How Set Theory Impinges on Logic

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Abstract

Standard (classical) logic is not independent of set theory. Which formulas are valid in logic depends on which sets we assume to exist in our set-theoretical universe. Second-order logic is just set theory in disguise. The typically logical notions of validity and consequence are not well defined in second-order logic, at least as long as there are open issues in set theory. Such contentious issues in set theory as the axiom of choice, the continuum hypothesis or the existence of inaccessible cardinals, can be equivalently transformed into question about the logical validity of pure sentences of second-order logic, where " pure" means that they only contain logical symbols and bound variables. Even standard first-order logic depends on the acceptance on infinite sets in our set-theoretical universe. Should we choose to admit only finite sets, the number of logically valid pure first-order formulas would increase dramatically and first-order logic would not be recursively enumerable any longer.

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