On the Inherent Incompleteness of Scientific Theories

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May, 2005[*]

We examine the question of whether scientific theories can ever be complete. For two closely related reasons, we will argue that they cannot. The first reason is the inability to determine what are "valid empirical observations", a result that is based on a self-reference Gödel/Tarski-like proof. The second reason is the existence of "meta-empirical" evidence of the inherent incompleteness of observations. These reasons, along with theoretical incompleteness, are intimately connected to the notion of belief and to theses within the philosophy of science: the Quine-Duhem (and underdetermination) thesis and the observational/theoretical distinction failure. Some puzzling aspects of the philosophical theses will become clearer in light of these connections. Other results that follow are: no absolute measure of the informational content of empirical data, no absolute measure of the entropy of physical systems, and no complete computer simulation of the natural world are possible. The connections with the mathematical theorems of Gödel and Tarski reveal the existence of other connections between scientific and mathematical incompleteness: computational irreducibility, complexity, infinity, arbitrariness and self-reference. Finally, suggestions will be offered of where a more rigorous (or formal) "proof" of scientific incompleteness can be found.

Omnia olim mortua sunt itemur vivent.

Main Introduction

There is much discussion in scientific and philosophical circles on the scope and limits of science. One aspect of this discussion is whether scientific theories can ever be final or complete. For example, Weinberg (1992) and Hawking (1988, 155-169) have argued that almost all the major experimental discoveries of physics have been made, and that a theory of everything (TOE) is not far off. Barrow (1991, 1998) and Lindley (1993) have challenged this position by arguing that experimental, computational, cognitive and fiscal impediments

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[‡] I thank Rocco Lo Bosco for his generous support throughout this project. Without his extensive feedback, editing and our discussions, this paper would not have come to fruition. I also thank Charles Lyons and Richard Mueller for their many helpful comments, and Jason M. Tyler for the Latin translation of the opening phrase, which reads: "All things once dead shall live again."

[*] This paper is a minor revision of the original, published online December, 2004.

may exist. These debates have extended beyond the domain of physics and to other disciplines in science (Horgan 1997). It has also been variously argued that parts of the world are inherently dappled (Cartwright 2001), stochastic or emergent and therefore rule out a complete scientific description based on fundamental, deterministic or reductive laws (Suppes 1978). While it is still debated whether parts of the world are actually so, it remains possible that adequately complete descriptions may still be afforded by advances within the emerging field of complexity (Waldrop 1992; Wolfram 2002). Elsewhere, others have suggested that since Gödel (1931) proved mathematics is incomplete, any physics based in mathematics is likewise incomplete (Jaki 1966, 129; Hawking 2002); but, some have retorted that certain parts of mathematics are complete and that physics may only depend on these parts (Barrow 1998, 224-227; Traub 1997a). In this regard, Wolfram (1985), Moore (1990), and Sommerer and Ott (1996) have shown that certain mathematical models of physical systems exhibit incompleteness (undecidability), but Traub (1997a, 1997b) has rejoined that it is unclear whether other mathematical or even novel physical models can circumvent these difficulties. Finally, epistemological and ontological issues within the philosophy of science, such as the observational/theoretical distinction, the Quine-Duhem (and underdetermination) thesis, the reality/anti-reality debate, and the nature of scientific explanation may impact this debate (see Klee 1997 for a summary).

The above arguments can be broken down into the following categories: (a) whether the world is inherently dappled, stochastic or emergent; (b) mathematical, computational and fiscal impediments; (c) cognitive and philosophical difficulties; (d) experimental difficulties. We now ask, regardless of the issues in (a) and (b), can the debate on completeness be settled? It is the intention here to sharpen the question, and not to negate the importance of (a) and (b). We put forth a thesis here that ties the remaining issues, (c) and (d), together and answers the completeness question in the negative.

We begin by defining what we mean by complete scientific theories. Some may argue that theories may have to meet certain requirements of explanation, such as determinism, for the theory to be complete. On this basis, Einstein, for example, famously argued that quantum theory is incomplete. Since then, this requirement has been relaxed, and many physicists are willing to accept quantum theory as a foundation of a fundamental theory of physics. For our definition, we therefore will not require a complete theory to satisfy any such “external” criteria, whether deterministic, continuous or, even, causal. We rather take our cue from the formal theories of mathematics. *A scientific theory on a given domain of empirical phenomena will be said to be complete if all questions constructible within the language of the theory are answerable within the theory, or, equivalently, if for all statements constructible within the language of the theory, it is decidable whether the statement is a true or false statement of the theory.*[1] On this definition then, the question of determinism, etc., are all matters to be settled by the language of the theory itself.

It is immediately obvious that this definition cannot be accurately applied today due to a lack of formality in the way scientific theories are expressed[2]. Therefore, we can only use it as an intuitive guide. Second, in many cases it is unclear what is the proper language of a theory, and hence what are considered valid questions or statements. For example, in the above case of quantum theory, if we believe that particle positions are part of the language of the theory, or even of an ambient theory (world view) in which we have assumingly couched quantum theory, then the question of the whereabouts of a particle is a valid question, and the theory may be rightly considered incomplete. If, on the other hand, we exclude particle positions and instead add the notion of positional probabilities (or state vectors) to the language, then the question is not valid. Thus the question of completeness depends on what we consider to be the proper language of a theory, and, if this issue cannot be settled (by scientific methodologies), then the theory is considered incomplete, a fortiori.

Even without the completeness definition and its accompanying difficulties, it is obvious today that every field of scientific inquiry stands incomplete. While some scientists believe that their fields may be approaching an end, none will say that the goal is finished. For instance, quantum theory, regardless of the issue of determinism, stands incomplete since we do not have a quantum theory of gravity and have still many unanswered questions surrounding the behavior of subatomic particles. Practicing scientists do not need to refer to the completeness definition to see if their theories are complete; they can simply refer to the consensus among their respective peer groups. As long as science maintains a rigorous

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[1] Though we have not given a precise definition of what is meant by a *domain of empirical phenomena*, the thesis to be presented here is not dependent on it, but see section 1.3 and also Ruphy (2003, 60-62).

[2] Whether such a formality is ever possible is presently unclear. This possibility is discussed further in section 3.6.1.

standard, we can FAPP say that the definition and the consensus opinion coincide. If a time comes when the consensus deems that a respective theory is complete, then we can look to the definition for verification; but, if a respective consensus remains negative, it is safe to say that the theory is incomplete. So while we assert in our thesis that scientific theories will remain incomplete in the sense of the completeness definition, practically we can assert that any scientific peer group will judge their respective theories to be incomplete, save for any brief periods of misjudgment.

Before giving our thesis exactly, let us elaborate on the difficulties of the cognitive, philosophical and experimental issues in regards to completeness. To begin with, the experimental dilemma concerns whether novel empirical observations on a given domain of phenomena will cease. If they never cease, then it is uncertain whether the respective theory can ever be complete. The newly gathered empirical data may continue corroborating the theory, but there is no guarantee of this—unless the theory itself can make such a guarantee. If novel observations do cease, the answer depends on how they cease. Lindley (1993) and Barrow (1998) have pointed out that novel observations may cease due to technological and fiscal impediments, and hence could prevent the testing and determination of correct theories. This could, for example, be the case with high energy particle experiments and superstring theory. On the other hand, if upon continued experimental probing they cease (henceforth, "naturally cease"), then assuming we can get past any other difficulties, complete theories are possible. For example, Hawking (1988, 167) has argued that once experiments reach the Planck energy level, then no more novel discoveries are possible and we should be in a position to determine the correct fundamental theory of physics. The viewpoint that novel observations will naturally cease is essentially that natural experience is ultimately finite and therefore can be made complete, a position closely related to Russell and Wittgenstein's idea of logical atomism. In what follows, we will establish Hawking's argument more thoroughly and show how the question of theoretical completeness turns on the question of experiential completeness.

Even if empirical data is finite, philosophers of science may argue that data underdetermines theory and therefore no final theory can be singled out. Though not a direct attack against completeness, underdetermination, via its antireality implication, does lessen the epistemological force that we may normally associate with complete theories. A similar argument may be put forth on behalf of the Quine-Duhem thesis. Additionally, the inability to clearly demarcate the lines between observational and theoretical terms, and sensorial and cognitive processes within philosophy and cognitive science, respectively, exacerbates the epistemological worries. (Henceforth, both dualisms will be considered to be the same[3].) In what follows, we will argue that the situation is more grave than this; underdetermination of theory (UDT), the Quine-Duhem thesis and the observational/theoretical (OT) distinction failure are all implicative of theoretical incompleteness. The validity of these philosophical positions, like the question on theoretical completeness, depends on the question of experiential completeness.

To answer the now larger question, we will introduce a notion that is only apparently new, but one that has been quietly stirring in the background: valid empirical observations. The main criteria that is used to judge scientific theories is empirical verification (say by Popper corroboration or falsification). Thus we need to make sure that our empirical observations are genuinely true, authentic or valid, and that we are not mistaken or fooled by seemingly similar or even false observations. One may initially wonder what is the relevance of this notion. For in many cases, it seems obvious that some observation has taken place (or not) and therefore the purported theory is to be accepted (or rejected); but, as the proponents of underdetermination know, there exist cases for which interpretational issues preclude a clear determination of observations. Even in the cases where a determination is clear, it would be reasonable to ask for some procedure, perhaps scientific, that could assure us whether said observations have taken place or not, in order to dispel any doubts left and safely ignore the lone sophist in the corner.

In what follows, we will argue that the acceptance of theoretical completeness and the OT distinction, and the denial of UDT and the Quine-Duhem thesis depend not only on experiential completeness, but also on the existence of valid empirical observations, two closely related notions. Finally, to argue our thesis that any scientific theory is inherently incomplete, we will show that there can exist no scientific procedure to determine valid empirical observations and, second, that there is "meta-empirical" evidence to support the

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[3] Indeed, both Jerry Fodor (1984, 1988) and Paul Churchland (1979, 1988) have used results from cognitive science to respectively argue for and against the observational/theoretical distinction; however, the still developing field of cognitive science may prove that an identity between the philosophical and cognitive dualisms is inaccurate. Nonetheless, a stance for or against either dualism will have an impact on the other.

thesis of experiential incompleteness.

The demonstration of the non-existence of a procedure to determine valid empirical observations is based on a self-reference Gödel-like (1931) proof. However, it is important to note that this demonstration is not based on the traditional argument stating that any math based physics (indeed science in general) is incomplete, owing to Gödel's proof that math is incomplete. It is a more fundamental argument we apply to all scientific theories regardless of whether their mathematical models exhibit undecidability or whether they even have mathematical models[4]. But in another sense, while we are not concerned with the traditional arguments of mathematical and computational difficulties, by examining Gödel's results in depth, we can gain a deeper understanding of scientific incompleteness. This endeavor will be taken up in part three of this paper.

The theorem of undefinability of valid observations (as it will be referred to henceforth) is actually more parallel to Tarski's theorem (1933) than Gödel's. Shortly after Gödel, Tarski used Gödel's methods of self-reference to show that there is no finite formal procedure to determine all true arithmetical statements (i.e., that all true arithmetical statements are not recursively enumerable). While this implies that arithmetic is not finitely axiomatizable (Machover 1996, 242), it comes short of actually producing an undecidable arithmetical statement, Gödel's accomplishment. Similarly, we do not produce an actual undecidable scientific statement, but only show that the notion of valid observations, the primary criteria to determine scientifically true theories, is undefinable. Moreover, we will argue that, like the implication from Tarski's theorem, our theorem implies that scientific theories are also not "finitely axiomatizable" and are therefore incomplete. The current inability to produce the scientific counterpart to Gödel's theorem and the lack of more formality in the demonstration of our theorem is due to the lack of a formal metatheory of scientific explanation. The possibility of such a metatheory and other routes for more rigorously establishing scientific incompleteness are discussed in section 3.6.

[4] There is a caveat concerning this statement, namely, that all scientific theories may ultimately turn out to be mathematical in character; moreover, they may all be mathematically rich enough (as the arithmetic of natural numbers) to manifest incompleteness. In this regard, the demonstration of undecidability in the mathematical models of certain physical systems, as mentioned in the first paragraph, may be relevant. However, the current difficulty in establishing undecidability in every mathematical manifestation of a theory and for every theory leaves us to consider the present line of argument. The prior possibility is further discussed in section 3.6, where we consider possibilities for more rigorously establishing scientific incompleteness.

We also note here that Breuer's theorem (1995), stating that no observer can distinguish all states of a system in which he is contained, which was proved using a Gödel-like self-reference argument, maybe related to our theorem, although this is not presently clear. What is clear, as Breuer points out, is that the language of scientific theories is capable of self-reference since it contains concepts and expressions referring to its own propositions: "If apparatus and object system, as well as their interaction, can be described by the theory, then the semantic concept of observation can be introduced into the language of the theory" (201). For our purposes, we will also add the sensorial-cognitive system of the human being to what can be described by the theory. Because it is this system that determines whether observations are valid or not, we can introduce the enlarged concept of *valid* observations into the language of the theory. In part three, we will show that the language of scientific theories meets other additional requirements for self-reference, such as a minimal level of complexity and infinity.

Closely related to the notion of valid empirical observations is that of complete observations. We will argue that they are mutually implicative. Therefore, evidence of observational incompleteness would, in addition to directly supporting the thesis of theoretical incompleteness, also remain consistent with the theorem of undefinability of valid observations. Furthermore, we will see that it fulfills the infinity requirement necessary for self-reference and incompleteness: natural experience is infinite. We will argue that there is meta-empirical evidence to support the thesis of observational incompleteness. The primary evidence is the precision gap that is always accompanied by any physical measurement. Experimental measurements can never be 100% precise; there is always room for improvement. It is in this room where novel experiences lie. We will also identify three other components of an observation that can contribute to novel experiences: perspective, interaction and range.

We point out here that novel experiences do not necessarily mean anomalous experiences. They simply mean previously unexperienced, for example, the discovery of a new particle or a more precise measurement of some physical property. These novel discoveries may still continue corroborating the current theories, or they may not (an anomaly). If they forever do, then novel experiences become a moot point. Presently no scientific theory, or meta-scientific theory, can guarantee that more precise measurements will continue corroborating a theory. Unless such a guarantee is given, it will remain uncertain whether improved measurement precisions will result in anomalies or not. Because of the theorem of undefinability of valid observations and other surrounding arguments given herein, we will assume that such a guarantee cannot be given and, hence, that novel experiences pose a relevant uncertainty for the completion of scientific theories. However, this assumption is not justified by the meta-empirical evidence alone. Nonetheless, future research may reveal empirical evidence that further justifies this assumption, and perhaps even show that novel experiences will necessarily result in anomalies.

Experiential incompleteness, or the novel experience problem, as it will be otherwise termed, while also supporting UDT and the Quine-Duhem thesis, clarifies some of their puzzling aspects. For example, how is it by UDT that there can exist empirically equivalent but theoretically distinct theories? We will show that it is only so because the empirical data itself is somewhat fuzzy or, otherwise, incomplete; if the data set on a domain is completely known, then only one isomorphic class of theories can exist on that domain.

Finally, we will show that scientific incompleteness, via UDT and the Quine-Duhem thesis, is intimately connected with the notion of belief, whatever form it takes, religious or otherwise. This is what we mean by an inherent incompleteness. Belief and understanding as cognitive capacities are related by the incompleteness of the latter. Faith and doubt are only possible because of the inherent incompleteness of our ideas about the world; or, vice versa, theories can’t ever be complete because of our fundamental cognitive capacity to doubt them and believe in some other theory. Already we can see that this is connected to the theorem of undefinability of valid observations: e.g., how do I know that this is truly a glass of water sitting before me? We can always ask what may lie behind our concepts and experiences. It will be shown that this is not such a trivial inquiry as, for example, Horgan (1996, 29-31) suggests in his discussion of ironic science and Francis Bacon’s *plus ultra* (“more beyond”). What is fact and what is belief can’t be so easily separated.

It is on this point that we will begin the paper. We start in part one by comparing the philosophical debate on God’s existence with the debate on whether a physical theory of everything is possible. By this comparison, we can gain an intuitive understanding of the connection between incompleteness and belief, and also of some of the other key ideas mentioned thus far. The theorem of undefinability of valid observations will be introduced at the end of this part. In part two, we will discuss in detail the relationship between scientific incompleteness and the above mentioned philosophical theses, and give a more thorough exposition of the novel experience problem. At the end of part two, we will present an informal quantification of our philosophy. Theorems will be introduced that quantify the relationship between the Quine-Duhem (and UDT) thesis and theoretical and experiential incompleteness. A number of other results will also be shown: no absolute measure of the informational content of empirical data, no absolute measure of the entropy of physical systems, and no complete computer simulation of the natural world are possible. Last, in part three, we will take up a thorough examination of mathematical incompleteness and establish the conditions under which incompleteness manifests, such as computational irreducibility, complexity and infinity, and show how these conditions exist in science. We will also show how mathematical self-reference and arbitrariness find scientific parallels in the OT distinction failure and the Quine-Duhem (and UDT) thesis, respectively. At the end of part three, we will offer some suggestions for where we might find an incompleteness theorem for science. For clarity, the structure of the paper charts an ascending road, from basic intuitive notions to robust philosophical ideas and finally to formal mathematical concepts.

Part One: TOEs and the Undecidability of God

1.1. Introduction. After centuries of debate, it is widely acknowledged in philosophical circles that the question of whether a God exists is ultimately undecidable. (By undecidable we mean that it is unprovable that God exists and also unprovable that he doesn’t exist.) Also, presently, there is some debate within the scientific and wider communities whether a theory of everything in physics (TOE) is possible. We will show that these two debates are not unrelated but share a common thread. A resolution to either one requires the mutual consideration of both. We begin therefore by examining the God existence debate within the empirical arena of science.

But first, let us briefly clarify our definition of God, as there are many. We are concerned with the definition (or part of the definition) of God as a supernatural, omnipotent being, a being who is able to defy the laws of the natural world, for example, the laws of physics. This definition can be associated with the Gods of western religions. We are not concerned here with definitions of God that, instead of assigning supernatural powers, may portray him, her or it as a passive observer, an all pervasive consciousness, a living but empty presence, etc., as can be found in many eastern religions. Although the latter definitions may have some peripheral bearing on this paper as a whole, it is not relevant to the central argument in this part.

1.2. The God Existence Debate. A decision is unreachable on two opposing counts: the inability to prove God does exist and the inability to prove God doesn't exist. Let us examine both counts briefly. First, why can't we prove God does exist? We consider three cases. Say we witness an unlikely, though not physically impossible event, such as a mist in the shape of a cross appearing on a window, water running from the eyes of a holy statue or the spontaneous organization of gas molecules into the corner of a room. The theist will argue that such occurrences are highly unlikely to the point of impossibility that they can only be caused by supernatural causes. But the atheist will counter and say that even highly unlikely events are not completely impossible and so does not necessitate belief in a God.

Next, consider the case where it appears that a physical law is broken (a miracle). Examples may include an object falling upwards in violation of the law of gravity or a particle annihilation event that violates the laws of particle physics. The theist will argue that only God, who is omnipotent, could break the laws of physics and cause such events to occur. But any atheist would counter that the laws of physics were never complete and therefore leaves room for surprises and new theories. She may cite as examples the startling predictions of the heliocentric, relativity or quantum theories. Once again, assent is not required.

Finally, consider the case where we witness the appearance of God, Herself. This may be a physical manifestation of any form, that also appears to be performing acts of physical impossibility. The theist will simply say, "What more proof do you need?" But alas, the atheist can counter with the possibility that some advanced alien civilization, whose knowledge of physical laws is far superior to ours, is orchestrating a complex technological drama for their curiosity or amusement. This counter argument goes through for the same reasons as the previous case: the knowledge of physical laws thus far is incomplete.

Of all three cases, the last two are the most interesting. This is because the breaking of physical laws would require assent, whereas the occurrence of an unlikely event would only suggest assent, albeit strongly.

Now let us examine the other side of the coin: why we can't prove that God doesn't exist? After considering the three cases before, we only need to consider one line of argument here. Unfortunately, for the atheist, the failure is due to the same reason she used in the previous two cases to argue against the existence of God: the knowledge of physical laws thus far is incomplete. For the theist will argue that since physical laws are incomplete, there are physical observations that it can't yet explain, and thus how do we know that God is not responsible for them? Of course the atheist will argue that prior observational anomalies have been resolved by new physical theories without the need for supernatural explanations. But the theist will then rejoin that the resolutions always left new anomalies unexplained.

Moreover, the theist will argue that since the whole of the physical theory is not complete, how do we in fact know that the theory is correct? How do we know that God is indeed not responsible for the whole of observational phenomena, not just the unexplainable ones? That is, how do we know that God is not the total and complete theory of everything? And, finally, therefore how do we know that God did not in fact cause the above mentioned unlikely or physically impossible events?

Let us now succinctly summarize both sides of the debate. Any empirical evidence for God is unconvincing because physical theories are thus far incomplete, and therefore loopholes allow for some yet unforeseen new theory to alternatively account for the said empirical evidence. On the flip side, the lack of a complete physical theory allows for God to slip in and account for the said empirical evidence. Precisely put: in lack of a complete fundamental physical theory, such as a TOE, empirical evidence cannot be said with certainty to fall within the purview of science or of God. The question of God's existence is empirically undecidable because of the incompleteness of physical theories.

1.2.1. Analysis of the Debate. At this point we are making no claim concerning the existence or non-existence of God. For instance, some may argue that empirical evidence is converging toward a complete theory, and that the God of the gaps is being squeezed out, once and for all. Others may maintain that there will always be new and unexplainable natural phenomena. Neither of these viewpoints are being disputed as of yet; we are only making a logical point, that as long as we don't have a complete fundamental theory that accounts for all natural phenomena, and there remains some gap, no matter how small, we must admit that the question of God's existence is undecidable.

With this in mind, we now ask two questions that does force us to take sides: is it possible to close the gap and achieve a complete physical theory and thus settle a debate which has been plaguing philosophy for centuries? Second, and more preliminary, does the question of God's existence, a religious question, and the question of achieving a TOE, a scientific one, really have any bearing on one another? That is, can't one maintain belief or disbelief in God irrelevant of physical theories? In what follows, we will see that these two questions are only apparently different.

Let us begin our analysis with the second and preliminary question. Can we still have faith in God in spite of a TOE? Does it necessarily exclude God from having any explanative role in our existence? For the theist may argue, so what if we have a physical theory that can explain all observable physical phenomena; how do we know that God is not still behind it all? How do we know that God is not responsible for seeding the big bang? How do we know that God is not ultimately the source behind the fundamental particles or entities, the quarks, superstrings or whatever we find them to be eventually? How do we know that God is not behind our senses and thoughts, behind our conscious awareness itself? By raising these questions, the theist is implying that there is always room for religion beyond science, that science is not the whole story, and that in the end science cannot cripple his belief in a deity or any other notion.

But, and in order to answer our question of whether science has any bearing on issues of religious belief, we need to ask ourselves are these questions raised by the theist truly beyond the reach of science and a TOE, or is that we have jumped prematurely in crowning a purported TOE, where in fact we have yet to produce a more thorough and fully complete theory that leaves no questions unanswered for the theist? That is, once we have a TOE, should there be any fundamental questions left at all? In order to answer this, we need to have a clearer understanding of what it is that we expect from a TOE. A clearer understanding of these expectations are also necessary before we can say whether we can achieve a TOE or not, and thus answer the first of our questions above.

1.3. Definition of a TOE. Before we start with our definition of a TOE, let us briefly address two preliminary issues. The first issue is the scientific scope of a TOE. Is it limited to fundamental physics, all of physics, or all of the sciences? Surprisingly (and as we will see in part two), for our arguments, it does not matter. Furthermore, it does not matter whether we are talking about a TOE or some other scientific theory which only addresses a limited (local) domain of phenomenological experience. The issue is whether the theory is complete on its given domain of phenomenological experience. It is easy to see that the above arguments concerning TOEs and God hold for these lesser grandiose theories. What complicates the issue is whether domains of phenomenological experience (physics, chemistry, biology, etc., and specialties within these fields) can be clearly distinguished. If they cannot, then theories, in order to be considered complete, may have to explain more than what was initially expected of them or require the support of auxiliary or background theories. For example, for a theory of chemistry to be considered complete, it may have to explain the behavior of subatomic particles and also complex organic molecules, such as DNA and proteins. Though reductionism—vertical and horizontal—has been invoked to answer this call, scientists and philosophers continue to debate its merits.

While we admit these questions to be open but impertinent to the core of our arguments, nonetheless, we take the viewpoint that all phenomenological experience is connected and that reductionism relative to a fundamental theory of physics holds. The former allows us to address the entire domain of phenomenological experience without worrying whether it can be precisely categorized; the later allows us to delineate our TOE requirements in a concise manner—since we limit ourselves to the domain of physics alone—and thus streamline our discussion. In what follows, one is free to just as well assume that our TOE concerns only the domain of fundamental physics and also that it is not a reduction of other scientific theories. The relevant question we are concerned with is can the theory be complete.

The second issue is whether mathematical and computational difficulties will prevent us from answering certain questions in spite of having achieved a TOE. An example of one such category of problems is what is know as intractable, a specific possible instance being that of how amino acid sequences fold into proteins (Traub 1996, 1997b). It is possible that some or all of these mathematical and computational difficulties can be overcome with new theoretical or technological breakthroughs (e.g., quantum computing). For our discussion we will assume hypothetically that all such technical difficulties can be overcome. We then ask are there still relevant questions remaining? For we are interested in whether a TOE, regardless of mathematical and computational difficulties, can answer all our questions (but see footnote four).

1.3.1. Unification. We now begin our definition of a TOE with some requirements that most physicists would agree on. It should unify all the four known fundamental forces or interactions: gravity, electromagnetism, strong and weak.[5] In this unification, there should be a reconciliation between the two main but disparate theories of modern day physics, Einstein's theory of gravity and quantum mechanics. This reconciliation should address large mass/energy phenomena, such as that which is theorized to occur in the singularities and vicinities of black holes, the earliest parts of the big bang and in particle physics extending to the Planck energy scale. The TOE should also answer all of the questions concerning the strong nuclear interaction which our current theory leaves unanswered. It should resolve the missing mass cosmological anomaly (Zwicky 1933; Babock 1939) and the recently observed acceleration of the expansion of the universe (Perlmutter 1988; Zehavi 1999). Additionally, any new interactions or processes discovered as a result of future research (e.g., during empirical verification of the TOE) should also be explained within the singular theory.

1.3.2. Uniqueness. Next, we continue with some TOE requirements that enjoy a less certain status among physicists. These comprise an explanation of the parameters and classifications of the Standard Model of particle physics, including the origin and masses of the elementary particles, along with their charge, spin, color and other quantum numbers; the relative field strengths of the four interactions; the preference for matter over antimatter; the various symmetries and symmetry violations; the values of the physical constants, such as the cosmological constant; the number of dimensions of our universe; finally, and if possible, how exactly the universe began and will end (what physicists term the boundary conditions—at present our theories can't account for the very beginnings of the big bang). Why the uncertainty over these criteria? For a long time it was assumed, or at least hoped,

[5] The electromagnetic and weak interactions were unified circa 1970 in the electroweak theory.

that a future TOE would provide some or all of these details as a by-product of unification, but the progression of theoretical research towards a TOE suggest that this may not necessarily be possible.

As an illustration of these reservations, let's consider the currently popular theoretical candidate for a TOE, string theory. This eleven-dimensional theory does a beautiful job of unifying the different fundamental interactions and the many elementary particles as the distinct features of a topological landscape of higher dimensions rolled up too tiny for us to see; however, there are tens and thousands of ways these higher dimensions can be rolled up or compactified, and only one of these will yield the specific details of the interactions and particles that we observe in our universe, i.e., the physical properties of our universe. The problem is that the theory does not pick out a unique compactification; there are many compactifications that will yield a consistent theory, albeit describing a universe with a different set of physical properties. Theorists are uncertain whether string theory, or some improved version of it (M theory) should necessarily predict the values unique to our specific universe.

Some believe that it should and are arduously pursuing this cause (Greene 2003, 218 & 360-361); some have even proposed plans for a solution. For instance, Hawking (2003) and others have argued that the physical properties are not uniquely determined just as the position of a subatomic particle is not; the different solutions are represented as quantum mechanical amplitudes with various probability outcomes. The idea is that many different types of universes, each with a different set of interactions and particles could have evolved at the earliest stages of the big bang, and it just so happens that ours was the one that did. Smolin (1997) has suggested that the physical properties have evolved via natural selection as universes continually give birth to new universes during black hole formations. Similarly, Linde (1994) has proposed that universes are continually sprouting new and different universes via the Guth inflationary process that is believed to have occurred during the big bang.

Others are not convinced that the uniqueness question can be answered and are simply hoping to find a match between one of the possible compactifications and the properties observed in our universe—a daunting mathematical task in itself (Greene 2003, 219-221). This group is flexible in admitting the anthropic principle (Susskind 2003) or religious explanations (Davies 1992, 161-193) as the determining reason. They are willing to accept the possibility that these details are simply beyond the reach of science, leaving an opening for the theist to enter his wedge of questions. Of course, the former group maintains that there is no opportunity for the theist here; the TOE is still incomplete. Who is right?

1.3.3. The Entities of String Theory. Let's for the moment assume that the uniqueness question can be answered satisfactorily. Then consider what we have thus far accomplished: a unification of the four fundamental forces, an elegant explanation of the many messy details of the Standard Model, an explanation of why we macroscopically observe three space and one time dimension, and, while we're at it, let's also hypothetically assume that somewhere in these explanations resides an account of the missing mass anomaly, the acceleration of the universe, black hole singularities, and how exactly the universe began and will end. Is that it? Have we succeeded in completing a TOE?

Perhaps some will feel that the theory should tell us what these eleven dimensional "strings" really are? What is the nature of their substance? Energy? What form of energy? Or is it holographic and informational in nature as some have suggested (see Greene 2003, 411 for references)? If so, how does information as a foundation give rise to material particles? Some might ask why specifically eleven dimensions (or 26, or however many we determine there to be)? Why should entities exist in any sort of dimensional space in the first place? What in fact are dimensions? Are the group of physicists who were earlier holding out for an explanation of the uniqueness of our universe now satisfied and willing to dismiss these new questions as truly beyond science?

1.3.4. Quantum Mechanics. Elsewhere, others might still question, even after almost a century of service, whether quantum mechanics is a complete theory and the appropriate foundation upon which to build a TOE. Even though it has proven to be a highly accurate theory, there are many mysteries surrounding it: (a) What actually happens during a superposition of quantum states? (b) Can the measurement (wave function collapse) problem be solved? (c) How are non-local correlations to be understood? Today it is unclear whether these mysteries are indispensable features of the theory, or whether they can be resolved by an alternative interpretation, or even by some new theory. Though Bohr (1949), Feynman

(1967, 129) and other physicists have felt that these mysteries are either irrelevant or have been already answered by quantum theory and its standard interpretation (the Copenhagen interpretation)[6], a recent poll has found that many physicists today feel that these mysteries are relevant and open (Durrani 1999).

In this spirit, there have been numerous attempts to fill the gaps. These include the novel reinterpretations of quantum theory such as the many worlds (Everett 1957; de Witt 1970), consciousness collapse (Wigner 1961), consistent histories (Griffiths 1984), quantum event (Hughes 1989), relational (Rovelli 1996), correlation/Ithaca (Mermin 1998) and information theoretic (e.g., Zeilinger 1999). Reinterpretations that seek to re-establish the classical deterministic view over the probabilistic one ushered in by quantum mechanics include the hidden variable theories as espoused individually by Bohm (1952), Kochen (1985), Healy (1989) and Dieks (1991). None of these reinterpretations have received full acceptance from the physics community; they are variously debated, considered incomplete (and perhaps inconsistent) and continue to be articulated and developed by their respective adherents. There also remains the possibility that these *reinterpretations* may actually be *new theories* that can be experimentally distinguished from the original quantum theory. The acerbity of this issue is obvious in attempting to interpret Hawking's quantum mechanical explanation for the uniqueness of our universe (section 1.3.2). For example, if we subscribe to the many worlds interpretation, then we hold that thousands of different universes were created with the big bang and we live in just one of them. Is this in fact the case, or perhaps some other interpretation or even some new theoretical explanation besides quantum mechanics? Or do we leave well enough alone and crown our TOE?

Others have concentrated their efforts on the measurement problem and have put forth theoretical addendums to quantum theory that provide novel experimental predictions. These include a modification of the hidden variable theory by Bohm and Bub (1966), the dynamic collapse theories (Ghiradi 1986; Pearle 1989; Gisin 1992), some still developing quantum gravity theories (Smolin 2003 for a summary) and the traditional decoherence theory. Though the hidden variable, dynamic collapse, and quantum gravity theories have yet to be experimentally corroborated, the decoherence theory has recently received some

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[6] Many may instead take von Neumann's operational interpretation ([1932] 1955) as the standard—perhaps more orthodox—or even conflate it with the Copenhagen.

corroboration from the remarkable SQUID experiments, which possibly demonstrate the existence of quantum superposition states for macroscopic size objects (Friedman 2000; van der Wal 2000). Despite this preliminary success, the theory's lack of a direct causal mechanism for wavefunction collapse leaves some to wonder whether it can provide a final resolution to the measurement problem (Adler 2003; Zeh 2003, 14-15).

Lastly, we mention some significant theoretical and experimental developments that cause great difficulties for a resolution to these mysteries in terms of our traditional, intuitive concept of properties (that belong to objects or physical systems). Gleason (1957) has implied and Kochen and Specker have shown (1967) that from purely mathematical arguments that the probability distributions that quantum mechanics predicts cannot simply be statistical distributions about properties that are themselves fundamentally non-statistical. This means that if the statistical predictions of quantum theory are as experimentally accurate as they seem to be, then electrons, for example, don't *inhere* with any definite momentum value. However, the fact that a value is produced upon *measurement* prompts many to insist that the electron has at least some *contextual* momentum value that depends on the measurement procedure. Additionally, Bell (1964) has shown that any physical theory that is non-contextual concerning properties (i.e., objective) and also local (i.e., does not allow faster than light connections between spatially separated particles) will produce statistics that are different than what quantum theory predicts for certain prescribed experiments. These experiments have been carried out by Aspect (1981, 1982) and others (Rowe 2001; Go 2004), and thus far support the statistics of quantum theory. Many take these results to imply that any interpretation, addendum or replacement of quantum theory will have to be non-local and treat physical properties as contextual at best, or disregard them altogether at worst; however, some still see possibilities for a traditional recovery (Barut 1995; Bub 1995; & other articles in the same volume).

The former viewpoint poses severe ontological difficulties for the future of quantum theory. First, some reconciliation is in order with the special theory of relativity, for although it has been shown that the non-locality of quantum mechanics does not allow *information* to be transmitted faster than light, there is a non-local influence of a subtle kind. Second, and more gravely, if particles and systems don't have objective properties, what do they have? Perhaps it is wise not to ask any more questions, as Bohr and Feynman felt, and simply get on with the application of the theory. Perhaps such questions are beyond the domain of science and we should be content with quantum theory as it is. Or perhaps they point towards a new physical theory in which long trusted concepts such as object, property, space and causation must be reconsidered.[7] Or even perhaps there may yet be a resolution couched in the traditional concepts.

1.3.5. Pre- Big Bang. Another area where we might question whether our TOE has fulfilled all its requirements is in the explanation of the universe's origin. Even if we hypothetically understand how the big bang started all the way back to time zero, can't we still ask what came before the big bang? It may be true that time in our universe started with the big bang. But is there a larger arena where the life and death of our universe takes place? Possibilities are the "multiverses" of Smolin and Linde, where individual universes created via black hole formation and inflationary expansion, respectively, are sprawled over vast stretches of space and time. So it may be a valid scientific question to ask what came before the big bang.

1.4. Resolution: Incompleteness. Although there may be further requirements for a TOE that we have omitted, we can already see from our discussion thus far that there is some uncertainty over the requirements considered here. It may perhaps help us to list some of these requirements in order of consensus among the physics community, with the first being unanimously agreed upon and the last being highly questionable: (1) unification of the four forces, and unification of quantum mechanics with general relativity, (2) an account of the uniqueness of the physical properties of our universe, (3) an explanation of the entities of string/M theory, (4) a better understanding of quantum mechanics and subatomic phenomena, (5) an account of what came before the big bang. This list is only based on what we know now and assumes that we can resolve all these present questions smoothly in the end; however, there is no guarantee that in our attempts to answer any of them, theoretically or experimentally, we won't encounter additional fundamental questions whose status as a

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[7] A hint of such a theoretical direction can be found in the quantum event interpretation (Hughes 1989), where the notion of property is obviated, and in the relational interpretation (Rovelli 1996), where the notion is contextualized relative to an observer or system. See also Mermin (1998). These ontological difficulties have also compelled many to re-examine the nature of scientific explanation, itself (Hughes 1989, 296-319; van Fraassen 1989, 1991; Bitbol 2001).

TOE requirement may also be debatable as the above requirements.

We now note that the above list loosely reflects some of the theist questions in section 1.2.1. Five can be compared with, “How do we know that God did not create the big bang?” Two and three can be compared with “How do we know that God is not ultimately the source behind the fundamental particles?” We once again ask are these questions the domain of science or religion? What ontological criteria can we use to make such a distinction? How is the question, “What is an atom made of?” distinguished religiously or scientifically from the question, “What is a string made of?”; or “What gives rise to mass?” from “What gives rise to dimensions?”; or “How was the universe created?” from “How was the big bang created?”? Without invoking something arbitrary, it appears that there can be no ontological criteria to distinguish the two domains. This immediately implies that these questions can fall on either side of the line. Such an implication however goes counter to our intuition of scientific facts on the one hand, and religious belief on the other.

A resolution to this conflict can be found along the following lines. For any given question about our phenomenological experience, we can always pursue a scientific course of inquiry; however, this effort can never yield a complete answer. Any answer given will always open up new questions, which can also be scientifically pursued, but again not to the satisfaction of a complete answer, and so on, ad infinitum. We will call this idea the principle of theoretical incompleteness. With openings continuously available, there is always room for invoking a religious explanation. This is what we meant earlier when we said that the question of whether we can achieve a complete physical theory is only apparently different from the question of whether physical theories have any bearing on religious belief. Faith and doubt, whether of a religious, scientific, or any other kind, is an implicit personal admission of the inherent incompleteness of our ideas about the world. This is why the theist insist that he can raise his questions even in the face of a purported TOE. If the TOE was truly a complete theory, the theist would not be able to raise his questions. They would all be answered by the TOE. A complete TOE is incompatible with the notion of God. We can see this even clearer by comparing God and the TOE.

By our chosen definition, God is an omnipotent being. Any experiences we may ever encounter can always be explained by God. “She” is almost like an ultimate scientific theory. It is as if we said the universe is made up of a single ultra-particle that can manifest a myriad of properties and is responsible for a myriad of interactions. In a way, this is what a TOE is after; in fact, there is a hypothesized particle, the Higgs boson, that has been bestowed this almost God-like property. In essence, God is like a TOE and the TOE is like a God. We have to pick one of the two; we can’t have both. It is this lack of awareness of the full import of a TOE that is at the root of crisis in many an ontological debates concerning God and science.

1.4.1. Arbitrariness. Another way to see the dilemma is in terms of arbitrariness, or the amount of assumptions one has to make for any given theory. TOEs, like other good theories, aim to be minimally arbitrary. This is why the Standard Model of particle physics, while a robust and useful catalog for the working physicist, is considered a bad theoretical description. It has some twenty arbitrary parameters that must be supplied from experimental data rather than being derived from within the theory itself. It is one of the goals of any future TOE to eradicate this sloppiness. Counterexamples to the Standard Model are Newton’s theory of gravity and his theory of motion, both of which, by themselves, unified the movement of the heavens with the common objects of the earth. The same laws that applied down here applied up there, thus eliminating the need for two separate sets of laws. Einstein’s theory of gravity went even further by unifying the law of gravity with the law of motion. Because of this, it is considered to be one of the most elegant and yet most powerful theories ever put forth. Quantum theory is mixed on this front. The primary reason being that there are two different laws to apply depending on the situation (non-measurement and measurement), without an explanation of why. The goal of an improved quantum theory or reinterpretation would be to eliminate this arbitrariness by means of a single principle that would account for both laws.

But no matter how much arbitrariness is eliminated from scientific theories, as long as some remains, there is an opportunity to raise questions. For example, in the case of Einstein’s gravitational theory, one can ask, why four dimensions instead of eight? What is so peculiar about the number four? TOEs, by aiming to be a fundamental theory, seek to eliminate such questions and arrive at the core truths. Indeed, something is fundamental exactly because it is not arbitrary or haphazard. But how far must we go in order to eliminate arbitrariness? How far till we can account for four dimensions? Till we can account for force? Till we can account for an atom, a nucleus, a quark, a string, or space itself? How far do we need to dig to get the fundamental answer(s)? Can any physicist provide a reasonable answer to this question? Once again, we find that there can be no natural ontological criteria where we can draw the line. A purported TOE or any other scientific theory will always contain some amount of arbitrariness, some set of assumptions upon which the theory is built, and this arbitrariness will leave it vulnerable to questioning. We will discuss arbitrariness more thoroughly in part two when we consider its formal manifestation within the philosophy of science, the Quine-Duhem thesis.

1.5. The Critique of Finite Experience. At this point, it is necessary to address a major critique that the above philosophy faces. Some may contend that there does exist a natural ontological criteria that distinguishes scientific questions from non-scientific ones, and that demarcate the line between fundamental physical notions and arbitrary ones. This contention rests on the notion that our experience of nature is finite or ultimately reducible to atomic components. This notion is espoused in Hawking’s earlier view (1988, 167) that physics stops at the Planck energy limit, that it will be physically impossible to probe nature past this energy barrier—incidentally, this was one of the reasons he was advocating a TOE, a position which he has since somewhat backed away from (2002). Though we have yet to realize any such limit, if it should exist, then we would be correct in saying that some questions are beyond science, since we have no way of experimentally examining it. Whatever entity we observe as our atomic experience or at the Planck energy level would be the fundamental entity of our TOE. This would allow us to eliminate arbitrariness and formulate a complete TOE, while also permitting us to believe in a heavenly deity. However, we will shortly see that a key component of our argument, which we have yet to expose, forbids us from subscribing to such an ideal position.

If we are then to reconcile with our original notion of theoretical incompleteness, we have no choice but to reject the notion of finite or atomic experience. We thus adopt the principle, herein referred to as experiential incompleteness, or the novel experience problem, which states that our experiences of any given phenomena will always be incomplete and augmentable. This is not purely an external characteristic of nature; it is an inherent feature of our experience. It goes hand in hand with theoretical incompleteness. The same sensorial and cognitive processing mechanisms that allow us to experience and form theories of nature also preclude these experiences and theories from being complete. An immediate and obvious example of experiential incompleteness is the precision gap that is always accompanied by any physical measurement. Experimental measurements can never be 100% precise; there is always room for improvement. It is in this room where novel experiences lie. In part two, we will take a closer look at this and other forms of experiential incompleteness.

1.5.1. The Apparentness of Convergence. The unawareness of the novel experience problem is also responsible for the illusion of convergence, the idea put forth that all our empirical evidence seems to be converging toward some final theory. If all there is the current empirical evidence for some domain of phenomena, then we are right in saying that they converge on some final theory for that domain, barring the Quine-Duhem thesis (a thorny issue which has much to say about our discussion, but which we will take up later in part two of this paper). However, the proponents of convergence recognize that for many domains of phenomena, there may still be some additional empirical evidence yet to be discovered, but they insist that the new amount will only be a fraction of the current evidence, and therefore their effect will be negligible on the current theory. What we must keep in mind is that at some point in the future, today's empirical evidence will only be a fraction of the then current total of evidence, and who knows what theory this running total will converge towards. The point is that convergence seems so only on hindsight.

Though we cannot make a case for the future, the history of science supports this viewpoint. A textbook example occurred at the turn of the last century, when physics was almost wrapped up, save two experimental anomalies: black-body radiation and the lack of detecting the ether. The first led to the discovery of quantum mechanics and the second to that of the special theory of relativity. Compare the amount of empirical evidence gathered since then with what existed before.

Of course, many feel that for fundamental physics it is different this time. The Planck limit seems unlikely to be penetrated. But is it really fair to make a judgement about a possible barrier that presently is so far off from our experimental reach? There is a vast energy difference between the level of our current particle colliders (10^4 GeV) and the Planck energy (10^{19} GeV). Additionally, to imply that any experimental discoveries made on the long road to the Planck energy will also converge toward our current theories is highly presumptuous. Already, experiments coming out of some of the newer and upgraded facilities are beginning to strain the Standard Model of particle physics (Choi 2003; Evdokimov 2004; Chen 2004). In part two, when we discuss the novel experience problem in detail, we will provide additional examples of empirical discoveries that challenge current theoretical models.

1.6. The Apparentness of Non-Scientific Questions. By our thesis, all questions about our phenomenological experience are amenable to scientific inquiry; nonetheless, it seems that some are far removed from the domain of science. Before coming to the coda of our argument, we pause to address this apparent difference. By looking at our debated list of TOE requirements (section 1.4), we see that the first question is obviously regarded by scientists and theologians alike to be an issue for science, but that as we progress down the list, this appears to be less and less the case; however, at the same time, we cannot think of any scientific or logical reason that would prevent us from theoretically or experimentally pursuing an answer to all of them. The source of this disparity in conviction then lies in the following. The first question seems readily to be one for science because there is an active and fruitful amount of theoretical and experimental work presently surrounding it; whereas we progress down the list, this becomes less and less the case. In some cases, it is not even clear where to begin an attempt, what direction the research should take.

To illustrate this point, let’s consider an earlier phenomenological question that is not on our list: How was the universe created? Prior to the twentieth century, such a question would have been considered almost solely the domain of religion. Scientists were hardly speculating on the issue, if at all. Their central concern was figuring out how the universe worked; they could not even begin to imagine from where it came. It was just assumed that it was always there, perhaps placed by God. It was only after further astronomical observation, combined with theoretical insight from Einstein’s theory of gravity, revealed that the universe was actually expanding that the question began to be considered scientifically, resulting in a tentative and still developing answer: the big bang hypothesis. Before these theoretical and experimental developments, there was simply no scientific framework in which to embed the question. A very similar parallel can be found in how the question of human creation turned from a religious one to a scientific one via Darwin’s research. In this light, is it fair to say that the question of where the big bang itself came from will never be considered a question for science?

As another example, look at question two on our debated list of TOE requirements (section 1.4), regarding whether the physical constants and parameters are uniquely determined. After much research into string theory, the possibility began to emerge that the theory may not uniquely predict these values. It was unclear how to go about finding a solution. This lack of direction led many to doubt whether a scientific answer could ever be found. However, with the suggestion that quantum mechanical probability amplitudes may play a role, renewed scientific interest flourished once again in the question.

As a final example, consider the questions surrounding quantum physics. When quantum theory first appeared on the scene, questions about superposition, the measurement problem and non-local correlations were strikingly novel. At that time, there was not even a clear formulation of the questions, due in part to the bizarre and mysterious nature of these phenomena. This ignorance was also kept on by the fact that the very success of quantum theory drew attention away from these foundational issues and towards the more productive application of the theory. Nonetheless, some physicists did work on these questions, with more joining in as time went on, and today we see that the questions once thought unapproachable have begun yielding to theoretical and experimental investigation, as outlined in section 1.3.4.

These examples follow the pattern of paradigm evolution and revolution as elucidated by Kuhn (1970). When a new theoretical paradigm answers the questions of an older paradigm, much investment is spent by the scientific community in explicating and applying the new theory to ever broader phenomena, until such application meets with resistance and questions concerning the new theory itself begin being investigated, and the cycle begins once again. The distant fuzziness of the questions that lie on the periphery of our knowledge appears inconsequential to the grand theoretical edifice presently in front of us. It seems only natural that they can be dismissed as unscientific or of only minor concern. In this regard, an earnest scientific investigation into questions three and five on our list of TOE requirements (section 1.4) won’t begin (if) until we have a successful string theory and a more thorough account of the big bang, respectively—although some have already begun speculating on these issues. We also note that a resolution to any of these questions does not necessarily lie in a new revolutionary theory. In many cases it is not clear till the end whether we need a new theory, more ingenuity in the application of our original theory, or even an addendum to or reinterpretation of it.

1.7. The Central Problem: Self-Reference and the Undefinability of Valid Observations.

Incompleteness, the novel experience problem, belief, arbitrariness, and the inability to find any natural ontological criteria to distinguish science from religion all arise from a more central problem. To see this, imagine how the debate over God's existence would turn out if we were to attain a TOE. It happens that such a state of affairs would be somewhat paradoxical for the debate. For we need a TOE in order to test for any miracle observations, but a TOE, by our given definition (section 1.3), should ultimately account for all our observations. Nonetheless, we can suppose that once we do achieve a TOE, then we have scientifically ruled out God; but, if at any time we observe some miraculous phenomena, then we are willing to acquiesce that a God exists. But now we are presented with a new problem: how do we know that the observation in question is truly a miracle or a natural observation that empirically falsifies our TOE?

To this extent, consider how we would empirically determine whether any given TOE is the correct one. By Popper's definition, the theory must be falsifiable in order to be empirically verified. Thus there must be a set of observations that the TOE predicts and a set that it doesn't. Say hypothetically we observe one of the observations that it doesn't predict. This would seem to indicate that the theory is incorrect. But of this observation we can ask how do we know that our observation is sound?

It is at this point, that the notion of valid observations, as mentioned in the introduction, becomes relevant. Why should we ask whether our observation is sound or valid? Isn't it obvious? For example, the lead ball fell down with a specific acceleration, or the electron was deflected at a specific trajectory. But what if someone insists that it didn't occur so? By what authority is she renounced as wrong? Popular consensus? No. If we are to be scientific about it, then we should ask for some scientific procedure that can establish whether the observation did indeed take place.

What sort of procedure would this be? Simply, it would be a procedure based on a scientific theory; a theory about the sensorial-cognitive interactions of a human being with its environment; a theory upon which when certain observable conditions occur within the sensorial-cognitive system of the human being, it can be said that the human being has indeed experienced a certain observation. We will call this theory, “the theory of valid observations.”

We will now show that such a theory cannot exist, and hence demonstrate one of our central results: the theorem of undefinability of valid observations. At this point, the route of our argument depends on what we consider to be the domain of a TOE. If we go with our prior assumption of section 1.3, in which science is reductive and that the TOE can account for processes beyond the domain of physics proper, including the behavior of complex biological entities, and, specifically, the sensorial-cognitive behavior of human beings, then the TOE should contain the theory of valid observations within it and therefore can tell us whether our observation is valid. But this puts us in an impossible situation: in order to determine that the empirical observation that falsifies a TOE is valid, and hence that the TOE is falsified, the TOE—which includes the theory of valid observations within it—must be true (or corroborated). This contradiction tells us that, even if a TOE can discourse about our sensorial-cognitive behavior, it cannot go so far as to tell us whether our experienced observations are genuinely valid. A TOE cannot contain the theory of valid observations within it.

If, on the other hand, we assume a TOE cannot (or should not) account for sensorial-cognitive processes, then it cannot discourse about our observations, and hence, a fortiori, is unable to determine whether our observations are valid. But then, in such a case, we can ask whether the theory of valid observations exists independently of a TOE? By similar arguments above, we show that such a theory cannot exist.

Assume that it does. Now recall, as in the case of a TOE, that the observations of a scientific theory, in general, fall into two categories: observations that corroborate the theory and ones that falsify it, thus allowing the theory to be subjected to the method of Popper falsification. To our theory of valid observations we put the question: Is an observation that falsifies the theory, itself, a valid observation? Assume it is. This then implies that our theory of valid observations is falsified. But this in turn implies that we are no longer certain that the falsifying evidence is valid, which further implies that we are no longer certain that the theory is in fact falsified. This is similar to the result we got above on the TOE. Both generated contradictions. These contradictions tell us that a TOE complete enough to guarantee the validity of our observations or a theory of valid observations, itself, cannot be empirically falsified, and therefore cannot be empirically tested; both are chimeras.

This is precisely why a TOE leads us to the paradoxical situation in our empirical debate over God's existence. Only a TOE complete enough to guarantee the validity of our observations can be used to test for God's existence, but such a theory can't ever exist. A TOE or any other theory cannot guarantee the validity of our observations nor disprove whether they are miracles. This is why the theist is always able to raise his doubts concerning the impregnability of a TOE, and this is ultimately why the debate over God's existence is empirically undecidable.

The inability of a TOE to secure the validity of scientific observations is core to our arguments. Without a theory to tell us what constitutes valid empirical data, we are unable to draw a clear boundary encompassing the hard facts of science. This failure undermines our ability to find any natural ontological criteria to distinguish questions of phenomenological experience into scientific and religious (or other) categories, or demarcate fundamental physical notions from arbitrary ones. At the same time, it undercuts the notion of a finite (or atomic) experience set. As will be seen in section 2.3, if experiences were finite, then we could determine their validity.

With the theorem of undefinability of valid observations, we can assert a general principle of scientific theories: *no empirically testable theory (TOE or otherwise) can guarantee the validity of its observations.* Lacking such a guarantee, we cannot then absolutely say whether some observation actually confirms or disconfirms a theory.

These last two statements may lead one to wonder (1) how is it that at many times it seems that some definite observation has taken place, and (2) how can science ever make any progress? The first question is answered in section 2.3, but for now we only say that the resolution lies in the principle of experiential incompleteness. The second question we answer now.

Before Popper, philosophers and scientists assumed that theories can be confirmed or disconfirmed by empirical evidence. Popper, of course, argued that this was a naive viewpoint, in that experiments cannot confirm a theory, but only corroborate it. However, Popper still held to the conviction that experiments can falsify, or disconfirm, a theory. Most scientists (if not philosophers) today hold to this position, and thus science can progress without experimental confirmation, as long as it is able to still disconfirm theories. But, of course, Popper's remaining conviction of falsification was challenged by the holistic turn in the philosophy of science led by Duhem and Quine. The stripping away of this last authority from empirical observations leaves us in the same quandary regarding the selection of theories as our theorem above. Holists and other philosophers have responded to this quandary with various answers ranging from aesthetic, pragmatic and social selection pressures (see also section 3.4.3). Here we respond with an answer arising from the principle of incompleteness. We pick theories based on our best theoretical and empirical knowledge at the time. Such knowledge may at times seem so definite as to determine a unique theory, but as both theory and experience are inherently incomplete, further theoretical and empirical discoveries may reveal our judgment to be inaccurate. This answer will be expounded in part two, especially section 2.5.

It is important for us to understand why a TOE or any other theory is unable to secure the validity of scientific observations. Scientific theories generally talk about, or refer, to the outside world, the world of observations. When we ask the theory to validate those observations, that is, examine the process by which we make those observations—this would include everything from our instruments to our senses, and to our final mental realization that those observations have taken place—we are turning the theory back onto itself, or more precisely, back onto us. This creates the dilemma of self-reference. However, it is unavoidable. We are a part of the physical universe that we are trying to describe. To give a complete description we have to include ourselves. Another equivalent way to see it is that a part of the universe—the sentient biological entities—is trying to describe the whole of it. The inability to extricate ourselves from this description pollutes any objectivity that such descriptions may have.

The problem of self-reference has been know to philosophers and logicians for quite some time, most notably as the liar or Epimenides paradox: "This statement is false" or "I am lying". But the issue was considered innocuous to theories of knowledge until its role in demonstrating the incompleteness of arithmetic was illustrated by Gödel (1931) in his celebrated theorem. However, even with this illumination, there was not an eye-opening realization that self-reference was a problem for our general understanding of the world;

discussions of it rarely went beyond the confines of logic and mathematics. Our findings here show that the thorn that has remained in our side all this time cannot be so easily removed. A resolution to the ontological and epistemological issues facing science and philosophy cannot be had without addressing the issue of self-reference. In part three of this paper, we will take a more in-depth look at self-reference and the similarity of its role in mathematical and scientific incompleteness.

Part Two: Philosophical Theory

2.1. Introduction. The above analysis of the God existence debate contains some core issues within the philosophy of science whose reach goes beyond TOEs to scientific theories in general. It will be worth illuminating their role here to give us an even deeper picture of the field that we are maneuvering in.

The impossibility of an empirically testable complete TOE, and thus the unresolvability of the God debate, rests on one central issue: the determination of valid observations. No physical theory can guarantee the validity of observations that are used to verify or discredit the theory itself. It is the assumption that the TOE could do that which led to the contradiction that a TOE can be empirically falsified if and only if it is corroborated. Like the theist, we can always ask what may lie behind our observations. Is it God? Is it some other theory? It is simply not possible to eliminate such doubts from an observation. What this means is that there is no such thing as hard observations or hard empirical data. This brings us to a famous issue within the philosophy of science: the observational/theoretical distinction.

Thus far within the philosophy of science there has been no successful attempt to develop a criteria for distinguishing observational terms from theoretical ones. In addition, the field of neural science, albeit young, has been unable to clearly demarcate the line between sensorial and cognitive processes. These results are not an accident. We postulate here that sensorial and cognitive processes are fundamentally inseparable, and that it is this inseparability that is the root cause of the observational/theoretical distinction failure and the theorem of undefinability of valid observations. The empirical undecidability of God’s existence then can be also seen as a consequence of the observational/theoretical distinction failure. With this insight, we see that the following four ideas are all implicative of one another: (1) the theorem of undefinability of valid observations, (2) the observational/theoretical distinction failure, (3) the unattainability of a TOE and (4) the empirical undecidability of God's existence (the possibility of belief). The implications do not end there. We will show that the following additional ideas are also implicated with the previous four: (5) the Quine-Duhem (and underdetermination) thesis, (6) the incompleteness of any scientific theory and (7) the novel experience problem. In what follows, we will detail these ideas more clearly and spell out the implications between them all. Before continuing, we would like to caution that the above ideas are not meant to encourage an attitude of science bashing, outlandish anti-realists positions, or other philosophical anarchy. More will be said on this in part three (3.4.3).

2.2. The Quine-Duhem Thesis and the Observational/Theoretical Distinction Failure.

We begin by considering the Quine-Duhem thesis, which states that any experimental data on a given domain of phenomena can always be accommodated by any scientific theory purporting to describe that phenomena. This is a more robust formalization of the intuitive notion of arbitrariness that we discussed in the first part of the paper: if scientific theories are not uniquely determined, then there is some arbitrariness in theory selection. The significance of the Quine-Duhem thesis is that it asserts that scientific theories cannot necessarily be falsified by empirical evidence. We want to show that this is only possible because of the observational/theoretical distinction failure. This can be seen most simply by noting that if observation always has some amount of theoreticalness to it, then by suitably modifying those theoretical aspects we can, in essence, modify what has been observed, and thus accommodate the observation to the theory.

A more sophisticated approach is through subscribing to Quine's holistic model of scientific theories: the web of belief. In Quine's web, scientific (and auxiliary) terms, concepts, laws and entire theories are interconnected so that their meanings don't stand on an island but depend on the meanings of other terms, concepts, laws and theories to which they are connected. In his web, Quine includes connections between observational and theoretical terms. By adjusting the meaning of, or adding additional theoretical terms to the web of a given theory, we can impact the meaning of observational terms, and thus affect what we observe. What is essentially happening is that the observation portion is being pushed around. The theory is tailoring the observation to suit itself. In some cases, the tailoring will be minor, in other cases, major. Examples from history include Ptolemy’s addition of epicycles to the geocentric model to accommodate the observed oddities of the motion of the other planets, and the inclusion of Lorentz forces to Newtonian mechanics to account for the negative results of the Michelson-Morley experiment to detect absolute space.

Some critics may raise the objection that the Quine-Duhem thesis (or a holistic model of science) does not require the abandonment of purely observational terms. They may argue that observational terms can be connected to theoretical terms without impinging on their sovereignty; that is, the meaning of the observational terms remain fixed while the theory or theories change around it. We will show that such a position is untenable. We will first show that the Quine-Duhem thesis necessarily implies that scientific theories are incomplete, and then how this incompleteness implies the observational/theoretical distinction failure.

2.2.1. The Quine-Duhem Thesis and the Underdetermination of Theory. There are two cases to consider here because the Quine-Duhem thesis is a generalization of the underdetermination of theory by observation (UDT). Before we begin then, let us clarify the exact nature of this generalization. UDT asserts that observations are compatible with more than one theory, and therefore an observation underdetermines multiple competing theories on a given domain of phenomena. We generalize UDT to the Quine-Duhem thesis by simply replacing the phrase “more than one theory” or “multiple theories” by the phrase “any theory.” The new statement, “any observation on a given domain of phenomena is compatible with any theory on that domain of phenomena” is the Quine-Duhem thesis. The Quine-Duhem thesis simply gives UDT unlimited compatibility.

The counterpart to (as opposed to the generalization of) UDT is the following: any theory on a given domain of phenomena can accommodate two or more seemingly incompatible observations on that domain. This will be correspondingly referred to as the “underdetermination of observation by theory” (UDO). This thesis emphasizes the flexibility of theories in accommodating multiple incompatible observations, while UDT emphasizes the flexibility of observations in accommodating multiple incompatible theories. We can generalize both theses to one, the Quine-Duhem thesis, by making their respective flexibilities unbounded so as to accommodate *any* observation (UDO) or theory (UDT). The Quine-Duhem thesis then can be seen to have two aspects: one touting the flexibility of theories, the other that of observations, depending on whether we wish to hold on to a given theory or undermine its sovereignty, respectively.

In what follows, we will make our arguments concerning UDO and UDT, the conjunction of which can be referred to as the "limited Quine-Duhem thesis." It is obvious that if the limited Quine-Duhem thesis implies incompleteness, then the much stronger Quine-Duhem thesis, itself, also does.

2.2.2. Underdetermination of Theory and Incompleteness. Does the underdetermination of a theory necessitate the incompleteness of the theory? We now show that it does. Recall the arguments above concerning God and TOEs. There we saw that in lack of a complete TOE, there is always the possibility for God or rival theories to exist. We also saw that the success of this argument rests on the critical realization that complete theories leave no explanative room for competing theories. That is, complete theories explain every aspect of physical phenomena; they are not found wanting anywhere in their explanation. It is easy to see that this realization is not limited to theories of everything, but also applies to theories which may address only a specific domain of physical phenomena. (Some contend that theories of specific domains are partial to begin with and therefore incomplete, a fortiori; however, this point has also been used to mount an argument against underdetermination—see the next paragraph.) Thus it is not possible to have two or more competing theories on a given domain and, at the same time, for those theories to be complete. Each of the theories must leave some room so that it is possible that the other theory can also be right.

We note here that incompleteness has also been used to argue against UDT. Hoefer and Rosenberg (1994), for example, point out that Laudan and Lepin's (1991) critique of UDT holds only in the case of partial, incomplete theories.[8] For brevity, we will crudely simplify the extensive and detailed arguments of the above authors: present incomplete theories are always subject to evolution in the future, and thus two incompatible theories that may be

[8] Hoefer and Rosenberg claim that theories of specific (localized) domains of phenomena are such theories, but their argument would also hold for (global) theories of everything that are incomplete.

empirically equivalent presently, and therefore underdetermined, may not be in the future.[9] Incompleteness, or auxiliary theories, is a double edged sword: it can be used to argue for or against underdetermination (similar to how it can be used to attack or defend theories, via UDT or UDO—as we will shortly see—respectively). However, while the above incompleteness argument allows for the possibility that certain rival theories on a given domain may not be underdetermined after all due to future changes; unless the incompleteness is eliminated altogether from the current reigning theory, there is always the possibility that some other theory is correct. And it is in this sense that we argue that incompleteness of a theory underdetermines that theory.[10]

2.2.3. Underdetermination of Observation and Incompleteness. Now we turn to the other half of the Quine-Duhem thesis. Does the fact that a given theory can accommodate multiple, seemingly incompatible empirical evidence require that the theory be incomplete? To answer this, let's consider a specific example. How was Ptolemy able to hold on to Aristotle's geocentric model in spite of the anomalies of the planets' motion that was observed at the time? Obviously he introduced the mechanism of epicycles to the simpler geocentric model. But why was he able to do this? It is precisely because the notion of a circular orbit was not defined enough in the original model. In that model planets travel in perfect circles around the earth. In Ptolemy's modifications, the planets still travel in perfect circles around the earth, except now there is more than one circle per planet. If the notion of a circular orbit had been completely or even more fully defined (e.g., there can only be one circle per planet), Ptolemy would not have been able to introduce his epicycles (unless by some other convoluted route perhaps); he would have to take advantage of an alternative hole in the theory.

As another example consider the explanation of light in terms of Newton's particle

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[9] The same argument can be made, as Laudan and Lepin originally did, without strictly requiring theories to be incomplete, but only that they be attached to auxiliary or background theories, which may change or be replaced, depending on future research—essentially Quine's web of belief. Though, in the case of global theories, incompleteness must be admitted, since global theories by definition include all auxiliary and background theories. Indeed, as it has already been stated and will become clearer later, incompleteness and the need for auxiliary theories are intimately related.

[10] Interestingly, the proponents of underdetermination, to undercut this argument against it, have moved the debate to the arena of complete (global) theories. We, of course, are arguing here that by giving up the sword of incompleteness, the proponents have no fight.

(corpuscular) theory and Huygens' wave theory. Newton was successful at explaining many phenomena of light in terms of his paradigmatic mechanical notions of particles and forces, but ran into difficulty with the phenomenon of refraction, a phenomena which Huygens' wave theory explained quite easily. To explain refraction, Newton ([1730] 1952) postulated the existence of attractive forces that act upon the light particles at the boundary between the two mediums where refraction occurs. This was already an added complication to his theory, for it is clear how a particle of light incident on a surface (e.g., a mirror) may reflect off of it, similar to how a billiard ball may bounce off the edge of the billiard table at the same angle that it strikes it; but what are the origins of an attractive force that would cause the light particles to bend away from its straight path as it enters a material (e.g., from air into water)? Nonetheless, such an explanation would not seem necessarily outlandish in Newton's time, for there was hardly a thorough understanding of how light interacts with matter; the gap in knowledge at the time could easily accommodate any such explanations.[11] However, the story is not over. In his own prism experiments, Newton discovered that not only did light bend as it enters materials, but that different colors of light bend at different angles, thus separating out the colors of white light into the well-known rainbow. To explain this, he postulated that light particles possess some additional property which determines how responsive they are to the previously postulated attractive forces (Hendry 1980; McGuire 1983). For example, a blue light particle, possessing more of this property is deflected at a greater angle than a red one, possessing less. Once again we find that, even though Newton was ultimately unable to give any definitive account of this novel property, a lack in understanding of what a light particle was at the time would certainly permit the possibility that it could exhibit such a property. The lesson to be learned is that the particle theory of light and its associated web of belief—a mixture of Newton's mechanical and gravitational theories combined with alchemy and religion—was never complete in its description of light and how it interacts with matter, nor could it answer all questions concerning the behavior of matter in general; thus it was possible that the yet unknown, missing theoretical descriptions could account for some new, seemingly anomalous observation. In this way, Newton's particle theory of light lasted for another hundred years.

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[11] For example, Newton postulated that light particles experience a gravity like force near the surface of the material. For a more thorough account of these explanations, see the citations in the referring paragraph.

As enduring as the particle theory of light was, further complications in the theory eventually gave way to the simpler wave theory. The wave theory faced its own trials at the turn of the twentieth century, and in an ironic twist of events, we get another example of theories being modified. The observation of the photoelectric effect, where light seems to be absorbed and emitted by matter in discrete chunks as if they were particles, caused renewed interest in the particle theory. At this point, the proponents of the wave theory introduced a modification that would allow their theory to also compete as an explanation: the wavepacket. Light is still a wave, but the amplitude of a given light wave smoothly diminishes at a given distance from either side of the central wave peak. This modification extends our notion of waves while still being compatible with its central ideas, albeit complicating it somewhat. Here too we find that the modification was only possible because the theory of light waves was incomplete. The theory never specified all the different ways in which a light wave can vibrate. Indeed, going further, can we ever give a complete formal definition of what constitutes a wave in general? For example, is a square wave really a wave even though it changes its amplitude discontinuously? What exactly qualifies something to be a vibration? How do we exactly define amplitude and wavelength? It is the contention here that such concepts can’t ever be given a complete definition. This applies to all physical concepts, whether wave, particle, force, orbits, electrical attraction, etc.

It is this incompleteness of definition that allows theories to be saved. When faced with some seemingly anomalous observation, we go in search of answers in the fuzzy, outlying areas surrounding our theoretical concepts. There we have some room to maneuver, some yet unexplored vistas that may afford us a solution to our crisis, a place where we can further mold and shape the terrain of our definition to suit our needs. Ignorance becomes a virtue instead of a vice, and flexibility becomes more valuable than stability. With this insight we see that theories can be modified when there is some room to maneuver within the theory, i.e., if there is some explanative room. This is consistent with the critical realization we recalled above: complete theories leave no explanative room for competing theories, even if some of those theories are modifications of the original theories. The conclusion is that complete theories cannot be modified, only incomplete ones can.

Without going into details, we briefly list some additional examples from the history of physics, which may be followed up in the accompanying citations: the many modifications of

Newton's mechanical paradigm to account for electromagnetic and light phenomena before the advent of relativity and field theories (see Einstein [1938]1966, 62-122 for a summary); an account of the perihelion advance of Mercury's orbit within the traditional Newtonian theory of gravity versus the modern account of Einstein's theory (Hall 1894; Campbell 1983); dark matter, dark energy and other proposals to account for the difference between what Einstein's theory of gravity predicts and some of the presently observed motions of galaxies and the rate of expansion of the universe itself (see van den Bergh 1999 & Caldwell 2004 for a summary); and, finally, the many hidden variable theories which attempt to save the classical deterministic paradigm in the face of anomalous sub-atomic observations (see section 1.3.4 for references).

Some may contend that in certain cases above, rather than modifying existing concepts, merely new ones were added, and thus we don't require concepts to be incomplete for UDO to be successful. Even if we allow this caveat, we do, nonetheless, require the theory to be incomplete, since we earlier established that complete theories are maximally explanative, and therefore leave no room for modifications or additions.[12] So whether we talk about extending concepts or adding new ones, this is only possible if the existing theoretical framework is incomplete.

As a note, we don't want to give the impression here that incompleteness is the sole reason why displaced theories have lasted as long as they have. Newton's mechanical paradigm, for example, was very successful in accounting for many phenomena and therefore has earned a certain level of credibility with scientists. In addition, social, religious, aesthetic and other influences may contribute to a theory's credibility. Naturally, how much anomalous evidence is needed before we are willing to abandon a theory depends on how much credibility the theory has with us. But, when faced with any such seemingly anomalous evidence, whether small or large, it is incompleteness that allows us to implicitly rationalize

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[12] But alas, this caveat does not hold. Concepts cannot simply be added to theories without affecting the meaning of other concepts within the theory. This is the critical realization behind Quine's web. Consider the case of dark matter. By adding dark matter to the web of Einstein's theory of gravity, we are extending the notion of what we mean by mass. Previously, our definition of mass included only the mass particles that constitute the standard model of particle physics, but by postulating dark matter, we are saying there may be new kinds of mass particles, for example, what some physicist have termed the weakly interacting massive particles (WIMPs). In Newton's explanation of refraction, the postulation of "attractive forces" impacted the notion of gravitational force, or force, in general. Thus additions to theories can equally be interpreted as extensions of existing concepts.

the correctness of a theory, or, equivalently, retain our faith in it.

2.2.4. The Quine-Duhem Thesis and the Undecidability of God. Before continuing with our argument, it is worth to pause and point out some interesting parallels between the God existence debate and the two aspects of the Quine-Duhem thesis. The inability to prove that God does exist can be likened to the inability to prove whether any given theory on a given domain of phenomena is the correct one. The failure is due to the underdetermination of theory. God and the purported TOEs are underdetermined by all available empirical evidence, just as the competing theories on a given domain are underdetermined by the evidence of that domain. Second, the inability to prove that God doesn't exist can be likened to the inability to prove whether any given theory on a given domain of phenomena isn't the correct one. This failure is due to the underdetermination of observation. We can always modify God or the theory to accommodate any empirical evidence, since neither God or the theory are completely defined. These connections throw light on ideas that have always been an integral part of science but has traditionally remained obscured: doubt and faith. The underdetermination of theory allows us to doubt scientific theories, and the underdetermination of observation allows us to have faith in them. In this respect, we see that science and religion share a certain bond.

2.2.5. Incompleteness and the Observational/Theoretical Distinction Failure. We now continue with our argument. We have shown that the limited Quine-Duhem thesis, and thus the Quine-Duhem thesis, implies that physical theories are incomplete. We now show that this incompleteness necessarily implies the observational/theoretical distinction failure, thereby demonstrating that the success of the Quine-Duhem thesis indeed rests on this failure. The argument is straightforward. The incompleteness of physical theories means, as given by our definition of completeness in the main introduction, that there are questions about the domain of phenomena under consideration about which the theory is unable to answer. This means that we cannot completely explain every aspect of a domain of observations; moreover, we cannot completely explain every aspect of any single observation. If we could in either case, then we would have a complete theory accounting for the domain of observations or for the single observation. (In the case of a single observation, we might be more inclined to call the theory an explanation or even a description, but the point is the same: the description, explanation, account or theory is incomplete.) Whether for a single observation or a group of observations, if the Quine-Duhem thesis holds, so does an incomplete account of the observation(s). If the description of an observation is incomplete, then we cannot give a complete list of conditions that must obtain in order for us to say that the observation has taken place. What this amounts to is that we cannot definitely determine whether the observation has taken place or not and thereby utter the appropriate observational term signifying our recognition of this observation. This clearly undermines our ability to assign unique observational terms to observations, hence the observational/theoretical distinction failure.

Another way to see this is to note that if the description of an observation is incomplete, then we can extend the description in a new way. This new description will also be a description of the observation, albeit different. The meaning of the observation has changed in a certain sense. This mutability is the real import of the observational/theoretical distinction failure. The meaning of the observation is dependent on how we describe it or, equivalently, the theoretical constructs we surround it with. We can paint the observation in a variety of ways and thus make it part of many different theories. The observation has no inherent meaning in itself; its meaning arises within our language, description or theory.

What we essentially have here is the notion of an observation as a cloud of theoretical constructs whose layers can be peeled or added to reveal different “observations.” New theories reveal the theoretical assumptions that were an implicit part of the observations of prior theories. For example, Copernicus’ theory revealed the implicit assumption that the earth stood still in observations of the retrograde motion of heavenly bodies; Galileo and Newton’s law of inertia revealed the assumption of frictionless motion; special relativity revealed the assumption of rigid measuring rods and ideal clocks; and quantum mechanics revealed the assumption of interaction (disturbance) free observations. Such peeling or revelations are only possible because the meaning of the theoretical constructs surrounding the observation are never completely defined. This incompleteness is what also precludes the observations from having definitive language terms assigned to them. Language cannot completely capture observations, and therefore, by conceding to use language to describe observations, we must allow for flexibility and mutability in its usage.

Within the philosophy of science there has always been a suspected link between the observational/theoretical distinction issue and the Quine-Duhem thesis, but it has until now remained unclear whether this was the case and what the nature of the supposed link was. What we see here is that there is a definite link: incompleteness. The underlying success of the Quine-Duhem thesis is due to the fact that the meaning of individual terms in a theory are incomplete, thereby allowing the theory to be adjusted to accommodate any data. In fact, this is the whole point of a holistic model of science. There is no reason to link together terms whose meanings are completely defined. Holists acknowledging that this can never be the case, recognize that terms must draw on other terms for their meaning.

But this does not mean that the meaning of any term, though not contained in itself, can be contained in a web of terms. No word or even a group of words can completely capture the meaning of observational terms. Such is the whole point of the Quine-Duhem thesis: a theory can accommodate any data, not just some finite range of data. Traditionally it has been argued that the Quine-Duhem thesis implies that no crucial experiment can debunk a given theory because the theory is part of a larger web of other terms and theories. What is important to recognize here is that a crucial experiment cannot debunk this larger web either. Holistic models don't merely grant scientific theories some finite range of flexibility, but an unbounded range.

2.3. Incompleteness of Observational Descriptions. How exactly are descriptions of observations incomplete, and why is it that we are unable to determine whether they have taken place? For example, it seems that there is a tree standing in front of me, and therefore I am right in uttering the observational term "tree". What we need to keep in mind however is that this is more of an agreement than a defacto truth. We all *agree* that there is a tree standing in front of me, thus making the statement appear true; but do any of us have enough of an understanding of the observable tree so that we are able to distinguish all observable phenomena into trees and non-trees? For example, is some observed large bush perhaps a tree? I may say yes, but you may say no, depending on the differences in definition that may arise beyond the borders of our common agreement and within the fuzzy incomplete areas. Moreover, is something that we all agree to be a tree really a tree? For even our agreed upon, common definition of a tree will be incomplete, and therefore we will face questions as we conduct further observations on our tree. For example, a tree has leaves. We need to make sure our tree's leaves satisfy the botanical definition of what a leaf is—so that we are not fooled by some plastic imitation tree or an even finer modern technological replica. But this definition itself is constituted of elements which have further definitions, and we will need to make sure the elements of our tree's leaves satisfy these definitions and so on.

The above analysis can be carried out for all observable phenomena ranging from biological species down to chemical, atomic and subnuclear elements and all the way up to astronomical objects such as planets, stars and galaxies. For example, how much genetic variation is allowed before we can distinguish between biological species? A definition based on sexual compatibility cannot be sufficient unless we can offer a complete definition of sexual compatibility itself. What about atoms? Is an isotope an atom? Most would say yes. Is an antimatter "atom" an atom? What about the possibility of atoms made out of some of the many other discovered nucleons? Is a neutron "star" a star? What about the forms of matter such as solids, liquids, gases and condensates? Can we clearly distinguish all such cases? There is currently much research in this field (Samuelsson 2002; Paredes 2004; Plazanet 2004). Even in the case of the current "fundamental particles", are we able to define their parameters enough so that when we observe readings in our particle detectors, we can clearly identify all cases which are, say, electrons? This is not possible as yet, because, for example, in the case of the electron, we do not know how it exactly behaves at higher energies and field strengths. This question is part of a current program of particle research (Anderson 2001).

Part of the reason we are unable to definitively identify observations is due to the inability to completely define observational terms within language. For any observational term, we will find that there will always be cases which will require us to further refine our definition in order to determine whether the observation has obtained or not. We may be led to believe that we can avoid such infinite regress in the definition of observational terms by directly associating the terms with experience. This is in the spirit of Wittgenstein's picture theory of language: we give meaning to a term by uttering it and then pointing to a phenomenon. Say, in such a manner, we assign the term "ugg" to the phenomenon of lightning strikes. The problem with such an approach is that there will be cases where we are not sure whether to utter "ugg", for example, if we observe the phenomenon of ball lightning.

Moreover, with such an approach, we can’t communicate with one another the intricacies of observations; we can’t communicate similarities and differences. We can’t create the notion of common properties that many phenomenon in this world share, such as round, soft, shiny, jagged, etc. That is, we can say we just experienced “ugg” but we can’t tell someone what that is. We must rely on the fact that they have also experienced “ugg” and therefore know what we’re talking about when we utter the term, but we can’t proceed to describe it to them. As soon as we do, then we end up with the problem of infinite regress that we had above.

With Wittgenstein’s picture theory of language, we see that we avoid the problem of infinite definitional regress, but still have the central problem of undecidable observations. However, there does seem to be a way out of the dilemma. One may contend that the problem above arose because we chose a complex observable such as lightning which has several processes going on. The picture theory of language will work if we can identify atomic, irreducible observations that are the components of the macro observations. Thus we won’t ever have any borderline cases; either something or some process is observed or it is not. This points to a reductionistic view of science in which there are fundamental physical entities and processes. (In fact, this is what TOEs promise.) It also points to a finite view of natural experience. It is because of this finiteness, that physical theories, being finitely described, can ever hope to completely capture our experience of nature. Furthermore, it is such finiteness that any correspondence theory of truth rests on.

But we see from our interaction with nature that such finiteness can’t ever come to exist. We can always probe nature deeper and deeper. This capability becomes obvious in any scientific experiment where the precision of measurements can always be improved—maybe not with today’s technology, but certainly with tomorrow’s. By increasing the precision, we are zooming in and getting a more detailed description of the observation. For any scientific predicted result, the corresponding observation will always be accompanied by a percentage error of measurement uncertainty. It is in this uncertainty that we find the undecidable cases of a given observation and further refine our observational definitions—the infinite regress. Whether a tree, an atomic nucleus, or a galaxy of stars, we can always ask for a more detailed picture, and we can get it by probing nature more deeply.

Cases from the mathematical sciences, like astrophysics, can serve to elucidate this point most effectively. Consider the motion of the planets around the sun. We can describe their orbits by finite mathematical descriptions. Initially, Copernicus described them as circles. This appeared to match what little data we had at the time and seemed like a good guess. With the arrival of the telescope, we were able to get more precise data, which prodded Kepler to describe the orbits in terms of ellipses. Then in the 19th and 20th century, with even more powerful telescopes, we saw that the ellipses precess, ultimately supporting Einstein's theory of gravity. What this shows is that finite mathematical descriptions do not fully capture observations. By improving the precision of the observation, we invite the possibility of new mathematical models. This implies that in the future there may be a new mathematical model of planetary orbits or some new theory that may supplant the notion of planetary orbits. The latter is possible because, as we improve our precision, we may find that the data no longer supports the notion of planetary orbits, but rather reveals planetary orbits to be some other existing or novel phenomena. (This is not unusual: the history of science is littered with ideas that no longer survive: epicycles, ether, phlogiston etc.) In general, we may expect improved measurement precision to sharpen, modify or even supplant prior observations.

2.4. Novel Experience Problem. This aspect of scientific experience, measurement uncertainty, we will fold into what we shall call the novel experience problem and give it a central role in our philosophy. What it states is that we can always have new experiences of any given phenomena. In this way, our experience of nature will always be incomplete. (We recall, once again, that novel experience is not synonymous with anomaly, as pointed out in the main introduction.) Besides measurement precision, there are other aspects of an observation that can be tuned to give us a new look at the phenomenon under consideration. We loosely identify three such other aspects: perspective, interaction and range. However, before turning to them, we will first discuss the issue of precision more thoroughly.

2.4.1. Precision. It is often remarked how highly accurate our current theories are. For instance, in his popular exposition of quantum electrodynamics (QED), the quantum theory of the electromagnetic interaction, Feynman (1985, 7) has pointed out that the theory has been successfully tested to a precision equivalent to measuring "the distance from Los Angles to New York to... [within] the thickness of a human hair." Let us explicate this statement more quantitatively. The quantity being measured is the anomalous magnetic moment (AMM) of a free electron. Feynman quotes the theoretical value to be 0.001 159 652 46, with an uncertainty of 20 in the last two decimal places. From these numbers, we can calculate the precision of the theoretical prediction: 17 millionth of a percent, or equivalently, 0.17 parts per million (ppm). Feynman quotes the experimental value to be 0.001 159 652 21 with an uncertainty of 4 in the last decimal place, corresponding to a precision of 0.03 ppm. Subtracting the theoretical and experimental values gives a disagreement of 0.000 000 000 25, with a joint uncertainty of 20 in the last decimal places, corresponding to an accuracy between theory and experiment of 0.22 ppm with a joint uncertainty of 0.17 ppm. Although the disagreement lies outside the joint uncertainty, it represents only a 1.2 standard deviation, considered to be in good agreement. The question we raise here is: does an agreement at such a high level of accuracy rule out that a more precise experiment will reveal a deviation from theory?

To this extent, consider the measurement of the AMM of the muon, a heavier cousin of the electron. But first, it is necessary to give some background concerning QED and the theoretical calculation of the muon's AMM. From experiments, it is known that for larger masses and energies, such as in the case of the muon, the electromagnetic interaction interplays with two of the other fundamental interactions (or forces) of physics, the weak and the strong. These interactions are beyond the scope of QED. It is therefore not the whole story when it comes to the electromagnetic interaction; in order to address the electromagnetic interaction at higher energies, QED must be extended. This has been accomplished for the weak interaction (the electroweak theory), but it remains one of the outstanding problems in fundamental physics for the strong interaction; moreover, the strong interaction, itself, is not that well understood. Part of the goal of a TOE is the integration and better explication of these interactions. The situation as it currently stands is known as the Standard Model. Though physicists acknowledge that it is incomplete, it is based on a certain theoretical foundation. It is this foundation whose experimental accuracy is being examined here.

Let us begin. (For brevity, the remaining numbers are given in scientific notation, the uncertainty is indicated by "±", and the precision or accuracy is in parentheses immediately following.) Though more difficult to measure than the electron, by 1999, the world experimental average for the muon's AMM (based on work done at CERN and Brookhaven National Laboratory) was $(1165920.5 \pm 4.6) \times 10^{-9}$ (3.9 ppm) (Brown 2000), with the theoretical value, as predicted by the Standard Model, at $(1165917.68 \pm 0.67) \times 10^{-9}$ (0.6 ppm) (Miller 2002)[13], giving a disagreement of $(2.8 \pm 4.7) \times 10^{-9}$ (2.4 ± 4.0 ppm), a 0.6 standard deviation, the disagreement being well within the joint uncertainty. We already see here that theory and experiment are accurate to 2.4 ppm—only an order of magnitude less than the electron's case—and found to be in agreement. By 2001, due to ongoing work at Brookhaven, the precision of the measurement was significantly improved, albeit not greatly affecting the value itself. The new world experimental average was $(1165920.3) \pm 1.5 \times 10^{-9}$ (1.3 ppm) (Brown 2001), with the theoretical value remaining the same.[14] The new disagreement was $(2.6 \pm 1.6) \times 10^{-9}$ (2.2 ± 1.4 ppm), a 1.6 standard deviation, the disagreement now outside the joint uncertainty. By January 2004, the Brookhaven team had further increased the precision, with a slight change in the value: $(1165920.8 \pm 0.6) \times 10^{-9}$ (0.5 ppm) (Bennett 2004). At this point, comparison with theory is complicated by some recent developments. The current consensus is that there are two possible theoretical values, the so called directly and indirectly calculated ones. The directly calculated value is $(1165918.1 \pm 0.8) \times 10^{-9}$ (0.7 ppm) and the indirectly calculated one is $(1165919.6 \pm 0.7) \times 10^{-9}$ (0.6 ppm) (Davier 2003), giving a disagreement of $(2.7 \pm 1.0) \times 10^{-9}$ (2.3 ± 0.9 ppm) and $(1.2 \pm 0.9) \times 10^{-9}$ (1.0 ± 0.8 ppm), with a 2.7 and 1.4 standard deviation, respectively.[15] While a 1.4 deviation is not a major concern, the 2.7 is; though, these theoretical calculations will most likely be updated in the future, hopefully settling on a single value. (We note here that the increase in deviation in the direct case is not due to the change in the theoretical value since 1999; in fact, the new theoretical value is closer to the latest experimental value than the 1999 theoretical value.[16])

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[13] This value represents a later correction of an error (Knecht 2002) that was found in the value at the time, $(1165915.96 \pm 0.67) \times 10^{-9}$ (0.6 ppm) (see Miller 2002 for further information and references).

[14] See previous footnote.

[15] One may wonder here why the new theoretical prediction is fractured and the precision not even improved. Certain portions of the calculation involving the strong interaction is vexed with technical difficulties. This has forced theorists to approximate these portions of the calculation based on experimental data, an ongoing process that is complex and not completely understood, and therefore prone to errors even outside the meticulously reported uncertainties. (For further information, see Davier 2003 and also the previous two footnotes.)

By looking at the above numbers we can see that, while the experimental precision continued to improve over the years, from about 4 ppm to 0.5 ppm, the accuracy between theory and experiment remained roughly the same, at about 2.3 ppm (excluding the case of the indirect calculation), thus contributing to the increased deviation between theory and experiment. Thus it is not necessarily the case that an agreement at an already high level of accuracy will preclude that a more precise experiment will reveal a deviation from theory. It is quite possible that a more precise measurement of the AMM of the free electron will result in a disagreement with the Standard Model.[17] In fact, the latest experimental (van Dyck 1987) and theoretical (Knecht 2003) results have increased the respective precisions from 0.03 to 0.004 ppm, or 930%, and from 0.17 to 0.021 ppm, or 830%, while increasing the accuracy between theory and experiment from 0.22 to 0.036 ppm, or 600%. Though not as large as in the case of the muon, the difference between the gain in precision and the gain in accuracy of the electron has increased the standard deviation from 1.2 to 1.7. It is suspected within the physics community that altogether these deviations may be hinting at new physics beyond the Standard Model, such as supersymmetry (Marciano 2003; Miller 2002).

We can make a similar case for another central pillar of modern physics, Einstein's theory of gravity, general relativity. Though gravitational effects are more difficult to measure, due to the relative weakness of the gravitational force compared to the other forces and the difficulty in isolating its effects from theirs, some highly precise measurements are possible. Gravity Probe B, an earth orbital space mission launched in 2004, will measure geodetic precession to a precision of 36 ppm or more (Turneaure 2003), compared to the current 0.7 percent (Williams 1996). Bertotti (2003) has improved the measurement of γ, the amount of space-time curvature, from 0.3% (LeBach 1995) to 23 ppm, and it is possible that Gravity Probe B (Weiss 2003) and GAIA (Mignard 2002), a future space mission, may

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[16] Some may feel that we should have been using these new theoretical values all along in our comparisons rather than making an historical survey. There are pros and cons to this argument, but, without digressing, we point out that if we had used the new theoretical predictions in the previous comparisons, it would have only made our case stronger since the new predictions are closer to the previous experimental values than their historical counterparts.

[17] The fact that the electron measurement needs greater precision than the muon before revealing a possible disagreement is similar to how the orbit of the outer planets needs to be measured more precisely than Mercury's before revealing a disagreement with the Newtonian theory of gravity. This indicates that parts of a domain are varyingly more sensitive to specific measurement anomalies in relation to a theory. This issue is discussed in section 2.4.4 on range.

improve the precision by another order of magnitude. Elsewhere, the planned MICROSCOPE space mission will test the weak equivalence principle to parts per 10^{15} (Touboul 2001) compared to the current 10^{12} level (Baessler 1999), and STEP, a proposed mission, aims to increase the precision to 10^{18} level (Mester 2001). Though no deviation of these values from general relativity has been detected thus far, it is certainly possible, that as we keep increasing the measurement precision, deviations will be found. Such deviations may be indicative of competing gravitational theories, quantum gravity, string theory or some revolutionary new theory (Will 2001—see also for further references on precision gravitational experiments).

Throughout science, better and better instruments are allowing for a more precise look at nature. In the field of astronomy, for example, more detailed observations of certain regions of space have recently prompted scientists to reconsider their theories of star formation (Cappellari 1999) and galaxy formation (Glazebrook 2004; Cimatti 2004). Also, continued improvement in the detail of the cosmic microwave background map, such as the results in 2003 from the new WMAP satellite, are keeping astronomers busy revising the age of the universe and when stars were first formed, and speculating on dark energy (Bennett 2003). In the field of particle physics, besides the AMM, there are many other measurements that scientists are continually working to improve the precision of, such as the extremely small mass of the neutrinos, the QCD lamb shift and the electric dipole moment of various particles, measurements that scientists speculate may lead to discoveries outside the Standard Model (see, for example, Andersen 2001, Hollik 2002, and the citations on AMM above). (The role of the latest high-energy particle colliders will be discussed in section 2.4.4 on range.) In the field of nanophysics, researches have been able to construct an electromechanical cantilever that is capable of measuring masses on the order of attograms (10^{-18} grams), a thousand-fold increase from the previous sensitivity, and are hopeful that the sensitivity can be increased another thousand-fold (Ilic 2004). Who knows what surprises such delicate measurements may reveal. Finally, and as a remarkable example of computer aided measurements, researches have developed a software controlled lens to correct the aberrations in a scanning transmission electron microscope, and thereby improved its precision twofold, to sub-angstrom levels—enough to resolve individual columns of atoms in crystals (Nellist 2004). To summarize, the highly precise experimental measurements of today do not rule out that further significant gains in precision can be made of the same measurements, nor that anomalies will be found with improved precision; they do not solidify the position of any their supported theories.

2.4.2. Perspective. The perspective is the vantage point, a position in space (or spacetime), relative to the given phenomenon. For example, we can observe the motion of the planets from earth or from an asteroid in space, or even from the event horizon of a black hole (perhaps one day). As a second example, consider the field of immunology. In studying a disease, our perspective is sometimes outside the body. By bringing a tissue or blood sample under a microscope, we can move the perspective to within the body, but still outside the infected area. Perhaps one day we can have micro-robots that can go into the infected area and make observations. The point is that different perspectives give different pictures, and may even suggest one theoretical model over another.

Moreover, from a prior perspective, what was theory for one observation, may become the observation itself from the new perspective. The heliocentric theory, for example, is a way to account for the observation of the planets as seen from earth. We can’t actually see the planets moving around the sun; it’s a theory that explains the data pretty well. But by shifting our perspective far outside the solar system, we can actually see the planets going around the sun, and our heliocentric theory now becomes an observation. This is yet another way to blur the observational/theoretical distinction. By shifting our perspective we can move between observation and theory.

2.4.3. Interaction. Another component of observation is interaction, the method by which we literally interact with nature: smell, sight, hearing, touch, and with technology, infrared, radio waves, X-rays, sonar, etc. In order to make any observation there has to be a physical interaction between the observer and the observed. The choice of physical interaction will, just as in the case of perspective, bring back a different picture of the phenomena under consideration.

For example, when we look at certain stars with a radio telescope, we can see that they pulsate radio waves, something that we would not see through a light receiving telescope of earlier years. If we use a UV, infrared, X-ray or gamma-ray telescope, we would see yet something different. Cosmic rays, composed of neutrinos, nuclei and other subatomic particles allow for forms of interaction besides electromagnetic radiation. What was once seen to be ordinary stars and galaxies are now seen to be novel and extraordinary stellar objects: pulsars, quasars, radio galaxies. By changing our method of interaction we have changed what we see, and what we have seen has in turn dramatically altered our understanding of the cosmos. More recent examples of novel astronomical interactions include the cosmic microwave background radiation (as mentioned in sub-section 2.4.1 on precision), the gravitomagnetic field, to be measured directly for the first time by Gravity Probe B, and gravitational radiation, awaiting to be detected by current and future detectors (LIGO, VIRGO, TAMA300, GEO600 and LISA). The later, once (if) detected, could afford us new visions of very massive objects, such as black holes, neutron stars and pulsars.

Outside astronomy, examples include X-ray scans, which allow us to see beneath the surface of things, and electron and atomic force microscopes, which allow us to use particles of matter to see the world with greater detail than traditional light based instruments. Additionally, various forms of matter can be used to directly interact with and detect novel particles or processes, for example: water to observe neutrinos and the yet undetected proton decay (SuperKamiokande); heavy water and heavy water with salt for more intricate neutrino observations (Sudbury Neutrino Observatory)—thus allowing for the solution of the solar neutrino problem (Ahmad 2001); germanium, silicon, sodium-iodide, xenon and calcium tungstate in the attempts to detect dark matter (DAMA, CDMS, HDMS, UKDMC, CRESST); and, finally, the various phases of different liquids and gases used in particle detectors, such as the cloud, bubble and wire chambers. Since all these interactions additionally depend on the existence and understanding of various subatomic particles, such as protons, electrons, and the W and Z bosons, of various forms of radiation such as gamma, beta and Cerenkov, and, lately, of computer technology, we can say that these observations actually employ very sophisticated interaction mediums involving many layers of matter, radiation and electronic circuitry. And as new forms of matter and radiation are discovered, they, in turn, can be used to detect newer forms.

We can also come to know the world through our touching, smelling and hearing. What sort of world does the blind person “see”? He uses his remaining senses to form images of the world around him. How do these images compare with the ones that we see with our eyes? And what more could he “see” if these other senses were enhanced by technology? To take the analogy further, imagine a planet where life evolved differently. Say the organisms there possess sensory organs radically different from ours. How does their world appear to them? How do they perceive the sun and stars? The “same” universe we live in: how does it look to them? The point is that it is not necessary that perception requires light or, even, electromagnetic radiation. Many forms of physical interaction can be used, and the form used cannot but effect the perceived observation.

2.4.4. Range. The last component of an observation we will consider is range. We can explain this best by giving some examples. Consider the motion of the planets in the gravitational field of the sun. By confining our measurements to the outer planets, we get an observation of gravitational phenomena that fits Newton’s theory. By extending our measurement to planets closer to the sun where the gravitational field is stronger, we start to find differences, for example, the precession of Mercury’s orbit. This novel observation lend support to Einstein’s theory of gravity, displacing Newton’s theory. Thus by observing gravitational effects at different field strengths, distances or masses, or, in other words, different ranges, we may find anomalies. Of course, by increasing the precision of the measurement in the original range, e.g., by observing the motion of the outer planets more precisely, we may improve our data and come to the same conclusion. Changing the range, however, will sometimes yield a result that is more obvious, though it may be more difficult to experimentally accomplish.

Although the Mercury measurement involves stronger gravitational fields than the outer planets, it is still considered a relatively weak field. For example, if g is the gravitational field on the surface of the earth, then the sun’s gravitational field within the solar system ranges from $7g$ (sun’s surface) to $0.004g$ (Mercury) to $4 \times 10^{-7}g$ (Pluto); by contrast, the field near a neutron star or black hole is on the order of 10^8 to $10^{11}g$ or greater. The limited observations of strong field effects currently support general relativity, though more observations are being planned in the future (Will 2001).

Conversely, we can ask how does gravity behave in extremely weak fields. By observing the orbits of stars at the distant perimeter of galaxies (Babock 1939; Rubin 1970) and the relative orbits and motions of galaxy clusters (Zwicky 1933; Smith 1936), where the fields are around $10^{-11}g$ or smaller, astronomers found that the motion is indicative of much more mass than indicated by the luminosity of the galaxies. Although, in a Quine-Duhem fashion, they have proposed new theoretical elements to account for these anomalies, such as the popular but yet undetected dark matter, others are toying with alternative gravitational theories that behave differently in extremely weak fields, the so called MOND (Modified Newtonian Dynamics) theories (Milgrom 1983a, 1983b, 2003; Soussa 2003).

Independently of the field strength, we can ask how does gravity behave at different distances. For example, though it has been difficult to conduct gravitational experiments at the sub-millimeter scale (see Long 1999 and Adelberger 2003 for a summary), many physicists speculate that quantum gravity and/or string theory effects may come into play (Damour 1994; Arkani-Hamed 1998; Adelberger 2003). The remarkable Eot-Wash experiments (Hoyle 2001) has managed to test the inverse square law down to 0.2 millimeters, when once the limit seemed to be more than a few millimeters. Though no deviation from theory has been found, the Eot-Wash team plans to improve to less than 0.1 millimeter. Elsewhere, Amelino-Camelia (1998, 1999) proposes testing quantum gravitational effects by making highly precise observations of astrophysical phenomena using space based gamma-ray and gravitational wave observatories. In one bit of exciting news, Nesvizhevsky (2002) and colleagues have preliminarily observed neutrons occupying discrete quantum states within the earth's gravitational field, at heights on the order of micrometers.

At the large scale, the above anomalous observations of stars and galaxies in a weak field may perhaps be due to the great distances involved, 10^9 to 10^{11} AU, where 1 AU is the distance from the sun to the earth, and 40 AU from the sun to Pluto. And at larger scales, recent observations of the velocities of distant (10^{14} AU) supernova (Perlmutter 1998) and of thousands of galaxies spread out over 10^{13} AU (Zehavi 1999) tentatively indicate that the expansion of the universe is surprisingly accelerating. Again, some theorists propose new theoretical elements, such as dark energy or quintessence, while others propose modifications to general relativity (Deffayet 2002; Moffat 2004; Caldwell 2004). (For further references on large scale gravitational experiments, see Fischbach 1999 and Adelberger 2003.)

We briefly give some additional examples of novel observations (or the possibility thereof) via range change, which may be more thoroughly followed up in the accompanying citations. Like gravity, the other fundamental interactions can also be measured at different ranges. For example, although QED is considered a highly accurate theory of the electromagnetic interaction, it has yet to be precisely scrutinized under strong electrical fields. For a survey of current and future strong field electromagnetic tests, including a strong field test of the electron’s AMM and binding energies (QED Lamb shift), see Anderson (2001). Similarly, although quantum physics is considered a highly accurate theory, it has yet to be tested over large scales of distance, time, mass, etc.; see Leggett (2002) for a survey. In the field of particle physics, experimentalists are always trying to build higher energy particle colliders so that they can look for new particles and clues on unification. Current colliders are already producing novel particles that challenge the Standard Model (Choi 2003; Evdokimov 2004; Chen 2004). The Large Hadron Collider at CERN, to come on line in 2007, will reach energy levels of 14,000 GeV, the highest yet, but still a vast distance from the Planck energy of 10^{19} GeV, where it is believed quantum mechanics and gravity merge. What novel discoveries await us on this long road, and then there after? As another example, consider the difference in the behavior of rods and clocks at small velocities and large. By observing matter at sufficiently large velocity ranges, we find that clocks run slower and objects contract. This difference supports the special theory of relativity over Newtonian mechanics. Other examples include an observation of a decrease in entropy in smaller systems over shorter time scales (Wang 2002)—a recently predicted violation of the second law of thermodynamics (Evans 1993); a reanalysis of the star formation time line by observing lighter mass galaxies (Heavens 2004); and anomalous observations of the cosmic microwave background radiation at larger spatial angular separations (Bennett 2003; Spergel 2003; Contaldi 2003), possibly supporting different geometric models of the universe (Efstathiou 2003; Luminet 2003). Generally speaking, for any physically observable quantity, such as mass, momentum, energy, distance, temperature, gravitational force, electromagnetic force, etc., we can conduct our experiments in wider and wider ranges as time and technology advance. By increasing the range it is always possible that we may get results that were not evident or obvious in previously observed ranges.

Precision, perspective, interaction and range: we have identified four ways in which an observation can vary. By modifying these components, we can always have novel and, perhaps, anomalous experiences of nature. This is a principal result, because it allows us to equate the notion of incompleteness in scientific theories with the notion of incompleteness in natural experience. We could not have one with out the other. That is, the novel experience problem is the experiential manifestation of theoretical incompleteness. This identity emphasizes the intimate connection between theory and experience, and at the same time, exposes the paramount significance of their inseparability.

2.5. Quantification of the Philosophy: an Information Metaphor. We can now paint a clearer picture of the role theory and experience play in the scientific enterprise. The interaction between nature (or the "outside world") and our sensorial-cognitive system results in the production of a flux of information. Our understanding of this flux as experiences, and on a more broader scale, as theories which integrate many different experiences, is an attempt at finding patterns within the flux. That is, science can be viewed as a problem of pattern recognition within a given information space of sensorial-cognitive flux. The incompleteness principle implies that such patterns can never fully represent all the information within this space. This is because the space has an infinite amount of detail.[18] At a specified level of detail, we may be able to find a pattern that most closely approximates the information. But if we zoom in to a higher level of detail, there is no guarantee that the found pattern will hold. Each higher level of detail arises in a non-algorithmic fashion.

2.5.1. Science as Pattern Recognition. We can make an analogy here using digital images. Imagine we take a digital photograph of some natural scenery. We can shoot the scene at various pixel resolutions. Someone looking at a low resolution image of the scene, who has never seen the scene with her naked eye, may perhaps interpret the image incorrectly. She may identify certain patterns of pixels that may not exist at a higher resolution. Moreover, the image will appear to represent many possible scenes. The lower the image resolution, the more erroneous of a guess is she likely to make and the more guesses is she likely to entertain. As the resolution of the image approaches closer to that of the human eye, she is

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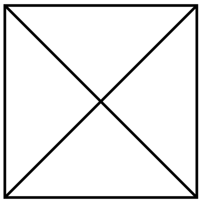
[18] Though it may be difficult to appreciate, mathematicians have learned that infinity comes in many sizes and that they do not encompass all there is in the mathematical world of objects. At present we are unable to identify the size of the infinity that we are dealing with; therefore, we will say it is at least the smallest infinity, $\aleph_0$, the size of the set of all natural numbers. (Though there is some reason to believe that it may be at least the next largest infinity, $\aleph_1$, the size of the set of all real numbers, a point to be explicated at a later date.)

able to narrow the possibilities and better identify the correct scene. In our case, stepping back away from the analogy, none of us have ever seen the “real” scene of natural experience. We are always looking at a resolution that is infinitely far below the “real” scene. We must take our best guess based on this resolution until technology allows us to look at the next higher resolution. Recall the example above where the data on planetary motion was described using mathematical models. At each higher level of detail, we found novel models arising in a non-predictable fashion.

Let us make this explication a little more precise. Consider an image which is being viewed at its full resolution, i.e., we know the color of every pixel in the image. To streamline our argument, let us simplify the image to a black and white one and consider the representation of geometric patterns instead of natural scenery. Even though we are now viewing a full resolution image, we may still give multiple correct descriptions of the image. We can say, for example, as a square with two diagonals drawn from pairs of opposite corners, or as two isosceles triangles, each perpendicularly bisected, or as four smaller isosceles triangles (see diagram). All descriptions, though in a certain sense being different, can be considered to be mathematically equivalent (isomorphic). Another way to see this is to write out the image as a binary number. For example, say our image turns out to be the binary number 1110. Translating into decimal notation, we get 15. We can express 15 in many ways: 3×5, $10 + 5$, $17 - 2$, etc. All these different expressions imply the same number. What we are getting at is that, given a full resolution image in which all the colors of the pixels are known, then there is only one isomorphic class of descriptions for that image. If, on the other hand, we are uncertain about the color of certain pixels—the ones that only come into view at higher resolutions—then there will be more than one isomorphic class of descriptions. For example, after translating an image into binary, we get 11?01?10, where “?” represents our ignorance of the pixel value, then this image has the potential to represent four different ones, corresponding to the four different binary numbers that could result after



filling in for the missing question marks.

If we now interpret the images as empirical data and the descriptions of the images as scientific theories, we get the following theorem:

> *UDT Requirement Theorem.* The number of isomorphically different theories on a given domain of phenomena is proportional to the amount of incomplete empirical data on that domain. Specifically, if the empirical data on that domain could be complete, then there would be only one isomorphic class of scientific theories for that domain. Moreover, all the theories in this class would be complete.[19]

This theorem is simply a more explicit rendition of the implication between the underdetermination of theory and the novel experience problem; it defines the applicability of UDT. UDT asserts the existence of non-isomorphic theories for a given domain of phenomena. By the above theorem, this is only possible if the empirical data is not completely known. This insight clarifies a puzzling aspect of UDT: how two or more mutually inconsistent theories can possibly describe the same empirical data (for example, see Bird 2003; Sankey 1997). It is only so because the data itself is inherently fuzzy, just as a low resolution image appears to represent distinctly different possibilities. The inherent incompleteness of empirical data will always allow for the existence of multiple theories.

One may wonder why we have put forth a theorem in which the incompleteness of empirical data can vary or even be zero after earlier asserting that empirical data is infinitely incomplete. We do stand by our earlier assertion, but this theorem helps us to gain a clearer understanding of the relationship between incompleteness and underdetermination. It also explains why, at times, scientists are willing to entertain multiple theories and, at other times, not, assuming that even all the technical problems have been resolved in these theories. Even

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[19] At this time, we are unable to offer a more precise relation of the proportionality. This is due to a lack of mathematically formal definitions for the concepts of empirical data and scientific theory. For example, by information theory, it may appear immediately obvious that the number of isomorphically different theories is proportional to n^2, where n is the number of incomplete bits of empirical data. However, the necessity to incorporate theoretical language when describing empirical data may preclude all the different n-bit combinations from only specifying purely data, and thus the proportionality may be less than n^2.

An example of isomorphically similar theories on the same empirical domain are the Schrodinger (wave) and Heisenberg (matrix) formulations of quantum mechanics.

though, by the principle of experiential incompleteness, empirical data is infinitely incomplete, at times, the amount of *perceived* incompleteness may vary. One reason for this variation is the cyclic behavior of scientific research as elucidated by Kuhn (1970). During revolutionary periods, it appears that there are holes in our empirical knowledge and that we are continually trying to fill them with discoveries, whereas during the stable periods, there is mostly supporting data being gathered. If scientists believe their empirical data is hardly lacking, then any claims of underdetermination would only fall on deaf ears; whereas, if the empirical knowledge of some domain is scant, they are willing to entertain many a possibility. The lesser the empirical knowledge, the greater the extent of underdetermination.

We can produce a similar theorem for UDO. But first we need to define the notion of "isomorphically different observations on a given domain of phenomena." We treat the totality of all observations on a given domain as a singular observation. For example, if there were ten different observables on a given domain, then each set of possible outcomes for all ten observables represents a singular observation and one that is different from another set. By isomorphically different, we mean that no two sets can be reduced to the same set; they are mutually incompatible (see the explanation a few paragraphs back regarding isomorphic different descriptions of digital images).

> *UDO Requirement Theorem.* The number of isomorphically different (mutually incompatible) observations that a theory on a given domain of phenomena can accommodate is proportional to the amount of the theory's incompleteness. Specifically, if the theory could be complete, then there would be only one isomorphic class of observations that can be accommodated. Moreover, all the observations in this class would be complete.

This theorem more clearly quantifies the idea expressed in section 2.2.3, concerning the relationship between UDO and theoretical incompleteness. There, we argued that UDO critically depends on theoretical incompleteness. This theorem quantifies that dependence. Also, just as in the case of experiential incompleteness, theoretical incompleteness is infinite, and therefore, as asserted by the Quine-Duhem thesis, a theory can accommodate any amount of seemingly incompatible observations; however, similarly to experiential incompleteness, the perceived amount of theoretical incompleteness can vary too.

The gist of the UDT and UDO requirement theorems is this: the extent of theoretical underdetermination by empirical observations and the flexibility of theories in accommodating novel empirical observations is dependent on the extent of the observational and theoretical incompleteness, respectively. UDT and UDO, and thus the Quine-Duhem thesis, holds no epistemological weight without observational or theoretical incompleteness.

2.5.1.1. Additional Theorems. One may wonder why we have not quantified the relationship between UDT and *theoretical* incompleteness, a relationship that was established in section 2.2.2, or much less, even mentioned the possibility of the same between UDO and *observational* incompleteness. The primary reason is that UDT and UDO assert underdetermination relative to observation and theory, respectively, and therefore it is their respective incompleteness that are relevant. Second, when it comes to theory, we may conjure up any theory we wish without regard to observation. In such cases, the amount of theoretical and empirical incompleteness may not match. Let us illustrate. Consider we have empirical data on the phenomena of gravity such that it is undetermined whether Einstein's theory or a modified Newtonian theory is correct, but that some other simpler theory *X* is ruled out. Someone may, nonetheless, attest that *X* is a theory of gravity. Thus if we were to assert in the UDT requirement theorem that the extent of theoretical underdetermination is proportional to the incompleteness of some theory on that domain, we would be wrong, for *X* is far more incomplete relative to the data than Einstein's or Newton's theory, and thus would accommodate greater theoretical underdetermination. We face a similar problem in the UDO requirement theorem: theory *X* can accommodate more observational possibilities than what is allowed by the present amount of incompleteness in the empirical data.

This situation can be remedied by requiring all theories to pay proper heed to the empirical data. Thus, in the case of theory *X*, we could add enough modifications to it—as we did for Newton's theory—so that it is a viable alternative.[20] The result of this requirement is that all the theories on a given domain of phenomena share the same amount of theoretical incompleteness. With this, we can then introduce the following theorem:

> *Theoretical and Empirical Incompleteness Equivalence Theorem.* Provided all theories on a given domain of empirical phenomena are viable, the amount of theoretical incompleteness is proportionally equivalent to the amount of empirical incompleteness.[21]

With this theorem, we can indeed extend the UDT and UDO requirement theorems:

> *UDT Requirement Theorem Addendum.* The number of isomorphically different theories on a given domain of phenomena is proportional to the amount of incompleteness of any of these possible theories. Specifically, if one of these theories could be complete, then its isomorphic class would be the only class of theories for that domain.

> *UDO Requirement Theorem Addendum.* The number of isomorphically different (mutually incompatible) observations that a theory on a given domain of phenomena can accommodate is proportional to the amount of incomplete empirical data on that domain. Specifically, if the empirical data on that domain could be complete, then there would be only one isomorphic class of observations that can be accommodated.

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[20] It may be confusing here as to why we are claiming that theories have to adhere to observations, when we have earlier argued that, by the Quine-Duhem thesis, a theory can accommodate any observation. The subtlety lies in that the Quine-Duhem thesis does not allow theories carte blanche accommodation to observations, but only if they undergo appropriate modifications. Thus any theory, even a seemingly silly one, can be accommodated to observations if the modifications are sufficiently strong. However, if the present amount of perceived incompleteness in the empirical data is small, then all the present theories, including the silly ones, will, after modifications, converge to a limited number of isomorphic classes. But because the empirical data can never be complete, it is not possible to determine the final isomorphic class. Thus it could turn out that some theory which has been greatly modified to satisfy the current level of empirical detail may still be a good candidate in a form closer to its original incarnation, given a greater level of empirical detail in the future—for example, closer examination of subatomic phenomena may indicate that space is discrete, as quantum gravity may suggest, which in turn may imply that Lorentz symmetry is violated, returning us to a pre-relativity theoretical framework (Smolin 2003 and references therein). It is in this stronger sense that the Quine-Duhem thesis asserts that any theory is compatible with any observation. To sum up, while we can’t just have any arbitrary theory, but must suit it to fit the current level of observational detail, it is always possible, because of the inexhaustible incompleteness of observations, that the original arbitrary theory may better fit the more detailed observations of the future than the currently accepted isomorphic class(es) of theories.

[21] There may be an yet undetermined proportionality constant relating these two values. See also two footnotes back.

If we assume the incompleteness equivalence theorem to hold, then by what we said in section 2.5.1, we can also assert the following obvious theorem:

> *UDT and UDO Equivalence Theorem.* Given proportionally equivalent amounts of theoretical and empirical incompleteness on a given domain of phenomena, the number of isomorphically different theories equals the number of isomorphically different observations.

2.5.2. Science as Data Compression. The science as information metaphor has more to teach us. Following the lead of Solomonoff (1964) and Chaitin (1970, 48-49), we now consider the idea of science as data compression. By recognizing patterns, such as a circle or ellipse, within a digital image, we are finding a compact way to express what would otherwise be just a long description of the colors and positions of each pixel. Similarly, by recognizing patterns in long strings of binary numbers, such as 010101010..., we can give more condense descriptions. Compression has become a measure of recognition or description quality: the best descriptions are the smallest ones.

In science then, experiential and theoretical descriptions are attempts to compress the vast amount of information arising in our sensorial-cognitive system, again the best descriptions being the smallest. For example, it is far more concise to give the initial position and velocity of a planet and Newton’s laws than to give an arbitrary large number of position and velocity values themselves. Similarly, it is far more concise to give the theory of the atom than to describe the behavior of the many chemical elements individually. All descriptions in science, from geometric models to equations of force, and from sub-atomic structures to biological structures, serve to compress large amounts of observational data into smaller, finite patterns. The adoption of computers in theoretical modeling further supports this metaphor: theories become programs and their outputs become observational predictions.

The data compression analogy allows us to reframe the question of what is the best theory on a given domain of empirical phenomena as what is the smallest compression that can be achieved on the corresponding empirical data set. Or, in other words, what is the absolute informational content of the empirical data set? By the novel experience problem, it is not possible to determine this. For assume we find some smallest compression algorithm (theory) to describe a series of binary digits (some finite data set). For example, if the binary digits are 001100111111001100111111, then the theory would be repeat 001100111111 two times. But upon probing nature deeper we find that the data set has grown to 0101010101010101010101010101010101010101. Even though the data set is larger, we can find a shorter description: repeat 01 twenty times. Unification examples from the history of science aptly illustrate this point. The informational content of Maxwell's electromagnetic theory is much less than the sum of its disparate predecessors: the electric, magnetic and light theories. The same can be said of Einstein's theory of gravity versus the disparate theories of gravity and motion of Newton's. In these cases, further details have emerged in our empirical investigations that allow us to simplify our descriptions, but it is not necessary that simplification is the rule. Generally, as the empirical data grows, it is not possible to determine the size of resulting theoretical models.

The above result can also be parlayed in terms of communication theory. We can view science as trying to accurately communicate our experiences of nature. Incompleteness then implies that there is no ideal communication code in which to convey these experiences. In fact, it is a well known theorem in communication theory that there is no absolute measure of information. Information can only be measured relative to some model. Depending on the model, the information content of a string of "*n* bits" can have a size from zero bits to any arbitrary number of bits larger than *n*.[22] Similarly, in science, we pick (knowingly or not) some model, interpretation or ambient theory in order to communicate our "raw experiences." This point has been recognized in Suppes (1962) and McAllister's (1997, 2003) idea of data models and data patterns, respectively. The insight to be gained here is that empirical data comes in multiple interpretations only because it is incomplete, the point made by the UDT requirement theorem.

2.5.3. The Entropy of Physical Systems. As with the information measure of empirical data, the novel experience problem also implies that the entropy of physical systems is not an absolute quantity either. The recognition of this point is to be credited to Weintstein (2003).

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[22] This result, like its scientific counterpart, is only possible because the information potential is infinite. If, in communication theory, we limit our message source to some finite size, then it is possible to put a cap on the information size of any particular message, even though we are unable to determine the exact size of the message. To do the latter, we need to, of course, pick some preferred model.

Although Wienstein does not explicitly state the novel experience problem or theoretical incompleteness, he argues that the entropy of a physical system depends on the description of the system, of which there can be no fundamental one:

> In conclusion, then, the entropy depends not only on the substance, but on the description of the substance, and there is no most-fundamental description. Which description is appropriate depends on which variables one is able to control, or interested in controlling. As is the case with algorithmic complexity and relative velocity, entropy can be contextualized, and entropy *relative to a description* can be considered an objective property of a physical system. But there is no most-fundamental description, just as there is no most-fundamental Turing machine or inertial reference frame. (1252, italics in original)

Again, as with the notion of data models, Weinstein's notion of contextualized entropy is only possible because the empirical measurements of a physical system can never be complete. If they were complete, then the entropy is absolute. Indeed, Weinstein recognizes this point:

> But the problem is that one can introduce further state variables as well, not only by discovering new sorts of properties, but by *manipulating the same properties in finer detail*...This process is limited only by the fact that, at extremely small scales, the system ceases to display equilibrium behavior. (1252, italics in original)

It is obvious from this passage that Weinstein implicitly assumes the novel experience problem to hold. Though, it is unclear whether he goes so far as to subscribe to the principle of theoretical incompleteness.

Also, as Weinstein points out above, it is unclear whether the notion of entropy continues to hold at extremely small scales due to thermal or quantum fluctuations. We also note that the Bekenstein (1974, 1981) bound, which limits the maximum allowable entropy per given volume of space, conflicts with the notion that the entropy of a system can be arbitrarily large. Presently, it is unclear how to resolve these issues. Perhaps a new understanding of entropy and information, say, founded on quantum informational approaches, may help (for example, see Zeilinger 1998, 1999; von Baeyer 2003).

2.5.4. Can a Computer Simulation Ever Represent Reality? By extending the digital images analogy we can examine an interesting question in ontology. For consider, instead of a 2D image, a 3D virtual reality (VR) computer simulation of our experience. This is identical in principle to a two dimensional digital photograph, only we are expanding the visual matrix into the full three dimensions (like a hologram), and adding digital "photographs" of the remaining human senses: hearing, touch, smell, and taste. The addition of physical (causal) laws into the program allow us to add the dimension of time into the VR world, making it dynamic instead of static.

Now, if we were immersed into such a virtual reality world without knowing it, we may be hard pressed to determine if the world is real or a simulation (a phony). The better the simulation, the harder it will be to make that determination. On the one hand, someone (a prankster) is trying to simulate observations, and, on the other, we are trying to determine whether they are valid.

This raises the classic question of whether this world, which we are all commonly experiencing, is a computer simulation or real (natural). Theoretically, there is a way to determine this. By the novel experience problem, we can interact with the real world on a ever deeper and deeper level, and thus always improve the resolution of our experiences. In any computer simulation, we will eventually hit a brick wall, or limit of interaction, because there is only a finite amount of data that was programmed into the simulation. That is, we will realize the simulation and probe past it into the real world. This contrast highlights a critical point that was just recently raised in our arguments: the natural world (as experienced through our sensorial-cognitive system) is infinite, while representations of it—theories, descriptions, computer simulations, etc.—are finite. It also gives us another angle on the incompleteness of scientific theories: theories are incomplete because they are always finite while nature is infinite.

This finiteness explains why we may have a hard time to determine valid observations. At any point in time, all our descriptions of observations are finite, however large or small, and thus the observations can be mimicked, given enough artistry or technology. Though, as we said, the real world is infinite, so if we probe the simulation deeply enough, we should reveal the illusion. But keep in mind, that at this point, we are also probing past the limits of our actual knowledge of the observation, and that if we were probing the real observation, we would be collecting new data and thus adding to our original description. If we interact with the phony observation within the finite range of its original description, i.e., in a normal, every day capacity, we will never notice that it is phony. Moreover, recalling the theoretical caveat above, the description, though being finite, may be arbitrary large, and thus we may have to probe the observation for an arbitrary long time to determine whether it is real or a simulation. Therefore, while we may be able to determine whether simulations that we, ourselves, create are phony—because we know the limits of our own knowledge and thus how far to probe, we may not be able to do so if an outside agent whose knowledge is greater than ours creates them. Thus to answer the question titling this sub-section, reality cannot be a computer simulation, but we can't ever prove that our reality so far experienced is not a simulation.

Some critics may counter that there are finite mathematical representations that, nonetheless, have an infinite capacity and thus may be used to describe the infinite depth of nature. Such examples may be fractals, cellular automata, or even numbers like pi and the square root of two. Let's consider cellular automata, as they are a more stronger example in favor of such arguments. Cellular automata are given by finite rules, yet calculating according to these rules, we can get an infinitude of unpredictable patterns. It seems that since we have an infinitude of patterns, we may be able to use such mathematical entities to give a complete description for certain physical theories, or provide an uninterrupted depth of interaction in a VR simulation. This is a subtle point. In mathematics, one learns that infinity does not necessarily mean everything. In spite of these mathematical entities being able to generate an infinitude of patterns, there are certain patterns that they cannot generate. More precisely, given a complex enough cellular automata, it will be undecidable whether certain patterns appear or not (Wolfram 2002, 755). (This point will be examined in depth in part three.) This undecidability will manifest itself as a limit of interaction in a VR program that employs cellular automata type rules. The only way to settle the question is to augment the rules of the cellular automata (increase the size of the VR program); but, then there will always be another undecidable pattern. In this way, the cellular automata is essentially incomplete in addressing the world of its patterns. So while a cellular automata may provide an infinite amount of description or interaction for a physical theory or VR program, respectively, it cannot provide a complete one. The bottom line is that all computer simulations, being necessarily given by a finite program, will have a limit of interaction.

Part Three: Mathematical Connection

3.1. Introduction. Our brief examination of cellular automata above glosses over some of the finer points of a milestone in mathematics: Gödel’s Incompleteness Theorem (1931). By examining these finer points, we can place our philosophical theory within the more familiar context of mathematical incompleteness and undecidability, and give it a formal dimension that has thus far been lacking. We will keep the discussion to a minimal technical level.

Gödel proved that the simple mathematical system of the arithmetic of natural numbers—the addition and multiplication of zero and the positive whole numbers—cannot ever be given a finite description. That is, no matter how sophisticated a system of axioms one comes up with, as long as it is given as a finite system, there will always be a statement pertaining to arithmetic that will be undecidable by the system. Immediately we see an analogy between Gödel’s theorem and the idea of incompleteness in scientific theories. This analogy is not a coincidence, but represents a strong connection between these two ideas. In what follows, we will show that the mechanisms that allow for scientific incompleteness finds a natural framework in the incompleteness theorems of mathematics.

3.2. Gödel’s Incompleteness Theorem and Computational Irreducibility. To begin, we need to find out why is it that such a seemingly simple system as the arithmetic of natural numbers cannot be finitely described. Despite remaining somewhat obscure and underappreciated after its initial shock, since his time, Gödel’s theorem has sporadically sparked further work in mathematical and related fields that has helped improve our understanding of the phenomena of incompleteness. For instance, it is now known that there are other systems that exhibit similar incompleteness properties. Shortly after Gödel, Turing (1936) showed that it is not possible to write a computer program that can decide whether all programs will ever halt or not (stop and produce some output or continue running forever). There will always be computer programs whose halting status is undecidable. Another example, due to Wolfram, includes certain class of cellular automata, as was mentioned at the end of part two. In more recent times, Chaitin (1971, 1975) has shown, via his algorithmic information theory, that given any algorithm or computer program, there will always exist numbers for which it is undecidable whether they are random or can be reduced to some pattern. Similarly, he has shown that the complexity of computer programs is an undecidable issue (1974, 2000).

From the study of these additional phenomena, researchers have been able to glean a common principle. We can state it briefly by echoing the words of Wolfram (2002, 788): there exists certain infinite processes that are computationally irreducible. Such processes occur in arithmetic, cellular automata and computers, among other systems. As a simple example, consider a finite two dimensional black and white matrix. Some matrices can be described without specifying the color of every cell in the matrix, such as a checkerboard pattern; but, if the matrix is completely random, then we have no choice but to specify the color of every cell. In the latter case, we say that the matrix has no pattern or is computationally irreducible. Now imagine that the matrix has an infinite number of cells. Like in the finite case, there exist infinite patterns—patterns that as you keep zooming in, repeat identically or in some sort of algorithmic or recursive fashion. However, if the matrix is infinite and random (more precisely if it has an infinite random portion, an infinite number of random finite portions, or an infinite number of patterned finite portions that vary randomly), then no amount of finite information is sufficient to describe it, and, in this case, we have non-finite computational irreducibility. Continuing this analogy, we can say that mathematicians knew they were dealing with an infinite matrix because the natural numbers go on forever, but it was not until Gödel that they realized that the infinite matrix could not be reduced to some finite set of patterns (finite set of axioms).

As a more robust example of computational irreducibility, let’s consider a direct example from the files of arithmetic. Specifically, we consider a class of arithmetical equations known as diophantine equations. (For the most part, these are just normal polynomial equations set equal to zero.) The equivalent of Gödel’s theorem here is that there exist diophantine equations for which it will be undecidable whether they have solutions or not. How is this possible?

Let’s think about what it means for us to say that it is undecidable whether or not a particular equation has a solution. This immediately implies that the equation in fact has no solution, but that we can’t prove it. (For if it did have a solution, then it wouldn’t be undecidable whether it has one or not.) To decipher the mystery we must focus our attention on why we can’t prove that it doesn’t have a solution. To this end, consider equations that don’t have solutions and for which we *can* prove it. Say $2x - 7 = 0$. No matter how many natural numbers we try, we will never find a solution to this equation; moreover, we can prove it. In attempting to solve it, we can simplify the equation to $2x = 7$. Seven is odd and therefore not divisible by two. Thus we can prove that the equation has no solution. Not all such proofs of non-solvability are this simple. Consider the famous case of Fermat’s last theorem, regarding the equation $x^n + y^n - z^n = 0$. Alas, Wiles (1995), building upon the work of many others, proved that any variation of the equation where n can be greater than two has no non-zero solution for the variables x, y and z. The proof was immensely long, taking hundreds of pages.

Both the above proofs are making assertions about the natural numbers: that none of that infinite set of numbers have a solution in the respective equations. These proofs or theorems of non-solvability serve to compress certain information about the infinite set of natural numbers into a finite form. Theorems of non-solvability have no choice but to do this, as they have to address all the infinite natural numbers, as all are candidates for a solution. The fact that we can’t find a non-solvability proof for the undecidable diophantine equation means that we are unable to find any such finite compression. The equation in question corresponds to some infinite process within the arithmetic of natural numbers which is not finitely compressible using the rules of the given arithmetical system. We can expand the rules to help us decide on this equation, but then Gödel’s theorem tells us that there will always be another undecidable diophantine equation. In fact, there are an infinite number of undecidable diophantine equations. We therefore see that the ultimate cause of Gödel incompleteness is that there exist infinite processes within the arithmetic of natural numbers that are simply not reducible to a finite description, no matter how large. This is the same reason why undecidable phenomena occur within cellular automata and computer programs. Any rules or descriptions we come up with to explain any of these systems will always be finite, but as there are computationally irreducible infinite processes going on within these systems, our finite descriptions of them will forever remain incomplete.

Chaitin's algorithmic information theory provides another take on the problem. Any theory or description contains a finite amount of information as is determined by its axioms. If put to the theory a question which contains more information than the theory—which is always possible in systems that contain a computationally irreducible amount of information—then it will be undecidable what the answer will be (Chaitin 1974). The only way to resolve the undecidability is to enlarge the axiom base and thus the information content of the theory. In this vein, we can say, for instance, that undecidable diophantine equations contain more information than the axioms of the given arithmetical theory, or that computer programs whose halting status is undecidable contain more information than the given computer program that was written to decide whether programs halt or not.

Gödel's incompleteness theorem, or its modern form, computational irreducibility, provides us with a natural mathematical framework for scientific incompleteness. We suggest that the interaction between nature and our sensorial-cognitive system involve computationally irreducible infinite processes that prevent us from ever achieving complete descriptions of any sentiently experienced phenomena. This fits in with the picture we painted earlier of science as a problem of pattern recognition within a given information space of sensorial-cognitive flux. As this space contains a irreducibly infinite amount of information, no amount of finite patterns can serve to completely describe it. This is also why no mathematical model or computer program, all of which are finite, can ever be used to provide a complete scientific theory or VR simulation.

At this point we must pause and say that we do not know how computationally irreducible processes arise within the sensorial-cognitive system. It is a future problem for neural and cognitive science. What we want to recognize here is the plausible case for the existence of such a process and what its consequences are for scientific understanding.

3.3. Requirements for Incomplete Systems. Let us continue our mathematical examination of scientific incompleteness and see how additional insights on the phenomena of mathematical incompleteness help us to better understand the scientific counterpart. To begin with, we can ask what is it about all these different systems that allow them to be incomplete and what is it that distinguishes them from other systems that are complete? It is clear from the previous discussion that incomplete systems are computationally irreducible, so perhaps it is more accurate to ask what gives rise to the latter. Both Wolfram (2002) and Hofstadter (1979), among others, have addressed this issue. At present, we can identify two main causes: (1) infinity and (2) a minimal level of complexity in the rules of the system.

3.3.1. Infinity. The first cause is almost obvious. Any finite system can always be finitely described, even if there is no order or pattern to it—though the description may be quite lengthy. Consider the system of arithmetic. If we limit the natural numbers to a maximum, say one million, then we can definitely prove whether all diophantine equations have a solution or not: we try all numbers from one to a million and see if we find a solution. Similarly, for computer programs, if we limit the memory of the computer to a maximum size, we can determine the halting status of all programs requiring memory up to that maximum size. Wolfram (2002, 255-260) has shown that no matter how sophisticated a cellular automata one has, if the width of the pattern is bounded to a maximum size, the automata will eventually repeat itself as it evolves along its length.

3.3.2. Minimal Complexity. But infinity is not sufficient to guarantee incompleteness. For instance, it has been proven that the arithmetic of natural numbers which excludes either addition (Skolem arithmetic) or multiplication (Presburger arithmetic) is complete (Skolem 1930; Presburger [1929] 1991). Wolfram (2002, 51-58, 231-235) has demonstrated that unbounded cellular automata with simple rules exhibit repeatable or algorithmically predictable behavior. In order for incompleteness to exist, the system should also have a certain level of complexity. For arithmetic, we can identify what is involved in achieving this level: it should be able to represent, what is known in mathematical terms as all the general recursive functions.[23] This basically amounts to the ability to add and multiply two numbers and compare two numbers to see if they are equal or if one is larger or smaller than the other. For computers, it involves the requirements as specified by Turing (1936), or equivalently, the capability to perform the Boolean-logical operations of AND, OR and NOT. Wolfram

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[23] Technically, unless arithmetic is infinite, it can't represent *all* the general recursive functions. Thus some might argue that the infinity requirement is part of the complexity requirement.

has thus far been unable to identify what exactly is needed for cellular automata to achieve incompleteness. Moreover, it is presently unclear what is the common mechanism that underlies the complexity of all these different systems.

There is strong reason to believe that there is such a common mechanism. For we know that some of these complex systems can, at least partially, simulate other such systems. A modern day example is computers representing other computers (the so called operating system emulations). Going further, the Church-Turing thesis stands unrefuted in its assertion that all computable arithmetical functions can be computed on any computer meeting the minimum requirements as stated above (Church 1936; Turing 1936). The reverse is seemingly true also. Since all computer program inputs and corresponding outputs can be viewed as natural numbers (in binary form), it is easy to see that all programs can be represented as an arithmetical function which maps one set of natural numbers onto another. The MRDP theorem of recursion theory, which establishes the equivalence of certain computable relations (recursively enumerable relations) to diophantine equations, also lends weight to these arguments (Machover 1996, 208). As an actual example, Chaitin constructed a diophantine equation 900,000 characters long with 17,000 variables which corresponds to a general purpose LISP program (Chaitin 1990, 17). Turning to other systems, in a remarkable series of examples, Wolfram (2002, 644-673) has demonstrated a variety of different systems simulating others, including cellular automata simulating other cellular automata and, even, computer and arithmetical operations. Armed with these examples, Wolfram claims that there is a certain sense of equivalency or universality in all computational systems that meet a minimal level of complexity. Whether this is the case or not, it is generally accepted that many a formal system with symbols and rules for the manipulation of those symbols can be represented on a Turing machine or within the arithmetic of natural numbers. This will turn out to have implications in our upcoming considerations on self-reference in formal systems.

3.3.3. Infinity and Complexity in the Human System. So while we cannot identify the common mechanism of complexity, what is clear is that infinity and complexity are involved in incomplete systems. There is then reason to suppose that they are involved in the processes of human experience and thought. From our discussion of the novel experience problem, we can see how infinity plays a role in experience: we can always improve the precision of our experiments. Similarly, for our theories, we can always ask what is behind any theoretical construct. Thus there is an unboundedness in our ability to probe both our experiences and thoughts.

How complexity plays a role is more difficult to address, partly because we do not yet understand the common mechanism underlying all the different complex systems. We can best surmise that (1) there are sufficiently complex processes going on within the human (and perhaps animal) sensorial-cognitive system as it interacts with its environment and (2) that some capability equivalent to, say, generating general recursive functions, must exist in any language strong enough to describe all our experiences. Simpler languages may exist that can completely describe some aspects of our experience, but such a language won't be strong enough to describe all aspects of it, just like the Presburger or Skolem formalism (see section 3.3.2) can describe some aspects of arithmetic completely, but can't describe all aspects.

3.4. Arbitrariness in Theories. When in incomplete mathematical theories we face undecidable statements, we said the resolution lied in expanding the axiom base or information content of the theories. What we failed to mention is how we go about such an addition. Do we make the undecidable statement an axiom of the theory, or do we adopt its negation or some other variation? What algorithms or formal guidelines do we use to make such a decision? The simple answer is that, since the statement is undecidable, there are no such guidelines. It is entirely arbitrary. We can add the statement, its negation or some other variation. The theory has nothing to say about the statement. It's the same way with the original axioms of the theory; the axioms are themselves undecidable statements. When we construct a theory, we start off by assuming a certain number of statements as true without proof, i.e. taken on faith, guts, intuition or on the basis of some other non-formalizable notion. Another way to see this is in terms of Chaitin's algorithmic information theory. There randomness is defined in terms of incompressibility: something is random if it can't be compressed into a shorter description (Chaitin 1970). The axioms of a theory serve to compress the larger information content of all its theorems into a smaller size (Chaitin 1975). That is, we can find common patterns among all the different theorems. However, since we can't compress the axioms, themselves, any further, we can find no common patterns among them; as Chaitin (1975) says, they are random.

3.4.1. The Role of Truth. Such arbitrariness in the expansion of a theory or in the selection of its axioms prompts one to question the role truth plays in theories. Before Gödel's theorem, the assumption was that there was some finite amount of mathematical truth awaiting to find an axiomatization. Gödel's theorem, of course, proved that such a finite axiomatization was not possible; but, in order to preserve the notion of truth, many have adopted a mindset that there is a permanent edifice of truth that, although could not be know entirely, could be revealed bit by bit. Such a Platonic outlook arose out of a misinterpretation of Gödel's theorem usually written thus: "Gödel's theorem states that there are true statements of arithmetic that are unprovable." Therefore, when we stumble across such statements, we should adopt them as axioms, for Gödel's theorem tells us they are true.

It is easy to see how such a misinterpretation may arise. If we hark back to the example of the undecidable diophantine equation, we saw there that the diophantine equation in fact has no solution. Isn't it therefore correct to adopt an axiom saying that the equation has no solution? This is fine, but as Gödel's theorem tells us that the statement is unprovable within the original theory, we know that its negation, instead, can also be added to the original axioms to create a consistent new theory. How is this possible? The confusion arises out of our assumption that we know all about the things the theory talks about, when in fact, we only know what the theory tells us. It is the Platonic idea that somehow we have an intimate and direct access to the world of arithmetic, and that we are just trying to find a description in the outside world of languages that matches this inner knowledge. If we assume arithmetic behaves one way, then there is no solution; but, if we assume it behaves another way, there is a solution. There is no ideal world of arithmetic that favors one assumption over the other. By extending the axiom base of a theory, we are extending the world that the theory talks about. Thus when we add the negation above to the theory, we are now saying that there is more to arithmetic than we thought, and this more part will allow the negation to be true within the new theory. For an in depth account of this point, see Hofstadter (1979, 451-456).

The above is not merely an abstract idea; we can find many examples of undecidable statements in mathematics that allow us to have to multiple versions of a theory. As a classic example, consider the fifth axiom of Euclid's geometry. This is the standard geometry we all were taught in school. As this is an axiom, it is undecidable within the theory comprised of

Euclid’s first four axioms. Therefore we should be able to create variations on Euclid’s original geometry by adopting different versions of the fifth axiom. Indeed such variations exist—spherical geometry, hyperbolic (Lobachevsky-Bolyai-Gauss) geometry, elliptic (Riemannian) geometry—and have practical applications on various curved surfaces or spaces, as in Einstein’s theory of gravity. An example from set theory, and a prime illustration of Gödel’s theorem in action, is the generalized continuum hypothesis. For a long time it was unclear whether the hypothesis or its negation could be proved from within the standard Zermelo-Fraenkel formulation of set theory. Eventually it was proven by Gödel (1940) and Cohen (1963, 1964) that the hypothesis was undecidable in the given set theory, thus bifurcating the theory. As another example from set theory, by negating what is known as the foundation axiom, mathematicians haven been able to arrive at a different version of the theory that allows them to deal with sets that possess an infinitely nested structure (Forti 1983; Aczel 1988). It is not all the time that, when we vary the axioms or other undecidable statements of a theory, we get a theory whose practicality or, even, comprehension is immediately obvious; but, we can definitely say that it is a consistent theory about something or another.

From these examples and the previous considerations, we see that the notion of mathematical truth is a profoundly relative one. Statements are only true relative to a theory. And any statement, as long as it is not logically inconsistent relative to the axioms of a given theory, is true within the theory comprised of those axioms and that statement. So what Gödel’s theorem should properly read is: “There are true statements relative to a given system of arithmetic that are unprovable.” But it is even more accurate to read it as: “For any (sufficiently complex) system of arithmetic, there exist undecidable statements.” The important thing to realize here is that, given an undecidable statement, we can extend a theory in many directions, not just one (true) direction. Failure to recognize that Gödel’s Incompleteness Theorem is a direct coup d’etat on the notion of absolute truth in mathematics would be a great oversight on our part. In fact, Tarksi’s Theorem (1933), a close cousin of Gödel’s Theorem, demonstrates that no sufficiently complex arithmetical system can provide its own truth definition. (We will examine Tarski’s Theorem in section 3.5.2.)

3.4.2. Arbitrariness in Scientific Theories. The rise of arbitrariness and the decline of truth in mathematical theories brought on by the development of Gödel's Incompleteness Theorem finds a parallel in scientific theories. However, here it was the recognition of arbitrariness that came prior to that of incompleteness. This arbitrariness came in the form of the Quine-Duhem thesis. As we are forced to recognize that there can be multiple theories of arithmetic, geometry and sets, so we are forced to recognize that there can be multiple physical theories on a given domain of experience—the underdetermination of theory; or alternatively, as we can modify a mathematical theory any number of ways by adding or deleting any number of axioms, so too we can modify a physical theory to accommodate any observation—the underdetermination of observation. It is as we said earlier, language cannot completely capture observations (or mathematical ideas), therefore by conceding to use language to describe observations (or mathematical ideas), we must allow for flexibility and mutability in its usage. The Quine-Duhem thesis rejects the notion of an absolute truth standard for scientific theories.

3.4.3. Formulating Theories in Lack of a Truth Standard. The rejection of truth and the acceptance of arbitrariness in scientific theories causes one to wonder what are the grounds for selecting correct or even good theories, and, second, does such a viewpoint support an anything goes attitude in formulating theories? We start with the second question. We are not, of course, advocating a philosophical position here that there is no world outside the mind, and that we can create any theories we want without regard for what is happening in the outside world. What we are saying is that we can't know this outside world directly, but only through our experiences, thoughts and language. Moreover, that there is something peculiar about this process of interacting with the outside world that disallows a one-to-one mapping between it and our theories about it, which immediately raises the scepter of incompleteness along with all of its entailments. To be sure, if the outside world was different than what it is now, our theories of it would be different also. We can only create theories using concepts that arise from our experiences (or if perhaps hard-wired genetically at birth). In this sense, we can say that our theories are conditioned by experience. Therefore, while we can create infinitely many correct theories, we cannot create just any theory (recall that infinity does not necessarily mean everything).

Another point in regards to this second question is that, while we don't advocate an anything goes attitude in formulating individual scientific theories, the incompleteness of scientific theories, particularly fundamental physical theories, forbids us from ever asserting that so and so physical phenomena is impossible, e.g. manned flight, teleportation, anti-gravity devices, etc. Incompleteness is not a license for anarchy, but a cause for hope. In this sense, negative results such as incompleteness theorems are not so pessimistic, but leave open the door for new possibilities in the future. Contrariwise, a complete TOE can actually be viewed as a limiting result, in that it says some things are possible but never others.

Now we turn to the first question raised above: If not truth, what standards do we use to judge theories? Aware that the thesis half-named after him left open this question, Quine (1992, 14-15), himself, suggested a pragmatic approach, which is succinctly summarized in his maxim: maximization of simplicity and minimization of mutilation. The basic idea is that among a pool of theories on a given phenomena, we should pick the one that is the most simple, requires the least modification of other theoretical constructs (in a holistic web), and has the broadest scope. An example par excellence of such a theory is, of course, Einstein's general theory of relativity, which unified the laws of gravity and motion into a dramatically simple framework, while achieving a tremendously broad scope in its predictive powers. (It is true that we had to undergo a drastic change in our view of the space-time continuum, but this mutilation is minimal when compared to the plurality of mutilations and complexity of theory we would have to endure in order to retain the Newtonian model.) Moreover, many a physicist (and non-physicist) would readily attach the adjective beautiful to the general theory. Pragmatic and aesthetic attributes, such as simplicity, symmetry, beauty, practicality, predictive powers, scope of the theory, consistency with other theories, among others, do play a role in the formulation and selection of theories. If a physicist was presented with a theory that she knew to be experimentally sound, but was enormously complex, not only would she be dissatisfied with it, but she would also doubt if there wasn't a simpler theory. Truth is not the only bread that scientists eat.

3.5. Self-Reference in Theories.

3.5.1. *Gödel's Proof.* The instrumental use of self-reference in Gödel's proof sheds much light on how self-reference is implicated in scientific theories. Gödel critically recognized that statements *about* arithmetic (meta-arithmetical statements) can be mirrored within arithmetic itself. That is, statements such as “2 is a factor of 6”, “7 is a prime number” and “2 + 3 = 5 is a theorem of the given arithmetical system” can all be represented as arithmetical formulas. This is possible because (1), as we pointed out in section 3.3.2, the arithmetic of natural numbers can be used to represent any formal system with symbols and rules for the manipulation of those symbols (as long as those systems’ complexity does not exceed that of arithmetic’s, not withstanding Wolfram’s claim of computational equivalence), and (2) meta-arithmetical statements are expressible in such a formal system. Formal systems that meet the basic requirements for achieving incompleteness—infinity and a minimal complexity—can represent not only other formal systems, but also themselves. The point is that these systems are capable of self-reference because they involve computationally irreducible infinite processes. We should point out here that, technically, even without meeting the requirements of infinity and a minimal complexity, formal systems can talk about themselves to a certain extent; however, in order to fully represent themselves and, moreover, to construct the self-referential Gödel formula below, infinity and a minimal complexity are required.

Having figured out this self-representation capability of arithmetic, Gödel was able to construct an arithmetical formula that when translated meta-arithmetically asserted its own unprovability. That is, the formula, say G, when translated meta-arithmetically reads, “G is not a theorem of the given arithmetical system.” A little reasoning then shows that the question of whether G is a theorem of the given arithmetical system is an undecidable issue.

The significance of Gödel’s method of proof is that (a) it demonstrates that certain sophisticated formal languages have the capacity for self-reference, and (b) that self-reference in languages leads to undecidable statements. We contend here that any language-cum-theory sophisticated enough to describe our experiences of nature also has the capacity for self-reference, and therefore leads to incompleteness in the theory. This is only consistent with our prior contention that any such language-cum-theory also involve computationally irreducible infinite processes.

A common example used by those explaining Gödel’s theorem is the self-referencing capability of a natural language (as opposed to a formal language), like English, as is illustrated in various versions of the Epimenides paradox, or liar paradox: “This statement is false" or "I am lying". Again, a little reflection shows that it is undecidable whether such statements are true or false. Although worth further study, this may be going too far off base for our purposes, as we are interested in languages that are limited to describing natural experience, i.e., languages of scientific theories.

3.5.2. Tarski's Theorem and the Theorem of Undefinability of Valid Observations. So far there has been no successful attempt at a formalization of languages for scientific theories. Whether such a formalization is even possible is furthermore not clear (or the even more interesting question of the formalization of any natural language). Because of this we are presently unable to offer a formal incompleteness proof of scientific theories. However, as a temporary measure, we offered, in section 1.7, an informal argument of the undefinability of valid empirical observations. This argument is similar to the proof of Tarski's Theorem (1933), which was mentioned earlier (in section 3.4.1). Shortly after Gödel, Tarski used self-reference to show that the notion of arithmetical truth (as opposed to provability or theorem-hood) is undefinable within any sufficiently complex arithmetical system. The counterpart to arithmetical truth in our argument is valid observations. As Tarski showed that there is no procedure for determining arithmetical truth, we showed that there is no procedure for determining valid observations. It is worthwhile to point out the parallels between his theorem and ours.

For his proof, Tarski assumed that there was a formal definition of arithmetical truth. From this assumption and using Gödel's method of self-reference, Tarski was able to generate a contradiction. He constructed an arithmetical formula that asserted, not its own unprovability as in Gödel's theorem, but its own falsity. That is, the formula, say *T*, when translated meta-arithmetically reads, "*T* is not a true statement of the given arithmetical system." This statement is exactly equivalent to the liar paradox; it is true if and only if it is false. This contradiction shows that the formal truth definition cannot exist. This can also be seen in another way: if the truth definition did exist, then it is incomplete, for *T* is an arithmetical formula whose truth value cannot be determined from the truth definition.

We can now trace the similarities between Tarski's theorem and ours (please refer to section 1.7). We assumed that there was a theory of valid observations. From this assumption and using self reference, we generated a contradiction: the theory of valid observations can be empirically falsified if and only if it is true (or corroborated). This contradiction shows that the theory of valid observations cannot exist as a scientifically falsifiable theory. Moreover, the observation that falsified the hypothetical theory of valid observations is itself an example of an observation whose validity cannot be determined by the theory.

So what we see is that self-reference in scientific theories precludes the attainment of a procedure for determining valid empirical observations, thereby leading to the inability of theories to absolutely classify empirical evidence as in favor or against the theories.

3.5.3. Self-Reference and the Observational/Theoretical Distinction Failure. The fact that self-reference plays a role in mathematical and scientific incompleteness is telling. When a sophisticated enough theory talks about its world (world of arithmetic, of geometry, of natural phenomena), then, because of an inherent self-reference, it is also talking about itself. That is, a theory can be interpreted on two levels: as statements about its world and as meta-statements about itself. In fact, we can reasonably assert that all axioms and postulates of such theories are none other than self-referring, undecidable Gödel-like statements. This inextricable mixing of reference in theories leads to a natural inseparability between the theory and its world. To put it in stronger terms, self-reference in a theory compromises the separation of the theory and its world—and therefore also excludes a one-to-one mapping between them, i.e., a correspondence theory of truth. In science, this inseparability manifests itself as the observational/theoretical distinction failure. We can’t simply put theories on one side and observations on the other; the self-reference inherent in our scientific languages won’t allow it. As a more general principle, we can conjecture that self-reference and inseparability exist between thought and experience, between language and thought, between language and experience, and, finally, between ourselves and the world outside ourselves.

3.6. On the Possibility of an Incompleteness Theorem for Scientific Theories. Our examination has revealed that many of the mechanisms that give rise to mathematical incompleteness also exist in scientific incompleteness. With the exception of complexity, we have been able to identify the common roles played by infinity, arbitrariness, and self-reference in both. Additionally, by employing self-reference, we have been able to construct the equivalent of Tarski’s Theorem for scientific observations. These similarities, while helping us to better understand the idea of scientific incompleteness, serve to reinforce the validity of the idea and give it a more formal dimension. Furthermore, they seem to suggest that an incompleteness theorem, of sorts, can be shown in science. In what follows, we will suggest some possible areas for where we might look for one.

3.6.1. Formalization. We begin with the most obvious route: a proof based on the formalization of scientific (or even natural) language. When (or if ever) we achieve such a formalization, we could bring the mathematical methods pioneered by Gödel and others to bear on our attempt. The biggest question here is whether and how such a formalization can be realized. The mathematization and informationization, if you will, of ever increasing areas of the sciences coupled with efforts to formalize natural language (for example, see Kracht 2003) seems to indicate that the goal of formalization is not unrealistic. We also hinted at this possibility in section 2.5.1 and 2.5.2, where we discussed the ideas of science as pattern recognition and data compression. From an information-theoretic/mathematical point of view, everything is just information or abstract (mathematical) structure; some things have little information or simple structure, while others have much information or complex structure. The argument we can put forth here is that scientific disciplines such as biology, or even more so, psychology, having a rich information content, will require considerably more effort to formalize than disciplines such as physics, which has comparably less information. Compelling as the goal of formalization may appear, unresolved issues of ontology and semantics cast doubt on whether even the discipline of physics can be given a total formal treatment. Ultimately, further advances in linguistics, mathematics, information theory and other fields are needed before we can consider a proof based on formalization.

3.6.2. Cognitive Science. Another area where we might find a proof is within the interdisciplinary field of cognitive science. Research into linguistics, neuroscience and artificial intelligence may reveal that theoretical incompleteness is part of a much broader principle that applies to sentient entities, or to living things in general. We might find that it is part and parcel of consciousness, that loaded word. Before this investigation though, we need to have a significantly better understanding of how cognitive processes work, specifically how sentient entities or artificial life forms model and interact within their environments. Clearly, this is going to require further experimental research.

With this said, an area of cognitive science that presently may offer at least a glimpse of the incompleteness phenomena is the behavioral approach. Maturana and Valera (1980) interpret (scientific) understanding as our attempt, as biological agents, to reach invariance or stability with respect to our environment. Then scientific incompleteness implies that biological agents can't ever reach 100% invariance or stability with respect to their environment. Using the more technical terminology of Maturana and Valera, we can say that an autopoietic system can never couple perfectly with its environment. For instance, and quoting Bitbol (2001) in his analysis of Piaget (1967), humans use gestures, including words or language, in an attempt to "[carve] out domains of invariance...[or] extract elements of stability and iterativity from the Heraclitean flux" for the purpose of reacting predictively to their environment. Then incompleteness implies that such gestures can't ever carve out domains of perfect invariance; in other words, the meaning of words aren't perfectly invariant.

Piaget, if not having come before Maturana and Valera, would say that the ultimate mode of coupling in humans thus far is the culmination of mathematics as a linguistic tool. This has implications for an incompleteness proof based on formalization. For if incompleteness implies that agents can never achieve a perfect coupling, it would mean, in the case of humans, our mathematical models can never completely capture the world. As an interesting possibility, this line of inquiry may benefit in collaboration with the search for a mathematical theory of life, a search first suggested by von Neumann (1966). The behavioral approach seems to offer some hope of a proof and, at the same time, a broader understanding of theoretical incompleteness; though, as is the case with cognitive science in general, considerable more research is required.

3.6.3. Quantum Physics. As a final suggestion for an area where might find a proof of an incompleteness theorem for scientific theories is within quantum physics, a bedrock of modern science. The field has already produced a limitative result in the form of Heisenberg's Uncertainty Principle. It is not unreasonable to think therefore that further refinement of the theory could yield additional limitative results, particularly an incompleteness theorem. Furthermore, quantum physics possesses many of the key traits found in mathematical and scientific incompleteness. For instance, the uncertainty principle and experiential incompleteness may be related. Also, the uncertainty principle dictates that the outcomes of individual measurements are random or arbitrary; though this form of arbitrariness is not the same kind as in incompleteness, where the arbitrariness manifests itself in deciding the postulates of a theory, there may still be a connection. Similarly, whether there is any connection between the infinities of quantum theory and those of incomplete systems is also unclear. While these parallels may not be convincing enough, the one drawn by self-reference is.

Perhaps the biggest reason to suppose that an incompleteness theorem could be found within quantum physics is because of its self-referential or relational feature. The theory is as much a theory of interaction and measurement as it is about subatomic phenomena. It incorporates within its theoretical framework the role played by the observer and her method of interaction with nature. However, it is not just that quantum physics is self-referential that gives us hope, but that it is primarily, if not purely, a theory of interaction that at its heart denies the traditional subject-object separation that has been the core presupposition of western thinking for over two millennia.

Though many have gradually come to accept the prominent role quantum theory has given the observer, they continue to retain the traditional view of an external world of objects that stand independent to the observer, i.e., the subject-object separation. In spite of the theory’s remarkable predictive powers, the mismatch between this traditional world view and the peculiarities of experimental outcome has nurtured a continuous philosophical crisis within the theory and spurred many attempts at reconciliation, most notably Bohm’s (1952). It is only in recent times, due to many theoretical and experimental advances (as outlined in section 1.3.4), that the heavy burden of any such reconciliation is becoming apparent. These advances make it highly difficult to sustain our most basic notions of object and properties (of objects) in any fundamental theory of subatomic phenomena. Some have taken this outlook as an indication that quantum theory may be a relational theory (Rovelli 1996; Smolin 1995; Bitbol 2001). That is, it does not purport to describe a world of objects and their corresponding properties, but rather a world of interactions or events relative to an observer. This point is what is relevant for our hopes in finding an incompleteness theorem. The possibility that quantum theory could be a relational theory makes it likely that our future theories will tell us more about the nature of our interaction or relationship with the world, and that somewhere in those theories we may find an incompleteness theorem for science.[24]

As an interesting note, Bitbol believes that the relational character of quantum theory is indicative of a strictly behavioral interpretation of cognitive science, in which cognition is not defined in terms of an agent's ability to represent its environment, but its ability to interact with it. On such a reading, a collaboration between quantum physics and cognitive science may be another route to follow. Additionally, the relational character of quantum theory and the interactive character of behavioral cognitive science seems to suggest that both theories favor the elimination of the observational/theoretical distinction.

The turn in science marked by the advent of quantum theory is not altogether surprising. In science, we form theories of our world based on our interactions with the world. Therefore, it is only a matter of time before we are forced to account for the role of our interactions in formulating those theories. In fact, we can postulate that any culture (or alien race) will eventually go beyond formulating merely physically descriptive theories and towards theories which account for the role of their interactions and the processes of ascription and measurement of physical properties. But this is not only the case with science. We can find an analogous situation in mathematics. Just as quantum theory was a reaction to the peculiar discoveries of subatomic phenomena, the meta-theorems (e.g., Gödel's theorem) and other foundational work in mathematics were a reaction to the discovery of paradoxes in set theory. These parallels tell us that, in general, we cannot continue to simply study a subject matter forever, but that foundational crisis will eventually force us to examine our own methodologies of inquiry, causing us to formulate more relational, holistic theories. In such a landscape, an incompleteness theorem and other meta-theorems will seem much more natural.

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[24] Equivalently, information theoretic approaches to quantum theory (e.g., Zeilinger 1999), in which the notion of knowledge is emphasized over the notion of relation, may be fruitful towards this goal.

Conclusion and Remarks

Implications of the Theory. The philosophical theory outlined here is only a preliminary result. Considerable more research is required in order to gain a clearer understanding of theoretical incompleteness, the novel experience problem, and their many related attributes. What these preliminary results do tell us is that there is something peculiar about the nature of consciousness, about the way sentient entities interact within and understand their environments, and that it is worth examining this peculiarity in more detail.

Second, one should not get the impression that this theory implies that there are absolute truths out there in the universe that we cannot know. Such an implication is indicative of a Platonic mindset, which, as mentioned in section 3.4, is contrary to the arbitrary nature of theory formulation. This philosophical theory does not draw boundaries on what knowledge is acquirable by humans or other sentient entities; rather it states that the nature of this knowledge, while it may extend to ever further and further reaches of our universe, will always be fuzzy around the edges. It may be the case that there is a limit to what we can comprehend—due to the limits of our own brain, for example—but this is not the point being raised here.

With these qualifications out of the way, we can now state what the theory does imply. The two main pillars of the theory, the incompleteness of scientific theories (theoretical incompleteness) and the persistence of the novel experience problem (experiential incompleteness), already say much. Let us then look at some other obvious or not so obvious consequences of the theory. To start with, we should be aware that the incompleteness of scientific theories manifests itself in any format of scientific description. In the definitional (or constituted) approach, incompleteness manifests itself as the inability to give a complete definition due to infinite regress. In the formal (or representative) approach, it manifests itself as the failure to set up a one-to-one correspondence between the symbols of a language and the experienced world. In the axiomatic (or related) approach, incompleteness manifests itself as the inability to specify completely the relations between all the primitive terms, i.e., the inability to give a complete axiom list. The reductionistic approach is incomplete on the account of the incompleteness of the definitional and representative approaches. A contextual approach is incomplete due to the inability to specify sufficient context for any scientific term. Finally, a computer model is incomplete due to a limit of interaction, as discussed in section 2.5.4. Similarly, for other scientific approaches, we can find the role played by incompleteness.

An obvious and fundamental implication, which follows directly from the observational/theoretical distinction failure, is that there is no such thing as pure data in science. This is nothing more than a restatement that there is no such thing as hard observations; but, nonetheless, this point needs to be reiterated, especially in modern times where scientific authority has achieved a preeminent status. When it is said that so and so data supports some scientific claim, we cannot simply take this at face value and assume the claim to be a scientific fact or truth. We must be aware that the corroborative power of the data is contingent upon the broader theoretical context in which it sits. Such a critical mindset allows scientists to seek alternatives to theories that, although “agree with the data”, disagree with their own scientific intuition.

The flipside to the above is the implication that there is no such thing as an absolute scientific concept. Just as there are no hard observations, there are no hard concepts either. The meaning of all concepts are incomplete and subject to change. We have seen this change happen many times in science, but still we continue to believe that there are at least some absolute concepts. A paragon of such an example is the concept of an object. As we saw in section 1.3.4 and 3.6.3, with the advancements of quantum physics, this concept is no longer sacred. Though we may continue to use the term particle to describe subatomic entities, such as electrons and neutrons, the present experimental landscape is making it difficult for us to envision these entities in terms of our traditional conception of localized objects; rather, it steers us toward an abstract quantum formalism, which has yet to receive a consensus interpretation. The concept of an object is a highly convenient theoretical construct based on prior experience. As we improve the resolution of that experience, by studying subatomic phenomena, we see that the concept is changing; we need a new form of data compression. In addition, the concepts of space, time and motion, and the entire metaphysical framework of causality are now being re-examined in the light of this new experience with the subatomic. Elsewhere, “fundamental” concepts such as light, mass, charge and gravity continue to evolve to this day. There is no Platonic world of scientific concepts. All concepts are founded on experience, and as experiences can change, so can concepts.

Connections with Other Ideas in Philosophy. In their search for truth, philosophers have been a constant witness to a dance between two main notions: rationalism and empiricism (or mind and body, subject and object, analytic and synthetic). Since the beginning of western philosophy, there have been those who advocated purely one or the other notion, but as time went on, there has been a gradual back and forth easing of such a strict dichotomy, culminating with some of the more recent ideas of the twentieth century. The observational/theoretical distinction failure and its many implications are a natural continuation of this recent theme of ideas. They all recognize that rationalism or empiricism as separate schools of philosophy are inadequate to address the central questions of epistemology and ontology. They call for a unification of the above dichotomies into a more holistic theory, and one that takes into account the role of the self.

The observational/theoretical distinction failure itself is comparable to Merleau-Ponty’s embodied mind, Heidegger and Gadamer’s hermeneutic circle, and Quine’s criticism of the analytic/synthetic distinction. Additionally, many of the implications these thinkers have drawn from their respective theses resemble those drawn from the observational/theoretical distinction failure, but in a broader context than that of just scientific theories. Merleau-Ponty ([1945] 1962) argues that because we cannot have disembodied experiences we cannot understand our experiences without ambiguity. Gadamer ([1960] 1975, [1967] 1976) sees understanding as an ongoing process of interaction within the hermeneutic circle of new experiences and prior knowledge, and therefore, like his mentor Heidegger ([1927] 1962, especially sections 32-44), rejects that understanding can ever be objective or complete. Furthermore, Winograd and Flores (1986, 35) interpret the hermeneutics of Heidegger and Gadamer to imply that interpretations are not unique, suggestive of the arbitrariness expressed by the Quine-Duhem thesis. The implications of Quine’s criticism resulted in his ideas on the philosophy of science, as have already been discussed; however, there is one idea outside the scope of science that is also worth mentioning. This is his thesis of indeterminacy of translation, which states that one can always produce different translation manuals between two natural languages which may be mutually incompatible in some of their translations (Quine 1960, 26-79). Quine’s thesis undermines the notion that a given sentence can have a singular meaning.

It is also worth mentioning the work of two other philosophers who have played a role in easing these dichotomies of the west. The latter Wittgenstein, in his advocation of a pragmatic approach to language over a representative one, recognized that for words to be practically useful they must be flexible in meaning. In the exposition of his concept of “language-games”, he argues that they can have more than one meaning and share in the meaning of other words—his “family resemblance” (Wittgenstein 1953, aphorisms 2-80). His desire to dull exactness and loosen boundaries in word usage hints at the holistic approach to scientific terminology that was on the horizon. Similarly, Derrida ([1967] 1973, [1967] 1974), in his deconstructive analysis of text, contends that the meaning of texts can never be explicitly clear, stable or without undecidability. Elsewhere, both Wittgenstein (1953, aphorisms 185-243) and Derrida (1994, 37; [1992] 1995, 77) respectively argue that rules do not determine a singular course of action and that decisions require a leap of faith beyond any informative facts. Both these ideas are suggestive of the undecidability of mathematical axioms and the arbitrariness expressed by the Quine-Duhem thesis.

The above philosophers and previously mentioned mathematicians have contributed immensely to philosophy in many different ways; however, a lack of unification among their various ideas have prevented philosophy from seeing more of their implications and feeling their greater impact. The forceful critique of language by Wittgenstein and Derrida, of the mind/body distinction by Merleau-Ponty, the subject/object separation by Heidegger and Gadamer, and the analytic/synthetic distinction by Quine all unifyingly raise a compelling doubt about our ability, as sentient subjects, to completely or uniquely represent our world. The meta-theorems of Gödel and others have materialized this doubt in the field of mathematics. It is the thesis of this paper that it has already materialized for the field of science. Thus it is most likely the case that no human endeavor is immune to theoretical incompleteness. This would then imply that any idea or concept cannot be completely defined, axiomatized or contextualized. It would also mean that a general correspondence theory of truth is unattainable and, moreover, that the notion of truth, itself, is undefinable.

Cause for Optimism: Lessons from Mathematics. The above remarks may convey an immediate negative impression, but, if we view the cup half full, the results obtained here are actually cause for optimism and excitement for not only the scientist, but the non-scientist as well. There are at least three reasons for this. One is that theoretical and experiential incompleteness entails that the scientist work is never done (half-empty); this implies that there will always be new and exciting fundamental discoveries to be made (half-full). A true explorer would only be saddened if she found out that there were no more worlds left to explore. Second, as was pointed out in section 3.4.3, theoretical incompleteness allows for novel technological possibilities in the future, permitting the human race to accomplish things that may have been previously thought impossible. Finally, just as the discovery of mathematical incompleteness did not make the mathematician's cause hopeless, but rather opened up whole new worlds of fruitful research (e.g., meta-mathematics), so will the discovery of scientific incompleteness. It suggests for us new questions within the field of linguistics, cognitive science, physics and other areas—foundational questions that bear equally on the issues of the physical world as well as the conscious mind. These questions are as exciting as any that has preceded the many productive periods of intellectual discovery in the past. The pursuit of truth, though it may continue forever, will never lead us to a dull road, but to always more and more adventurous ones.

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Note: For translations or reprints of well known publications, although the original date is given in brackets, "[]", the original title and publication information is not given, as it can be readily found (on the Internet). For less well known publications, this information is provided.

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