Maximal Beable Subalgebras of Quantum-Mechanical Observables

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Abstract

The centerpiece of Jeffrey Bub's book Interpreting the Quantum World is a theorem (Bub and Clifton 1996) which correlates each member of a large class of no-collapse interpretations with some 'privileged observable'. In particular, the Bub-Clifton theorem determines the unique maximal sublattice L(R,e) of propositions such that (a) elements of L(R,e) can be simultaneously determinate in state e, (b) L(R,e) contains the spectral projections of the privileged observable R, and (c) L(R,e) is picked out by R and e alone. In this paper, we explore the issue of maximal determinate sets of observables using the tools provided by the algebraic approach to quantum theory; and we call the resulting algebras of determinate observables, "maximal *beable* subalgebras". The capstone of our exploration is a generalized version of Bub and Clifton's theorem that applies to arbitrary (i.e., both mixed and pure) quantum states, to Hilbert spaces of arbitrary (i.e., both finite and infinite) dimension, and to arbitrary observables (including those with a continuous spectrum). Moreover, in the special case covered by the original Bub-Clifton theorem, our theorem reproduces their result under strictly weaker assumptions.

This added level of generality permits us to treat several topics that were beyond the reach of the original Bub-Clifton result. In particular: (a) We show explicitly that a (non-dynamical) version of the Bohm theory can be obtained by granting privileged status to the position observable. (b) We show that Clifton's (1995) characterization of the Kochen-Dieks modal interpretation is a corollary of our theorem in the special case when the density operator is taken as the privileged observable. (c) We show that the 'uniqueness' demonstrated by Bub and Clifton is only guaranteed in certain special cases -- viz., when the quantum state is pure, or if the privileged observable is compatible with the density operator.

We also use our results to articulate a solid mathematical foundation for certain tenets of the orthodox Copenhagen interpretation of quantum theory. For example, the uncertainty principle asserts that there are strict limits on the precision with which we can know, simultaneously, the position and momentum of a quantum-mechanical particle. However, the Copenhagen interpretation of this fact is not simply that a precision momentum measurement necessarily and uncontrollably disturbs the value of position, and vice-versa; but that position and momentum can never in reality be thought of as simultaneously determinate. We provide warrant for this stronger 'indeterminacy principle' by showing that there is no quantum state that assigns a sharp value to both position and momentum; and, a fortiori, that it is mathematically impossible to construct a beable algebra that contains both the position observable and the momentum observable.

We also prove a generalized version of the Bub-Clifton theorem that applies to "singular" states (i.e., states that arise from non-countably-additive probability measures, such as Dirac delta functions). This result allows us to provide a mathematically rigorous reconstruction of Bohr's response to the original EPR argument -- which makes use of a singular state. In particular, we show that if the position of the first particle is privileged (e.g., as Bohr would do in a position measuring context), the position of the second particle acquires a definite value by virtue of lying in the corresponding maximal beable subalgebra. But then (by the indeterminacy principle) the momentum of the second particle is not a beable; and EPR's argument for the simultaneous reality of both position and momentum is undercut.

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