On Symmetry and Conserved Quantities in Classical Mechanics

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Abstract

This paper expounds the relations between continuous symmetries and conserved quantities, i.e. Noether's ``first theorem'', in both the Lagrangian and Hamiltonian frameworks for classical mechanics. This illustrates one of mechanics' grand themes: exploiting a symmetry so as to reduce the number of variables needed to treat a problem.

I emphasise that, for both frameworks, the theorem is underpinned by the idea of cyclic coordinates; and that the Hamiltonian theorem is more powerful. The Lagrangian theorem's main ``ingredient'', apart from cyclic coordinates, is the rectification of vector fields afforded by the local existence and uniqueness of solutions to ordinary differential equations. For the Hamiltonian theorem, the main extra ingredients are the asymmetry of the Poisson bracket, and the fact that a vector field generates canonical transformations iff it is Hamiltonian.

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