Twin Paradox and the Logical Foundation of Relativity Theory

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Abstract

We study the foundation of space-time theory in the framework of first-order logic (FOL). Since the foundation of mathematics has been successfully carried through (via set theory) in FOL, it is not entirely impossible to do the same for space-time theory (or relativity). First we recall a simple and streamlined FOL-axiomatization SpecRel of special relativity from the literature. SpecRel is complete with respect to questions about inertial motion. Then we ask ourselves whether we can prove the usual relativistic properties of accelerated motion (e.g., clocks in acceleration) in SpecRel. As it turns out, this is practically equivalent to asking whether SpecRel is strong enough to "handle" (or treat) accelerated observers. We show that there is a mathematical principle called induction (IND) coming from real analysis which needs to be added to SpecRel in order to handle situations involving relativistic acceleration. We present an extended version AccRel of SpecRel which is strong enough to handle accelerated motion, in particular, accelerated observers. Among others, we show that the Twin Paradox becomes provable in AccRel, but it is not provable without IND.

Keywords:	twin paradox, relativity theory, accelerated observers, first-order logic, axiomatization, foundation of relativity theory
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