

A Topos Perspective on the Kochen-Specker Theorem:

I. Quantum States as Generalised Valuations

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Abstract

Any attempt to construct a realist interpretation of quantum theory founders on the Kochen-Specker theorem, which asserts the impossibility of assigning values to quantum quantities in a way that preserves functional relations between them. We construct a new type of valuation which is defined on all operators, and which respects an appropriate version of the functional composition principle. The truth-values assigned to propositions are (i) contextual; and (ii) multi-valued, where the space of contexts and the multi-valued logic for each context come naturally from the topos theory of presheaves.

The first step in our theory is to demonstrate that the Kochen-Specker theorem is equivalent to the statement that a certain presheaf defined on the category of self-adjoint operators has no global elements. We then show how the use of ideas drawn from the theory of presheaves leads to the definition of a generalised valuation in quantum theory whose values are sieves of operators. In particular, we show how each quantum state leads to such a generalised valuation.

A key ingredient throughout is the idea that, in a situation where no normal truth-value can be given to a proposition asserting that the value of a physical quantity A lies in a set D of real numbers, it is nevertheless possible to ascribe a partial truth-value which is determined by the set of all coarse-grained propositions that assert that some function $f(A)$ lies in $f(D)$, and that are true in a normal sense. The set of all such coarse-grainings forms a sieve on the category of self-adjoint operators, and is hence fundamentally related to the theory of presheave

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