the role of mathematics and reason in Western and world culture (and its role in power structures). I see Bachelard, but no Foucault!

My other regret is the book's lack of engagement with the modern anti-absolutist literature in the philosophy of mathematics. There is no mention of Lakatos, Davis, Hersh or Tymoczko. Kitcher is only mentioned in passing in the introduction. An Anellis reference is drawn from the important special issues of *Philosophica* (42-42, 1988-89) devoted to new approaches to the philosophy of mathematics, but there is no reference to the other contributions. Such developments *are* relevant; after all, the modernist, logical-foundationist, project for mathematics had one goal in mind: to establish its absolute certainty. Rejection of that goal by philosophers and mathematicians trying to reconceptualize the philosophy of mathematics is surely central to this book's project? Surely the meta-rationale for logic in the foundations of mathematics was that it would systematically deliver certainty, security and safety?

Let me end by stressing again what is both the central point and the greatest achievement of the book. It argues that the image of logic as reason, closed, mechanical, unimaginative, is miles away from the creative and imaginative nature of mathematical practice. Thus the foundational rhetoric of logic and metamathematics falsifies the nature of mathematics. Mary Tiles deconstructs the concept of reason and shows it has two warring parts, the one logical, systematic, careful, predictable, closed, left-hemispherical and anal-retentive. With its meaning subtracted this is the image of mathematics and reason offered so often in school. The other is wild and woolly, open, accepting, inspired but often unsystematic (the spirit of mathematics!). Both accept the disciplines of coherence and consistency, but remain poles apart. For this carefully argued insight alone, presented in a forceful and novel way, this book deserves to be more widely known and read.

Steve Fuller, SOCIAL EPISTEMOLOGY, 1988, Indiana University Press, Bloomington and Indianapolis, (paperback 1991, 0-253- 20693-6)

One of the important developments in recent decades has been the breaking down of some of the barriers that separated the different disciplines concerned with the nature of knowledge. Subsequently there have been a number of significant contributions to the sociology of science and knowledge. Many refer to developments in the philosophy of science, but few of them count as a contribution to philosophy, as this one does. In fact, it is an interdisciplinary contribution which is so wide ranging, that all I can do in a pocket review is to pick out a few themes that grabbed me.

The title 'Social Epistemology' takes us to the nub of the matter. Is knowledge warranted, by the presence of some extra- human set of logical reasons, tests, or states of affairs in the world? Or is knowledge, in the final analysis, what human communities of thinkers (and doers) accepts as plausible, justified and well-founded? A social epistemology takes the latter route. But this is where the philosophical problems begin. For the former route has the weight of philosophical tradition behind it; by demarcating different disciplines and areas of concern, excluding the human from logic and mathematics, it arrives at a very neat and tremendously powerful account of objective knowledge. A social epistemology has to tackle this monolith head on, and has to use the conceptual tools and language of that perspective, or else it is not even acknowledged to exist.

Steve Fuller begins by distinguishing *normative* philosophy and epistemology from that which is *naturalistic*. The former legislates about what should be the case, and partitioning up the field of enquiry, excludes such issues as arise from the context of discovery. This is typical of the traditional approaches described above. Fuller locates himself amidst naturalistic approaches, and wants to explore many of the correlates and contexts of knowing, without restricting his enquiry to Popper's World 3 of objective knowledge. Indeed, he challenges the pre-emptive imposition of such divisions of knowledge. He points out that a weakness of the normative approach to epistemology is that it fails to give an adequate account of the scientific method; it is incomplete, not only in its treatment of rhetoric, technology, administration, but also with regard to education. The assumption is that the Method is 'self-certifying', in that rational beings need only see it to recognize it for what it is and to follow it. (The reality is that power and coercion play a significant role in the reproduction of scientific culture.) Fuller argues that there is a chasm between this Method and the means by which its acceptance is brought about, with the latter requiring some transformation of the social environment. Thus the social is implicated in epistemology.

A key issue treated is that of the differences between disciplines, 'bounded' by their procedures for adjudicating knowledge claims; and the variety of justifications given for them. Corresponding to a discipline is a community of knowers. Thus another central issue is that of how groups come to accept knowledge; how they achieve consensus. An elaborated account is given, which problematizes: how the consensus is reached; who is entitled to join in? and what is the boundary of what it is legitimate for them to consider (and who decides it?). Fuller criticizes one perspective of epistemic communities given by an authoritarian theory of knowledge. Unwarranted knowledge is irrational belief, and only experts have a full range of warrants for specialist knowledge. Therefore it is less rational to think for oneself than to defer to the authority of experts. Indeed, skepticism thus becomes irrational. It is good to see this view analyzed (and criticized) explicitly.

Little of this book directly touches on mathematics or its philosophy. Yet there is much that is vital for any social or evolutionary view of mathematical knowledge. It is erudite, far-reaching, asks deep questions. An important book in my view

Donald Gillies (ed.) REVOLUTIONS IN MATHEMATICS. 1992. Clarendon Press, Oxford. ISBN 0-19-853940-1 (Hardback)

Following the editor's introduction, the contents are as follows:

M. Crowe, Ten 'laws' concerning patterns of change in the history of mathematics (1975)

H. Mehrtens, T. S. Kuhn's theories and mathematics: a discussion paper on the new historiography of mathematics (1976)

H. Mehrtens, Appendix (1992): revolutions reconsidered.

J. Dauben, Conceptual Revolutions and the history of mathematics: two studies in the growth of knowledge (1984)H. Mehrtens, Appendix (1992): revolutions reconsidered.

J. Dauben, Appendix (1992): revolutions revisited.

P. Mancosu, Descartes's Géométrie and revolutions in mathematics

E. Grosholz, Was Leibniz a mathematica revolutionary?

G. Giorello, The 'fine structure' of mathematical revolutions: metaphysics, legitimacy and rigour. The case of calculus from Newton to Berkeley and Maclaurin

Y. Zheng, Non-Euclidean geometry and revolutions in mathematics

L. Boi, The 'revolution' in the geometrical vision of space in the nineteenth century, and the hermeneutical epistemology of mathematics

C. Dunmore, Meta-level revolutions in mathematics

J. Gray, The nineteenth-century revolution in mathematical ontology

H. Breger, A restoration that failed: Paul Finsler's theory of sets

D. Gillies, The Fregean revolution in Logic

M. Crowe, Afterword (1992): A revolution in the historiography of mathematics?

In POMEnews 4&5 there was a short discussion on whether 'Kuhnian' revolutions take place in mathematics. A diversity of opinion was represented, including the claim that such revolutions take place on the meta-level, in the epistemological framework and foundations of mathematics. This volume represents the thorough and historically well-grounded discussion that this issue demands. Seminal papers by Crowe, Mehrtens and Dauben on the controversy are reprinted. Each of these authors also adds a 1992 dated piece reflecting on the controversy, and reporting their current views. This makes the volume essential reading for anyone interested in the history and philosophy of mathematics, and the issue of revolutions. Dauben also considers the case of non-standard analysis as a example of a revolution in mathematics, an issue discussed in POMEnews 4&5.

The issue under dispute is of course a matter of history, or rather of the interpretation of history, and so a number of case studies of particular candidates for revolution from history are particularly welcome. Another issue of note is the fact that not all references to revolutions in science stop at Kuhn's theory. Reference, albeit infrequent is made to other like F. Enriques, G. Bachelard, M. Foucault, L. Fleck, who made significant contributions to the history of ideas, and who developed theories of 'ruptures' or discontinuities in the history of science.

Many of the chapters are worth individual reviews, but I shall select just one, which reflects an issue mentioned above, due to lack of space. Caroline Dunmore argues in her chapter that mathematics is conservative on the object-level and revolutionary on the meta-level. At the former level mathematics is cumulative, and undergoes evolutionary changes in its concepts, terminology and notation, definitions, axioms, and theorems. However at the latter level, comprising the metamathematical values of the community that define the *telos* and methods of the subject, and encapsulate general beliefs about the nature of mathematics, the subject undergoes revolutionary changes. Dunmore illustrates her thesis with historical examples including incommensurable line segments, negative and imaginary numbers, non-commutative algebra, and other meta-level revolutions including Cantorian set theory.

Overall, this is a most valuable book, doubtless destined to become the standard reference on its topic. The UK price hardback price is high (£55), but it is very well produced and essential for every good library.

C. Ormell, (Ed.) NEW THINKING ON THE NATURE OF MATHEMATICS, 1992, available

from MAG, School of Education, University of East Anglia, Norwich, NR4 7TJ, U.K. for £5 (ISBN 0-907669-20-4)

This is a valuable collection of short pieces on the nature of mathematics with some attention to educational issues. A provocative and interesting book.

CONTENTS

M. Nickson and S. Lerman.(Eds.) THE SOCIAL CONTEXT OF MATHEMATICS EDUCATION: THEORY AND PRACTICE, 1992. Available from Southbank Press, 103 Borough Road, London, SE1 0AA, UK.; ISBN-1-874418-03-9; Price £13.95 (+postage).

The book contains papers from the Group for Research into Social Perspectives of Mathematics Education. The groups focus is the mathematics classroom as a socio-cultural setting and the papers reflect the growing interest in the area within the mathematics education community. Contributors include people drawn from the fields of mathematics, mathematics education, sociology and philosophy.

The four sections of the book are:

The National Curriculum in mathematics: a critical perspective

The social context of mathematics education

Theoretical frameworks and current issues

Political action through mathematics education

The contributions include the following topics: the sociology of the mathematics classroom; ethnomathematics; discourse analysis; gender issues; cultural origins of mathematics; problem-posing; multicultural mathematics; political implications of the National Curriculum; barefoot mathematicians. Overall, a very interesting and likely to be important collection.

A SEMINAR ON HISTORY AND MATHEMATICS EDUCATION (SHEM)

Has been taking place in the Institute of Mathematics, Statistics and Computer Science (Brazil) since 1988, discussing and studying the role of the History of Mathematics in pedagogical use. The participants are mathematics teachers from all three phases of education and the product of the seminars has appeared in the form of papers, conferences, postgraduate programmes, and most importantly, in the development of a new conception of mathematics education through the history of mathematics.

For more information, please contact: Eduardo Sebastiani, IMECC-UNICAMP, C. P. 6056, 13.081 - Campinas - SP - Brazil. Fax (0192) 39-5808.