Numbers 4/5 (DoubleEditor: Dr. Marilyn Nickson

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This Newsletter is the publication of the

PHILOSOPHY OF MATHEMATICS EDUCATION NETWORK

Organising Group

Aims of the Newsletter

The aim of this newsletter is to foster awareness of philosophical aspects of mathematics education and mathematics, understood broadly; to disseminate news of events and new thinking in these topics to any persons interested in education and mathematics; and to encourage international cooperation between scholars engaged in such research.

Subscriptions

POMEnews will be published twice annually, in the Spring and Autumn (Fall) of each year. (Occasionally these may be combined into a double issue.) There is an annual subscription charge, running for the calendar year. Please send subscriptions to the following according to which currency is most convenient (made out in the name of the person to whom it is sent).

UKœ 5 (5 pounds sterling) to Paul Ernest (address below)

US$ 10 (10 dollars USA) to Stephen I. Brown, Graduate School of Education, SUNY at Buffalo, Buffalo, NY 14260, USA.

AUS$ 10 (10 Australian dollars) to Kathryn Crawford, Faculty of

Education, University of Sydney, NSW 2006, Australia.

All subscriptions received in 1991 will be deemed subscriptions for 1992.

Colleagues in Africa, Central & South America, Eastern Europe, the Middle East, and the Far East who have any difficulties in obtaining these currencies are currently welcome to receive the newsletter free of charge.

Editor of next issue of POMEnews

The editor of POMEnews 6 will be Paul Ernest. Please send any correspondence, responses, polemics, opinion pieces (up to 1000 words), news items, comments, reviews, books/papers for review, and announcements to him. Controversial or dissenting pieces are especially welcome.

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PHILOSOPHY OF MATHEMATICS EDUCATION TOPIC GROUP AT ICME-7

This is Topic Group 16 at the 7th International Congress of Mathematical Education (ICME-7), Quebec, August 16-23, 1992 (ICME-7 Secretariat, Universit, Laval, Quebec, QC, Canada, G1K 7P4). The chief organizer of the topic group is Paul Ernest. It is planned to have 2 X 90 minute plenary sessions, with the following speakers and organisation.

SESSION 1: The Philosophy of Mathematics and its Educational Implications

Kathryn Crawford CHAIR for session 1

Paul Ernest INTRODUCTION (5 mins)

4 CONTRIBUTORS (10-15 mins each)

Reuben Hersh Recent Thinking in the Philosophy of Mathematics

Philip J. Davis The Raised Eyebrows of Mathematics

Sal Restivo The Sociology of Knowledge and Mathematics Education

David Pimm Language, Psychoanalysis and Mathematics

OPEN DISCUSSION (20-40 mins)

David Henderson REACTOR (5 mins)

SESSION 2: Philosophical Issues in Mathematics Education

Anna Sfard CHAIR for session 2

4 CONTRIBUTORS (10-15 mins each)

Ernst von Radical Constructivism and the Philosophy of Glasersfeld Mathematics

Stephen I. Brown The Nature of the 'Problem' in Mathematics

Education: Its Philosophical Basis

Christine Keitel Critical Mathematics Education: Its Philosophical Basis

Paul Ernest The Philosophy of Mathematics Education

OPEN DISCUSSION (25-45 mins)

Ole Skovsmose REACTOR (5 mins)

As they say in show business, this is a great line-up, and we look forward to two exciting sessions. Many more colleagues have kindly written in to offer their services and we regretfully have had to decline. There will be time for short statements by the 'audience' during the open discussion time. Paul Ernest is planning to edit a book containing the plenary presentations and chapters by other attenders and members of the POME Network, in cooperation with Sal Restivo, in his Science Technology and Society Series at SUNY Press. If you would like to contribute, however tentative your plans, please write in with an indication of your intended paper or chapter (e.g. tentative title plus four sentences).

PHILOSOPHY OF MATHEMATICS EDUCATION DISCUSSION GROUP AT BCME

This took place at the British Congress of Mathematics Education, Loughborough, U. K., July 13-16, 1991, organized by Paul Ernest, with the assistance of an organizing group comprising Ricky Lucock, Sue Sanders and David Wells. David did sterling work as the recorder of the three sessions, and the following is his impression of the discussion group.

INSERT DAVID WELLS BCME RESPONSE HERE

copy sent direct to Marilyn

NEUTRALITY OF MATHEMATICS SYMPOSIUM GROUP AT AMET CONFERENCE

This took place at the annual conference of the Association of Mathematics Education Teachers (the British association for teacher educators) at St. Martins College Lancaster, 9-12 September 1991 (For conference buffs: the cuisine was excellent!). It was organized by Paul Ernest. The following report was published in the proceedings.

INSERT 3: NEUTRALITY OF MATHS REPORT HERE

THE FUTURE OF BRITISH MATHEMATICS TEACHER EDUCATION

One of the central issues discussed at the AMET conference is that of the future of the British teacher education system, as there had been warnings that the government was about to intervene in a heavy handed way. Ken Ruthven, with the assistance of a few other members, undertook to draft an AMET policy on mathematics teacher education. This is a statement of criteria for teachers' entitlements in terms of a quality initial and in-service teacher education. Interested colleagues can contact Ken at University of Cambridge, Department of Education, 17 Trumpington Street, Cambridge, U.K.

In January 1992 Kenneth Clark (Education Secretary) announced plans to move the bulk (80%) of graduate initial teacher education from university departments, polytechnics and teachers' colleges to schools, with corresponding cuts in undergraduate courses to be worked out. His rationale is that teachers need practical experience not theory; that all they need apart from a degree in a specialist subject is a grounding in the 3 Rs, to be able to stand at a chalkboard and lecture, and to be able to control children.

There were a number of responses, although the media gave far more space to the proposals and claimed ills than to reasoned responses. An example is the following letter which was sent to 8 key politicians, all the national 'quality' and educational press, and 2 local (South West England) papers. It was printed in one local paper (Western Morning News).

10 January 1992

Dear Sir,

At Stockport Kenneth Clark spoke of his plan to shift teacher training from universities and colleges to schools. This, I regret to say, is based on folly and ignorance.

His folly is in listening to the uninformed opinions of Sheila Lawlor, Dennis O'Keefe and others of their ilk. They are determined to promote their extreme views, no matter what the cost to the nation. Sheila Lawlor admits that she has not visited a single training college or department. Her claims are based on prejudice alone.

Kenneth Clark's ignorance is shown first, in believing that all teacher trainers are left wing 'progressives'. This is arrant nonsense. Teacher trainers have been in the front line of preparing new teachers to teach the National Curriculum, not subverting it. They are in the main naturally conservative, just like university lecturers and doctors.

Secondly, it is ignorant to believe that the best of our hard-pressed schools can take on the training of 10-20,000 new teachers each year. Their business is teaching children and raising educational standards, and should remain so. Will parents stand for thousands of apprentices teaching their children as they 'learn on the job'? I think not.

Thirdly, Kenneth Clark is ignorant of the importance of the professional knowledge of teachers. Just as doctors, nurses, lawyers, architects and engineers could not begin to practice without it, so too it is essential for teachers.

Any over-theoretical 'ologies' and 'isms' of the 1960s were done away with by the end of the 1970s. The content of courses is strictly regulated by the government's Council for the Accreditation of Teacher Education. This recognises the essential role of professional knowledge, which must be introduced in the quiet of the lecture room, before being used in the classroom.

For example, recent research has classified the errors children make in subtraction and provides strategies for remedying them. Without this professional knowledge teachers cannot diagnose the many possible errors, which follow hidden patterns, and correct them. So the mistakes persist into secondary school, and standards fall. But research also shows that this professional knowledge is not learned in the classroom. Recent studies at Exeter found that 50% more of the total sample of teacher trainees could diagnose subtraction errors after the one year PGCE course than they could at the beginning, when they were fresh graduates. Clearly teaching mathematics effectively depends on professional knowledge of its classroom application, as well as knowing the subject!

If Kenneth Clark persists with his plans, he risks destroying a national resource, one which is widely admired abroad. He will probably also drive educational standards down. I ask him to think again. By all means reform teacher education. Any professional training system needs periodic review. But base any changes on informed opinion and evidence please, and not on folly and ignorance!

Yours faithfully

Dr. Paul Ernest

CONFERENCE & ORGANISATIONS NEWS

The 16th Annual Meeting of the International Group for the Psychology of Mathematics Education (PME-16).

This will be held at the University of New Hampshire, 6-11 August 1992. For further information contact William E. Geeslin (PME), Mathematics Department - Kingsbury Hall, University of New Hampshire, Durham, New Hampshire 03824. You may call Bill at 202-357-7538 (NSF, Washington, DC).

As Ken Clements once said: "PME is the place where serious mathematics educators get together every year!"

Mathematics Applicable Group (MAG)

This group has been meeting for some years concerned both to develop applications works in school mathematics at all levels, as well as to reflect on the nature of mathematics that underlies its applications. The proceedings of the conference New Thinking on the Nature of Mathematics are about to be published (papers by C. Ormell, P. J. Davis, P. Ernest, etc.). For more information contact Chris Ormell, MAG, School of Education, University of East Anglia, Norwich, NR4 7TJ, U.K.

Chris Ormell is one of the few mathematics educators who has maintained a consistent interest in the philosophical basis of school mathematics for several decades. He has published a number of significant analyses of the nature of mathematics and the relationship between pure and applied mathematics, a relationship which is so often neglected by philosophers. Many of his relevant publications are published by MAG and anyone wishing to know more should write to Chris and ask for the MAG list.

The British Society for Research into Learning Mathematics will next meet at Oxford University, Department of Educational Studies, 15 Norham Gardens, Oxford OX2 6PY on Saturday April 4th. The day meeting will be free of charge to members and includes a free lunch (courtesy of the Chairperson and host, Dr. Susan Pirie, I presume). Since annual subscriptions are only œ5, now would be a good time for non members to join! Who says there's no such thing as a free lunch? The membership secretary is Pat Booth, Westminster College, Oxford, OX2 9AT. Cheques to BSRLM. [BSRLM was originally the UK Chapter of PME, and it remains affiliated, but it changed its name so as not to be overly restrictive in content.]

The Second International Conference on the Political Dimensions of Mathematics Education (PDME-2) is planned for April 1993 in South Africa. For further information contact Jill Adler, Department of Education, University of the Witwatersrand, PO Wits 2050, South Africa, Tel. (011) 716-5257; Fax (011) 339-3956, E-mail jadler@worknet.alt.za. PDME-1 was an exciting invitation only conference held in London, Easter 1990. One of its strengths was that all participants shared a commitment to social justice and social change through mathematics education. PDME-2 will undoubtedly retain this character, but the emphasis is likely to shift to include, at least in part, the mathematics education concerns and needs of the oppressed peoples of colour in South Africa.

THE CRITICALMATHEMATICS EDUCATORS GROUP NEWSLETTER

Contact person: Marilyn Frankenstein, College of Public and Community Service, University of Massachusetts, Boston MA 02125.

POMEnews 3 included the first part of a definition of criticalmathematics educator from CmEG newsletter. Marilyn justifiably points out that detaching the first (more frivolous) part of the definition leaves out the more important sections that follow and tends to trivialize it. So here is remainder of the definition (from CmEG news 1, page 5):

criticalmathematics educator [These are mathematics educators in dialogue working to connect their concerns in mathematics education with their critique of society and their commitments to social, political, economic and cultural empowerment.]

INSERT: 1 from CmEG NEWS 1, PAGE 5 HERE

The Humanistic Mathematics Network shares many of the concerns of this network, and publishes a substantial newsletter, more akin to a journal. Alvin White, Harvey Mudd College, Claremont, California 91711, USA.

Second International Conference on History, Philosophy and Science Teaching will take place in Canada on 11-15 May 1992. Write for details to Prof. Skip Hills, Faculty of Education, Queens University, Kingston, Ontario, Canada K7L 3N6. Fax: 613-545-6584. This should prove to be an exciting conference, and reflects the thinking of the science education community which parallels that of the POME group, but is more advanced in its organisation. The conference will include papers directly relevant to mathematics education and related philosophical issues. Some of the prospective participants are Jere Confrey, Paul Ernest, Ernst von Glasersfeld, Sandra Harding, William Higginson, Michael Matthews, Joseph Novak and Lauren Resnick, out of a possible total of 400.

NEW JOURNAL: SCIENCE & EDUCATION - Contributions from History, Philosophy and Sociology of Science and Mathematics edited by Michael R. Matthews, Education Department, University of Auckland, Auckland, NEW ZEALAND.

This journal represents an exciting new development for readers of this newsletter, for the focus is on just those issues of our concern, but encompassing science as well as mathematics. Each issue will have articles targeted at mathematics orientated readers. Kluwer kindly offered reduced subscriptions to recipients of this newsletter, but they would have to total at least 100 in number, and be distributed by us. But all is not lost, because members of the International History, Philosophy and Science Teaching Group also get a reduced subscription (US$35 including membership), so readers are urged to write to Michael Matthews and join. Delegates at the Kingston conference (see above) also get the journal as part of their registration fee.

The journal has a distinguished editorial committee including the following mathematicians and mathematics educators; Ubiratan

D'Ambrosio, Paul Ernest, John Fauvel, Ernst von Glasersfeld, Victor J. Katz and Nel Noddings.

ARE THERE REVOLUTIONS IN MATHEMATICS?

Thomas Kuhn's Theory of the Structure of Scientific Revolutions, first published in 1962, heralded both a renaissance and a shift in the philosophy of science. The main tendency had been towards Logical Positivism and its successor Logical Empiricism, with an emphasis on the logical structure of scientific theories, shown in the work of Carnap, Frank, Hempel, Nagel and others. This was revitalized with the English publication of Popper's Logic of Scientific Discovery in 1959. It was not until after the impact of Kuhn that the philosophy of science became thoroughly cognizant of developments in the history of science (although there were precursors, such as Hanson). Kuhn offered a powerful new synthesis of pre-existing elements (some perhaps unknown to him) such as Wittgenstein's notion of a 'paradigm' and Bachelard's concept of 'epistemological rupture' in the history of ideas. He constructed profound theory in the philosophy of science, the influence of which, controversy notwithstanding, has reverberated through many other fields of enquiry since.

According to Kuhn, science does not grow by a simple accumulation of knowledge. Instead, it alternates between periods of 'normal' and 'revolutionary' science in its development. During a period of 'normal' science, new knowledge is accumulated by accretion, as a dominant theory and paradigm of inquiry are followed and used as a model. Anomalies and contradictions in the dominant paradigm lead to a period of revolution in which competing camps of scientists promote alternative theories (including the falsified old theory). A new theory comes to be accepted and gradually becomes the new paradigm of explanation and enquiry. In the shift to the new theory many of the concepts involved change meaning (e.g. mass and length in the transition from Newtonian Mechanics to Relativity Theory). Kuhn's controversial claim is that the old and new theories are 'incommensurable', and that their supporters may not be able to understand each other.

The Kuhn-Popper debate in the philosophy of science hinged on the issue of rational versus irrational criticism of scientific theories. Popper's position is prescriptive, and he posits falsification as a rational criterion for the rejection of a scientific theory. Kuhn, on the other hand, proposes a more descriptive philosophy of science, which while treating the growth of objective knowledge acknowledges that rational features are neither necessary nor sufficient to account for theory acceptance or rejection.

Although it is beside the point, there is a fascinating analogy between Kuhn's theory of normal and revolutionary development, and Piaget's theory of assimilation and accommodation in cognitive growth, respectively. This lends some support to the thesis that individual conceptual development mirrors that of humankind as a whole (the Phylogenetic Law). It represents the application of the evolutionary maxim 'ontogenesis recapitulates phylogenesis' to the intellectual plane. This is a strongly heuristic analogy which provides a rationale for the use of history in the teaching of mathematics and science (although ultimately the analogy breaks down).

The claim is made in this and earlier issues of the newsletter that the philosophy of mathematics is currently undergoing a Kuhnian revolution, with the rationalist Euclidean paradigm of mathematics as an absolute, incorrigible and logically and hierarchically organised body of knowledge increasingly under question. A number of mathematicians, philosophers and educators are taking mathematical practice and history as central to any account of mathematics, in place of the traditional narrow focus of the philosophy of mathematics on the foundations of pure mathematical knowledge and the existence of mathematical objects. This new 'maverick' tradition, as Kitcher terms it, regards mathematics as quasi-empirical and fallible, a view which is supported by an examination of the history of mathematics.

A key question concerns the applicability of Kuhn's theory of scientific revolutions to mathematics. Is this theory applicable to mathematics? Does mathematics have revolutions? H. B. Griffiths (1987: 71) questions the applicability of the notion of revolutions to mathematics, and argues "it is doubtful whether Kuhn's notion of a paradigm applies to mathematics in the same way that it does to other sciences". Griffiths makes this point in the context of an extended review of a book on mathematics education. He argues that incompatible theories and indeed paradigms can coexist in mathematics, unlike in science, where all theories purport to describe the same underlying objective reality. This is a point well made. Any over-facile parallel with Kuhn must fall foul on this issue. He argues that the 'overthrow' of the paradigm of Euclidean geometry by that of non-Euclidean geometry does not force mathematicians to reject it, as physicists reject Newtonian theory in favour of Relativity theory.

On this basis, it must be accepted that not all major changes or developments of new theories in mathematics deserve the epithet of 'revolutionary'. Nevertheless, I still want to argue that some radical changes or global restructuring of the background epistemological and scientific context of mathematics can be described as Kuhn-type revolutions. Such changes result in a profound re-orientation of mathematics, which can lead to as much 'incommensurability' as is found in science.

Some possible candidates for mathematical revolutions are the following. First of all, infinitesimal based proofs in analysis were universally accepted, despite Berkeley's (1734) pungent criticism, until they were banished by new standards of mathematical rigour in analytic proofs introduced by Cauchy, Weierstrass and Heine in the nineteenth century. This change reflects a shift in the nature and standards of proof from those based on geometric intuition, to those of arithmetical argument (Boyer, 1968). Another chapter in this story is the re-introduction of infinitesimal based arguments in the proofs of non-standard analysis (Robinson, 1966). This reflects a further change in the nature and standards of proof accepted in analysis, from those based on arithmetic to those of axiomatic first-order logic (Lakatos, 1978; Robinson, 1967). Many other such examples can be cited. These include in the late nineteenth century, the shift of geometric demonstrations from those relying on spatial intuition to a reliance on an axiomatic logical basis (Hilbert, 1899; Richards, 1989); the move to an axiomatic basis in arithmetic proofs (Peano, 1889); and the axiomatic rigorization of deductive logic itself (Frege, 1879).

To dwell a little longer on an example, a further example of a 'revolution in mathematics' is the shift of standards of proof in algebra in the nineteenth century. These changed dramatically from intuitive generalisations of arithmetic to a deductive axiomatic basis (Richards, 1987). The conceptual difficulties in making this transition should not be underestimated. The rigid attachment to the field-structure of number, crystallized in such laws as Peacock's 'principle of the permanence of equivalent forms' constituted what Bachelard terms an 'epistemological obstacle' to reconceptualizing the nature and epistemological basis of algebra. It took the mathematician Hamilton over ten years to overcome this obstacle in inventing his non-commutative ring of Quaternions. In doing so, he enabled a reconceptualization which heralded a revolution in the nature of algebra and the basis of proof in the subject.

This and the above examples illustrate a global restructuring of a branch of mathematics that might in my view legitimately be termed a 'revolution in mathematics'. What they illustrate is not the replacement of one mathematical theory by another. Instead they record a revolutionary shift in the background scientific and epistemological context, its constituent proof criteria and paradigms, and the associated meta-mathematical views. Changes in the background context can involve a changed pool of problems, concepts, methods, informal theories, the language and symbolism of mathematics, proof criteria and paradigms. It will also include a shift in the meta-mathematical views accepted by the mathematical community, including accepted standards for proof and definition, views of which types of inquiry are valuable, and views concerning the scope and structure of mathematics. Such changes can result in a profound re-orientation of mathematics.

The outcome of this radical restructuring is a new or revised scientific and epistemological context for mathematics. In particular, it represents a global restructuring of the epistemology underlying mathematics, and the way truth, proof and meaning are conceptualized by the mathematical community. In the examples cited, not only did the standards of proof change. In addition the criteria for evaluating mathematical theories changed, for these themselves are largely based on the proof and definition standards employed in the formulations of the theory. Such shifts do seem to correspond well to Kuhn's notion of scientific revolution, and would not appear to admit multiplicity as is the case with mathematical theories. In other words, like scientific theories, multiple epistemological frameworks cannot consistently coexist in mathematics, justifying the extension of Kuhn's theory to mathematics.

A number of other authors have also suggested that there are revolutions in mathematics, including Kitcher (1984), Gillies (forthcoming), and McCleary (1989). Overall, whilst agreeing with Griffiths that Kuhn's Theory of Scientific Revolutions cannot be directly applied to mathematics, my claim is that a transformation of it directed at the underlying epistemological context, instead of just at mathematical theories, does offer a valuable insight to the history and philosophy of mathematics.

A final aside is that the above argument offers grounds for a criticism of Lakatos (1976). For Lakatos' Logic of Mathematical Discovery only treats mathematical innovations at the micro-level, and does not accommodate macro-level changes such as the mathematical revolutions described above. (Elsewhere, in recognition of this deficiency, I propose a the Generalized Logic of Mathematical Discovery, see Social Constructivism as a Philosophy of Mathematics, forthcoming, SUNY Press.)

Paul Ernest

Ernest P (forthcoming) Social Constructivism as a Philosophy of Mathematics, SUNY Press, NY.

Gillies D Ed. (forthcoming) Revolutions in Mathematics.

Griffiths HB (1987) Looking for Complex Roots, Journal for Research in Mathematics Education, Vol. 18, 58-75.

McCleary J (1989) 'A Theory of Reception', Rowe DE & McCleary J The History of Modern Mathematics, 1, 3-14, Academic Press.

It was Brian Griffiths' review (cited above) brought to my attention by Geoffrey Howson (in a review of a publication of mine) that inspired the above piece, so I sent it to Brian and Geoff for their comments. Geoff was kind enough to note down two critical points in a letter:

1. That I confuse 'advance' with 'revolution',

2. That my example of non-standard analysis (NSA) merely reinforces Brian Griffiths' arguments. It has not changed the teaching of standard analysis one iota. They had a third-year option on the topic and they were interested in seeing if an approach through NSA might prove more comprehensible to students. They were not convinced that it was.

Brian was also kind enough to reply. He sent me the following letter, which although not intended for publication, contains just the kind of response I was hoping for. (It is included here with Brian's permission.)

INSERT 5: BRIAN GRIFFITHS' LETTER HERE

Brian Griffiths, Faculty of Mathematical Studies, University of Southampton, Southampton, SO9 5NH, U.K.

THE POPULAR IMAGE OF MATHEMATICS

The popular image of mathematics is that it is difficult, cold, abstract, ultra-rational, important and largely masculine. Many persons operating at high levels of competency in numeracy, graphicacy, computeracy in their professional life still say "I'm no good at mathematics, I never could do it." They perceive mathematics to be alien too themselves and their professional concerns.

For many people the image of mathematics is associated with anxiety and failure. When Brigid Sewell asked adults on the street if they would answer some mathematics questions, 50% fled. (She was gathering data on adult numeracy for the Cockcroft Committee of Inquiry.) Extreme mathephobics are undoubtedly a small minority in Western societies, and may not be significant in other countries. But their existence, and that of the popular image of mathematics raises a number of important questions. How widespread is the popular image described above? Does it correctly describe mathematics? What causes it? Can any change in educational practices alleviate it? Is there any hidden agenda behind the popular image?

It could well be that the popular image of mathematics is the single most important issue of concern for the POME network. Important in terms of social significance. For mathematics serves as a 'critical filter' (to use Lucy Sell's term) controlling access to many areas of advanced study and better-paid and more fulfilling professional occupations. If its image is an unnecessary obstacle which blocks popular access to mathematics, then it is a great social evil. Of course, changing the image alone may do little to solve the problem. That is the politician's and advertiser's view. It may be that the nature of the populace's encounters with mathematics also need to be changed, to be humanized.

These insights are increasingly widespread. Alvin White has founded the Humanistic Mathematics Network and has been actively promoting mathematics as a humanistic discipline through the network's conferences and newsletter. ICMI sponsored a conference on the popularization of mathematics in 1989 in Leeds, England. An outcome was the volume The Popularization of Mathematics edited by A.G. Howson and J.-P. Kahane in the ICMI Study Series, Cambridge University Press, 1990. This book offers valuable insights about the problems of mathematics described above, and a range of possible measures to address them. The upcoming conference ICME-7 has a Working Group 21 on the Public Image of Mathematics and Mathematicians, with Thomas J. Cooney as chief organizer.

Given this attention, what can POME uniquely contribute to the understanding and solution (or rather initial steps towards the solution) of this problem? It could be argued that even if POME cannot offer something unique, the problem is of such importance that all efforts directed at it are valuable. However POME does have something unique to contribute. On the one hand, the image of mathematics, the nature of mathematics, conceptions of mathematics all find their most systematic treatment in the philosophy of mathematics. On the other, their promulgation, dissemination and re-creation is largely affected through education. Hence the study of the intersection and interaction of these two fields, which is the concern of the Philosophy of Mathematics Education, has a central role to play.

To return to the questions listed above: How widespread is the image of mathematics as difficult, cold, abstract, ultra-rational, important and largely masculine? To answer it, first the distinction must be drawn between mathematics as a discipline (what professional mathematicians understand as mathematics) and school mathematics. As it happens, both of these can share the popular image described above, at least to outsiders. Such an image is associated with negative attitudes to mathematics. However, research on children's attitudes towards mathematics in the past two decades shows fairly widespread liking of the school subject, certainly in the years of elementary schooling. In the later years of schooling attitudes become more neutral, although extreme negative attitudes are relatively rare. Presumably this downturn in attitudes is due to such things as adolescence, peer-attitudes, the impact of competitive examinations, not to mention the image of mathematics conveyed in (and out) of school. According to this image, school and the discipline of mathematics are all of a piece, beginning in school, and then rising like a ladder to dizzy heights of abstraction. In contrast, numeracy, contextual mathematics, even ethnomathematics are perceived to be quite distinct form 'academic mathematics', presumably because of the differences in context and surrounding practices.

Does the popular image indicated above correctly describe mathematics? The answer to this is both Yes and No. First the Yes part. The experience many learners have, and certainly in the West virtually all citizens go through the educational process for many years, confirms this image. Teachers and others and the experience of learning itself all confirm this view. Secondly, the No answer. The image of mathematics is not as described in many enlightened schools and colleges, and certainly does not have to be that way. This pertains largely to school and college mathematics. What about the discipline of mathematics itself? This is where the philosophy of mathematics enters directly into the picture. Philip Kitcher has described a 'maverick' tradition in the philosophy of mathematics which emphasises the practice and human side of mathematics. This has been termed variously Quasi-empiricist, Fallibilist, and anti-foundationalist, and has been associated with Constructivist and Post-Modernist thought in education, philosophy and the social sciences. Such mathematicians and philosophers as Lakatos, Putnam, Hersh, Davis, Tymoczko, Kitcher have been at the forefront of these developments. The maverick tradition is represented by Tymoczko's anthology 'New Directions in the Philosophy of Mathematics' (Birkhauser, 1986) and more recently has found expression in Philosophica volumes 42 (1988) & 43 (1989) edited by Jean Paul Van Bendegem (to be expanded and reprinted in book form in Sal Restivo's series Science Technology and Society, SUNY Press).

The point of this story is that this maverick tradition rejects the image of mathematics described above as unnecessary, mistaken and downright false. To use Reuben Hersh's image: mathematics has a front and a back. In the front, the public are served perfect mathematical dishes, like in a classy restaurant. In the back, the mathematicians cook up new knowledge amid mess, chaos and all the inescapably associated human striving, successes, failures (and displays of ill temper!).

Can any change in educational or other practices alter the popular image of mathematics? Presumably change is always possible, or else we would all give up! The first step must be to raise consciousness about the nature of mathematics, and about the fact that there are alternative and competing conceptions of it. Promulgating such views within educational circles and beyond in society at large are vital. But the final question must be asked. Is there any hidden agenda behind the popular image of mathematics? If there are, then strong resistance to change can be expected. The status quo always has its own momentum, and is difficult to change. But there is a more radical view that the kind of popular image of mathematics described here serves conservative interests in the mathematics community and in society in general. For if mathematics is viewed as difficult, cold, abstract, ultra-rational, important and largely masculine, then it offers access most easily to those who feel a sense of ownership of mathematics, of the associated values of western culture and of the educational system in general. These will tend to be males, to be middle class, and to be white. Thus the argument runs that the popular image of mathematics described above sustains the privileges of the groups mentioned by favouring their entry, or rather by holding back their complement sets, into higher education and professional occupations, especially where the sciences and technology are involved.

This argument is quite radical, and may involve assumptions unpalatable to some. It may not be accepted that the popular image of mathematics has a hidden agenda or serves particular interests. Even so, it should be conceded that the type of popular image of mathematics described obstructs the full participation of all sectors of the population in higher education and professional occupations involving mathematics, especially in science and technology. It may also prevent citizens in modern society from developing critical numeracy and the mathematical confidence needed to understand the social uses of mathematics and to question statistics, whatever their source. Thus even from a traditional liberal perspective it can be argued that the common, popular image of mathematics impedes both industrial and technological development and the full expression of democracy in a mathematically empowered citizenry.

Paul Ernest

PHILOSOPHICAL AND PUBLIC CONTROVERSY - A RESPONSE TO PAUL ERNEST

Paul Ernest seems to be correct in drawing attention to an important political and social controversy concerning mathematics education, but not in describing the part that the philosophy of mathematics has to play in resolving this controversy.

The controversy is related indirectly to a fundamental ambiguity in the term 'mathematics', which is not shared to any confusing extent by such terms as 'physics' or 'science'. No confusion arises when we wish to talk about stars on the one hand or about astrophysics on the other (though we can 'talk astrophysics', which IS talking about stars). Similarly we can talk about anatomy as a study without danger of our words being misapplied to a passing elephant. The sciences are able to distinguish with reasonable ease between a study and the object of that study. Of course, it is possible for philosophers to use the subtleties of their trade to blur the edges of any distinction at all; but in this case I suspect that an averagely intelligent person would, at least initially, find the philosophers' views and motives difficult to comprehend.

Kuhn and Popper can discuss revolutions in science and (even if their views are mistaken) people can comprehend them without even considering the possibility that after a scientific revolution the world actually behaves differently. Quantum measurement theory may suggest that interaction with the world by way of measurement or observation is bound to change the world; but when we theorise in a new way no such interaction occurs.

With mathematics things are less simple. We suffer from a general failure to distinguish between the study of mathematics and that which is studied. This failure is partly attributable to the apparently abstract nature of the subject matter, which, unlike elephants and stars, is not obviously something other than mental. In part the failure is due to the excessive concentration on the subject matter by many philosophers in the past, countered but not balanced by an equally excessive concentration on the human processes of the study by many philosophers of the present century.

It may be extremely difficult to say much which is uncontroversial about truth in mathematics; but it is fairly easy to say a few things about falsehood. In mathematics it is possible to get things wrong either on a small scale by stating, e.g., that 4 x 5 = 30, or on a large scale by stating, e.g., that there can be no coherent geometry which does not include something equivalent to Euclid's fifth postulate. Even those who wish to emphasise the human activity of mathematicians must face the fact that something in mathematics is beyond our control. Intersubjectivity is not enough, though it may be possible to spread subtlety and confusion so thick that for a time it appears adequate. Any adequate philosophy of mathematics must include an account of its subject matter and its objectivity.

Revolutions in mathematics are revolutions in the study and not in the subject matter. The revolution may turn the study to new aspects of the subject matter. New problems seem important. New approaches are used for old problems. But a mountain is not changed simply because it becomes fashionable to climb it by a different face or to use new equipment.

Revolutions even in logic do not change the subject matter. Something which is absolutely the case is not so because of my reasons for believing it to be so. This elephant is walking past quite independently of any logical or observational reasons that I may have for thinking that it is walking past. An omniscient God might ever geometrise; but she would not prove theorems. For the angle in a Euclidean semi-circle would just be a right angle. Any logical links between this and any other geometrical statement occupying the deity's mind would doubtless come to her attention but would not be any sort of warrant for her knowledge of the theorem. A more controversial issue is whether such a deity would need generalisations at all. Would she find that in every individual case the angle found in a Euclidean semicircle just was a right angle, quite independently of the fact that all other such angles were also right angles?

I suggest that philosophy can have a role in public controversy which parallels the role of mathematics in the now commonplace model of mathematical modelling. We commonly take the application of mathematics to a real problem as involving (a) translating the problem into mathematics, (b) solving the mathematical problem, (c) translating the mathematics back into a real world solution. Similarly philosophy can be useful if we translate a problem which is causing trouble in more immediate educational debate into a philosophical problem. We can then be as subtle as we wish until the problem is solved, or (more probably) dissolved. Our (dis) solution can then be presented in popularly intelligible, rhetorical terms and possibly (dis)solve the problem facing the educational community.

In this model it is inappropriate for the general public to be faced with the internal wranglings of the philosophical community. Between ourselves we may be as subtle and/or polemical as we wish; but only when we think we have something to offer which resolves a point of public debate should we go public and attempt to intervene in the real world. No applied mathematician is tempted to present newspaper readers or politicians with abstruse mathematical formulae and computations instead of waiting until there is something intelligible to pass on. We should not be mislead by the apparently (and deceptively) less obscure and formal nature of some philosophical language and allow ourselves to be tempted to present the public with technical philosophical ramblings, however clear and incisive we consider them to be.

I suggest that before we do have anything to offer which will have any effect on those who have real power in education (government office holders or classroom teachers) we need to evolve some form of theory which does full justice to the manifestly (or, at least, apparently) timeless objectivity of the subject and to the manifestly messy historical development and social practice of its study.

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Philosophy, Rationality, and Individual Differences by D. Wells

Since the late 1960s I have been writing about an epistemology in which individuals construct their own markedly different worlds, including the idea that rather than everyone speaking 'the English language' more or less similarly, everyone speaks a different idiolect, and communication and use of language consist of the interaction of these subtly different idiolects. This theory, which I now label idiolectal interactionism, has appeared in Studies of Meaning, Language & Change, (SMLC) which I have published since 1977.

Such a theory inevitably emphasises individual differences, especially in use and perception of language and in what may be called epistemological style. (Hadamard is a relevant example here, in his well-known incredulity at Max Muller's view that no thought is possible without words. (The Psychology of Invention in the Mathematical Field.)

Such differences raise philosophical problems. Thus: are arguments over Platonism genuinely 'rational' arguments about what exists 'somewhere or other', or do they contain the personal artifacts of individual philosophers' ways of perceiving and structuring their worlds ? If philosophers genuinely 'see' mathematics differently, they can hardly be expected to construct the same philosophies of mathematics: but philosophers naturally do not acknowledge the possible influence of individual differences on their work - how could they ?

Philosophers have traditionally emphasised rationality, by implication potentially public and shareable, and never even considered that some of their differences might be due to deep differences in personality between them.

This type of argument links to another interest on mine, the study of the differences in modal psychology between different societies, or between groups within one society, researches which have resulted in the book, Russia and England and the Transformations of

European Culture, (Rain Press, 1990).

Popper is a good example here, simply because he has written widely on political as well as philosophical themes. I would argue that he represents a particular type of personality, common in England and the United States, which strongly emphasises order, rationality, objectivity and the role of society in the construction of the world - it can be interpreted as anti-Romantic - and that these features permeate his philosophy. Naturally, anyone of a very different personality type will hardly be inclined to accept his conclusions, and will have no difficulty in undermining his arguments. In particular, more individualistic - in a Romantic-schizoid sense - philosophers are hardly likely to be happy with the idea that what is or is not a 'proof' is decided by 'the mathematical community'. I would argue that such communities, like the notorious 'language community', disintegrate when you look at them more closely. The question of what is or is not a proof is popular at the moment: how can it be resolved without taking the variety of individual differences into account ?

Looking cross-culturally, the very ideas of proof and certainty and logical inference in a Greek sense can be interpreted as relating to the modal psychologies of the cultures in which they arise. The Japanese, pre-Meiji, had no Western conception of proof, but a highly developed mathematics. Littlewood doubted that Ramanujan, an Indian, had any real conception of proof.

Does this potentially affect mathematics education ? Yes, because we not only live in very mixed communities, but even individuals in our classrooms have very different personalities, and their perceptions and constructions of mathematics will tend to vary accordingly. So perhaps we should pay more attention to individual pupils' constructions of their experience than to

philosophies of mathematics that claim to a generality, even perhaps a universality, that is illusory. (More generally, I will claim that any sufficiently 'radically' constructivist epistemology is a threat to traditional twentieth-century modes of philosophising.)

Another cross-cultural angle: in sufficiently coherent societies, the idea of 'truth' naturally incorporates the twin ideas of sharedness among the community, and effectiveness in relation to the world of experience. In our less coherent society, 'truth' can be interpreted as dichotomised into two logically independent factors, sharedness and effectiveness. (SMLC 12, Epistemological privilege, 1981.) This interpretation is also highly relevant to arguments over truth and proof. Philosophers, once again, who make proof depend on 'the mathematical community' can be interpreted as overemphasising sharedness, (which often does not exist anyway) at the expense of effectiveness.

On another tack, I am interested in interpreting mathematics as game-like, in the first instance in order to solve the problem of the ontological status of mathematical entities, without falling back into Platonism, or mysticism. Most philosophers of mathematics refer to the game analogy, only to dismiss it. I think that is a mistake. (Epistemology of Abstract Games and Mathematics, SMLC 20 and 21, 1987-8, and Games as a metaphor for mathematics BCME, 1990.) The game-analogy has powerful implications for the classroom, in countering an over-emphasis on mathematics as science-like, and in justifying a creative approach to algebra and 'technique', so often dismissed as 'mere technique'.

There is tremendous pressure at the moment for teachers to accept Lakatosian ideas of mathematics as quasi-empirical, as fallibilistic. I would be the first to insist that mathematics is fallible, not least because abstract games are fallible, but, unfortunately, fallibilists tend to ignore the immense difference between the subjective fallibility of conclusions within the game of chess, say, and fallible conclusions in the physical sciences. Given a suitably simple position, the conclusion that, say, White is checkmated, may be as subjectively certain for any 'competent' player as 2 x 17 = 34 is for anyone who can do elementary arithmetic - and both conclusions are incomparably more certain that any of the physical theories that pupils learn about in school.

The philosophies of Popper and Lakatos do not satisfy me for another reason: they ignore the role of affect, including experiences of beauty, in the development of mathematics and the sciences. There is ample evidence in the history of science that scientists have relied on aesthetic criteria, in the context of justification, as well as in mathematics. Yet few philosophers pay any attention to aesthetics in this context.

Rationality is all ! But rationality, far from being all, is frequently influenced by aesthetic considerations, and no one has ever succeeded in putting aesthetic judgements on a rational basis, and the simplest experiments (Are these the most beautiful ?, Mathematical Intelligencer, 12-3, 1990,) suggest that, once again, idiosyncratic differences play a very large role.

Mathematicians, pure at least, rely heavily on aesthetic criteria in the process of doing mathematics. Once again, claims for the high role of rationality in human thinking is undermined. But how can these aspects of mathematics be brought into the classroom, for most of our pupils ?

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(An annotated bibliography is available on request.)

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But all that mathematics is still tricked out in, its absolute character and perfect accuracy, its generality and autonomy, in a word, its truth and eternity, all this (if I may be forgiven the expression) all this is pure superstition!Mannoury