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A General Measure of the 'Effective' Number of Parties in a Political System

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Abstract

This paper proposes a general measure of the effective number of parties, based on the family of *generalized entropy* inequality indices. This measure encompasses existing measures in the sense that these can be derived, through an appropriate configuration of parameter values, from this general measure. The proposed measure has attractive properties both in terms of interpretation and in terms of aggregation. In terms of interpretation, this measure always yields a value between 1 and N (N=the number of parties contesting) and takes one, or the other, extreme value depending on whether vote (or seats) are monopolized by a party or shared equally between the contesting parties. In terms of aggregation, it is always the case that the effective numbers of parties at sub-national levels can be aggregated to yield a national figure. The aggregation is effected through weights which, themselves, have an appealing interpretation in terms of the different sub-national contributions to overall inequality in the distribution of votes (or seats). The use of this general measure is illustrated by applying it to the results of the 1997 and 2001 Parliamentary (Westminster) elections in Northern Ireland. The central message of the paper is that the construction of indices or measures which purport to give scalar representation to vectors of distributive outcomes cannot be wholly based on 'objective' considerations. This observation applies in full to the measurement of the effective number of parties in a political system.

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1 Introduction

An important aspect of the analysis of electoral outcomes is the evaluation of the distribution of votes and/or seats across political parties. For example, assessing the proportionality of an electoral system requires a comparison of the distribution of seats with the distribution of votes (Gallagher, 1991, Lijphart, 1995). Another example is provided by measures of the 'effective' number of parties in a system. As Lijphart (1995) notes, the problem here is how to count parties of different size and the assumption in the comparative politics literature is that some kind of weighting is required to solve this problem.

In essence, calculating the effective number of parties in a system involves assigning a scalar value to a vector of inter-party distribution of seats or votes. At one extreme, this scalar value will (should) equal the number of parties that actually contest the election. This will occur when votes (or seats) are distributed equally among the parties. At the other extreme, this scalar value will (should) equal unity. This will be the case when all votes (or seats) accrue to just one party. In between the these extremes, the effective number of parties will be greater than one, but less than the number of parties contesting.

The general theme that underpins both examples is that of inequality analysis. Indeed, some of the well-known measures of both proportionality and effective number of parties are based on inequality indices. For example, the Lassko-Taagepera (L-T) measure of the effective number of parties (Lassko and Taagepera, 1979) - which is regarded as the "widely accepted formula" for such calculations[1] - is based explicitly on the Herfindahl (1950) index. However, one of the drawbacks of existing analysis of such problems is that it does not do more than scratch the rich vein of methodology that the study of inequality provides. To use the language of Cowell (1995), many of the measures, that are currently widely-used, came about more or less by accident with concepts borrowed from statistics being pressed into service as tools of proportionality measurement.

[1] Chhibber (1999), p. 54.

Against this background, this paper proposes a general measure of the effective number of parties, based on the family of *generalized entropy* inequality indices. This measure encompasses existing measures in the sense that these can be derived, through an appropriate configuration of parameter values, from this general measure. The proposed measure has attractive properties both in terms of interpretation and in terms of aggregation. In terms of interpretation, this measure always yields a value between 1 and N (N=the number of parties contesting) and takes one, or the other, extreme value depending on whether vote (or seats) are monopolized by a party or shared equally. In terms of aggregation, it is always the case that the effective numbers of parties at sub-national levels can be aggregated to yield a national figure. The aggregation is effected through weights which, themselves, have an appealing interpretation in terms of the different sub-national contributions to overall inequality in the distribution of votes (or seats). The use of this general measure is illustrated by applying it to the results of the 1997 and 2001 Parliamentary (Westminster) elections in Northern Ireland.

2 Generalized Entropy Inequality Indices

There are N parties contesting an election. Let $v_i \geq 0$ represent the vote share[2] of party i where $\sum_{i=1}^{N} v_i = 1$. If the function $h(.)$ is defined, for $z \geq 0$, as:

$$h(z) = \frac{1 - z^{\beta}}{\beta} \text{ if } \beta \neq 0 \text{ and } h(z) = -\log(z) \text{ if } \beta = 0 \tag{1}$$

then the family of *Generalized Entropy (GE)* Inequality Measures, $G(\beta)$, is obtained in any of the following *equivalent* ways (Cowell, 1995):

$$G(\beta) = \frac{1}{1+\beta}\left[\sum_{i=1}^{N} \frac{1}{N} h\left(\frac{1}{N}\right) - \sum_{i=1}^{N} v_i h(v_i)\right] \tag{2a}$$

$$G(\beta) = \frac{1}{1+\beta}\sum_{i=1}^{N} v_i \left[h\left(\frac{1}{N}\right) - h(v_i)\right] \tag{2b}$$

$$G(\beta) = \frac{1}{\beta+\beta^2}\sum_{i=1}^{N} v_i \left[v_i^{\beta} - N^{-\beta}\right] = \frac{1}{\beta+\beta^2}\left(\sum_{i=1}^{N} v_i^{1+\beta} - N^{-\beta}\right) \tag{2c}$$

[2] Though the analysis works equally well for seat shares

The logic of this class of measures is as follows. Suppose a random variable x can take values $x_1 ... x_N$ with probabilities $p_1 ... p_N$, $0 \leq p_i \leq 1 \sum p_i = 1$. Hence the information content h_i of observing x take the value x_i can be regarded as a decreasing function of p_i: if p_i is large/small, then it would not/would be a surprise if $x = x_i$ and so the 'information content', h_i, of the observation would be small/large (Renyi, 1965). A measure of the 'expected amount of information' or *entropy* conveyed by the observations, $x_1 ... x_N$ is $e = \sum p_i h(p_i)$ and equation (1), above, represents a formulation of the 'information content' function, $h(.)$ in terms of a parameter β.

The measure of inequality $G(\beta)$, in equation (2a), is obtained by subtracting the actual entropy of the distribution of votes across the N parties, $v_1 ... v_N$, from the maximum possible value of this entropy which obtains when every party gets an equal share of votes ($v_i = 1/N \ \forall i$). The expressions for $G(\beta)$ in equations (2b) and (2c) are derived from that in equation (2a), using - in the case of equation (2c) - the expression for $h(.)$ from equation (1).

When $\beta = 0$, from equation (2b) we have:

$$\begin{aligned} G(0) &= \sum_{i=1}^{N} v_i \left[\log(v_i) - \log(\frac{1}{N}) \right] = \sum_{i=1}^{N} v_i \log(v_i) - \log(\frac{1}{N}) \\ &= \frac{1}{N} \sum_{i=1}^{N} \frac{V_i}{\bar{V}} \log\left(\frac{V_i}{\bar{V}}\right) \end{aligned} \tag{3}$$

where: V_i is the number of votes received by party i and $\bar{V} = N^{-1} \sum_i V_i$ is the mean of the votes received by the parties. The expression for $G(0)$ in equation (3) is Theil's inequality index (Theil, 1967)

For $\beta \neq 0$, under perfect equality ($v_i = 1/N \ \forall i$), $G(\beta) = 0$ and, under perfect inequality ($v_i = 1$ for some i), $G(\beta) = \frac{1}{\beta + \beta^2} \left[\frac{N^{\beta} - 1}{N^B} \right]$; for $\beta = 0$, the corresponding values are $G(0) = 0$ under perfect equality, and $G(0) = \log(N)$ under perfect inequality.

3 Generalized Entropy Measures of the Effective Number of Parties

Now consider the linear transformation:

$$H(\beta) = a + bG(\beta) \tag{4}$$

such that: $H(\beta) = 1/N$ if $v_i = 1/N$ $\forall i$ (that is, there is perfect equality); and $H(\beta) = 1$ if $v_i = 1$ for some i (that is, there is perfect inequality). If one could effect such a transformation then a good measure of the *effective number of parties* (in constituency j) would be:

$$\Omega(\beta) = \frac{1}{H(\beta)} \tag{5}$$

Such a measure would have the following appealing properties:

1. Under perfect equality, with all parties getting an equal number of votes, $H(\beta) = 1/N \Rightarrow \Omega(\beta) = N$: the effective number of parties would be equal to the actual number of parties.

2. Under perfect inequality, with all votes accruing to a single party, $H(\beta) = 1 \Rightarrow \Omega(\beta) = 1$: the effective number of parties would be 1.

3. As inequality increased/decreased (the value of $G(\beta)$ and, therefore, of $H(\beta)$ rose/fell), the effective number of parties would fall/rise: $d\Omega(\beta)/dH(\beta) < 0$

4. Properties 1-3, above, would be valid for all values of β

In order to realize the transformation of equation (4), with its attendant properties, set $v_i = 1/N$ $\forall i$ to obtain:

$$\begin{aligned} H(\beta) &= a + \frac{b}{\beta + \beta^2} \sum_{i=1}^{N} v_i \left[N^{-\beta} - N^{-\beta} \right] = 1/N \\ &\Leftrightarrow a = 1/N \end{aligned} \tag{6}$$

and, then, set $v_i = 1$ for some i to obtain:

$$\begin{aligned} H(\beta) &= \frac{b}{\beta+\beta^2}(1-N^{-\beta})+N^{-1}=1 \\ &\Leftrightarrow\ b=\left[\frac{N-1}{N^\beta-1}\frac{N^\beta}{N}\right](\beta+\beta^2) \end{aligned} \tag{7}$$

If $\beta = 1$ then $b = 2$ and so, from equations (2c), (4) and (5):

$$G(1)=\frac{1}{2}\left[\sum_{i=1}^{N} v_i^2-\frac{1}{N}\right]\ ,\ H(1)=\sum_{i=1}^{N} v_i^2 \text{ and } \Omega(1)=\frac{1}{\sum_{i=1}^{N} v_i^2} \tag{8}$$

where: $H(1)$ is the Herfindahl index (Herfindahl, 1950); and $\Omega(1)$ is the Lassko-Taagepera (L-T) measure of the effective number of parties (Lassko and Taagepera, 1979). It is important to emphasize that the L-T measure is a special case of the general measure proposed, obtained by setting $\beta = 1$ in equation (5)[3] . The significance of this restriction is discussed in the following subsection.

For $\beta = 0$, the linear transformation:

$$H(0)=a+bG(0) \text{ where: } a=\frac{1}{N} \text{ and } b=\frac{N-1}{N}\frac{1}{\log(N)}$$

ensures that, under perfect equality, $H(0) = 1/N$ and, under perfect inequality, $H(0) = 1$. Consequently, the effective number of parties when $\beta = 0$ is:

$$\Omega(0)=\frac{1}{H(0)}=\left[\frac{1}{N}\left(1+\frac{1}{\log(N)}\frac{N-1}{N}\sum_{i=1}^{N}\frac{V_i}{\bar{V}}\log\left(\frac{V_i}{\bar{V}}\right)\right)\right]^{-1}$$

where Properties 1-3, above, also hold for $\Omega(0)$.

4 The Interpretation of the β Parameter

All inequality indices should embody the *weak principle of transfers.* In the case under discussion, this principle (also known as the Pigou-Dalton property: Dalton, 1920) requires that a transfer of votes from a 'larger' to a 'smaller' party should cause the value of the inequality index $H(\beta)$ to fall. More precisely, consider two parties, one with V votes and the other with $V + \delta$ votes. Then,

[3] In turn, the L-T measure encompasses Rae's (1971), index of 'party system fractionalisation', $F = 1 - \sum_i v_i$ since $\Omega(1) = 1/(1 - F)$.

by the weak principle of transfers, a transfer from the second ('larger') to the first ('smaller') party of ΔV votes, $\Delta V < \frac{1}{2}\delta$, will cause inequality to fall and, as a consequence, the effective number of parties to rise. Since $G(\beta)$, as a *bona fide* inequality index, satisfies this principle, so does its linear transform, $H(\beta)$.

But by *how much* the value of the inequality index will fall - and, therefore, by *how many* the effective number of parties will rise - following this 'egalitarian' vote transfer, will depend upon the value of the parameter, β. The value of β, therefore, measures the 'transfer sensitivity' of the inequality index: the larger the value of β, the greater will be the fall in inequality - and the rise in the effective number of parties - following a a transfer of votes from a larger to a smaller party. In the context of the 'social welfare' approach to inequality measurement (Atkinson, 1970), the value of β represents society's degree of 'inequality aversion'.

More formally, as a consequence of a transfer of Δv of vote share from a 'larger' party $(i = 2)$ to a 'smaller' party $(i = 1)$, the inequality indices $G(\beta)$ and $H(\beta)$ will fall by:

$$\begin{aligned} \Delta G &= \frac{1+\beta}{\beta(1+\beta)}(v_1^\beta - v_2^\beta)\Delta v = \frac{1}{\beta}(v_1^\beta - v_2^\beta)\Delta s \\ &= \frac{1}{\beta}[(1-v_2^\beta) - (1-v_1^\beta)]\Delta s = [h(v_2) - h(v_1)]\,\Delta v \qquad (9a) \\ \Delta H &= b\,[h(v_2) - h(v_1)]\,\Delta v \qquad (9b) \end{aligned}$$

If one defines a *distance measure*:

$$\lambda(\beta, v_1, v_2) = h(v_1) - h(v_2) = \frac{v_2^\beta}{\beta} - \frac{v_1^\beta}{\beta} \geq 0$$

then the *strong principle of transfers* requires that the reduction in inequality, following an egalitarian transfer of Δv (that is, a transfer from a larger to a smaller party), depends *only* upon the distance between two shares, *regardless* of the parties between which the transfer is made. An egalitarian transfer of Δv from party 4 to party 3 will have the same effect on reducing inequality as an egalitarian transfer from party 2 to party 1 if, and only if, the 'distance' between v_4 and v_3 is the same as the 'distance' between v_2 and v_1 or, more formally, if and only if:

$\lambda(\beta, v_4, v_3) = \lambda(\beta, v_1, v_2)$. The *greater* the distance between two vote shares the *larger* will be the fall in inequality following an egalitarian transfer of Δv.

The family of GE inequality indices - discussed above - satisfies the strong principle of transfers. For the purposes of this discussion, the relevant point is that, given the vote shares of four parties, $v_1 < v_2 < v_3 < v_4$, such that $v_4 - v_3 = v_2 - v_1$:

$$\begin{aligned}
\lambda(\beta, v_1, v_2) &> \lambda(\beta, v_2, v_4) \text{ if } \beta < 1 \\
\lambda(\beta, v_1, v_2) &= \lambda(\beta, v_2, v_4) \text{ if } \beta = 1 \\
\lambda(\beta, v_1, v_2) &< \lambda(\beta, v_2, v_4) \text{ if } \beta > 1
\end{aligned}$$

so that, if $\beta < 1$ / $\beta = 1$ /$\beta > 1$, a transfer of Δv from party 2 to party 1 will cause inequality to fall (and, therefore, the effective number of parties to rise) by an amount greater/equal/smaller than the amount by which it would fall if the same transfer was effected between parties 4 and 3.

This follows because, from equation (1):

$$\frac{dh}{dz} = -z^{\beta-1} < 0 \text{ and } \frac{d^2h}{dz^2} = -(\beta-1)z^{\beta-2}$$

so that $h(.)$ curve is: linear if $\beta = 1$ ($d^2h/dz^2 = 0$); convex to the origin if $\beta < 1$ ($d^2h/dz^2 > 0$); and concave to the origin if $\beta > 1$ ($d^2h/dz^2 < 0$). These outcomes are shown in Figure 1. The top curve relates to $\beta = 0.5$ and it is convex to the origin; the curve below it - the middle curve - relates to $\beta = 1$ and it is linear; the bottom curve relates to $\beta = 1.5$ and it is concave to the origin.

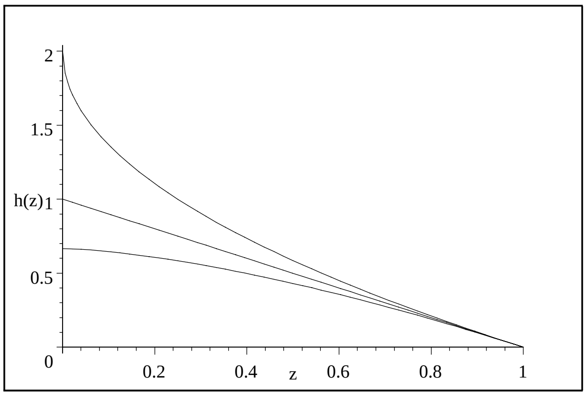

Figure1: Distance and values of β

Figure 1 shows that the distance between the shares 0.6 and 0.8 is the same as/less than/more than the distance between the shares 0.2 and 0.4 when $\beta = 1/\beta < 1/\beta > 1$. Moreover, as the value of β decreases from $\beta = 1$, the curves will become more convex, and as the value of β increases from $\beta = 1$ they will become more concave. Therefore, the gap between the distances will increase with higher (absolute) values of β : as β increases (from $\beta = 1$) through negative[4] /positive values, we become increasingly more approving of egalitarian transfers at the lower/upper end of the distribution. It is in this sense that β may be thought of as a 'transfer sensitivity' (or, equivalently, the 'inequality aversion') parameter.

When $\beta = 1$, $\lambda(\beta) = h(v_1) - h(v_2) = v_2 - v_1$. Therefore, the implication of $\beta = 1$ is that the distance between a party with a a 40% share of the vote and another with a 35% share is the same as the distance between a party with a 7% share of the vote and another with a 2% share of the vote. Consequently, an implication of the L-T measure of the effective number of parties is that a given change in vote shares, regardless of whether it occurred between a pair of mainstream parties or between a pair of fringe parties, would cause the effective number of parties to change by the same amount, provided the difference in vote shares was the same between the two sets of pairs.

However, if $\beta < 1$, then, using the preceding example, the distance between the mainstream

[4] 1, 0.5, 0, -.5, -1 etc.

parties would be smaller than the distance between the fringe parties and if $\beta > 1$, it would be greater. Consequently, if $\beta < 1/\beta > 1$, a given change in vote shares would cause the effective number of parties to change by more/less if the change occurred between a pair of fringe parties than if it occurred between a pair of mainstream parties. This is the point made by Wildgen (1971) when he proposed a 'hyperfractionalisation' measure of the effective number of parties which accorded a higher weight to smaller parties. In the context of the above analytical framework, he was simply proposing a $\beta < 1$. On the other hand, Molinar (1991), who wanted a higher weight to be assigned to the largest party, was arguing for a $\beta > 1$.

The restriction implied by $\beta = 1$ is not always appealing. For example, if, in Britain, the Conservative share of the vote increased by 2 percentage points and the Labour party's share fell by a corresponding amount then it would be hard to argue that the effective number of parties had changed. On the other hand, if the British National Party - a right-wing, anti-immigrant party - increased its vote share by 2 percentage points, at the expense of say, the Green Party, then that would be likely to increase the effective number of parties in Britain in the sense that the voice of both the British National Party and the Green Party would be heard in the nation's political debate. The general measure of the effective number of parties proposed in this paper - offering, as it does, a menu of choices between different degrees of transfer sensitivity - is free of such restriction[5] .

5 Aggregation Issues

The effective number of parties in a political system can refer to a variety of geographical areas, ranging from a country, to regions within a country representing conglomeration of constituencies, down to the individual constituencies themselves (Chhibber and Kollman, 1996; Chhibber and Nooruddin, 1999). This section draws out the relationship between the effective number of parties in, say, a country and the effective number of parties in the regions of that country.

[5] Which is not say that one may not, if it is deemed appropriate, wish to adopt such a restriction.

There are M $(j = 1...M)$ regions in the country with N_j parties contesting the elections in region j. Let $v_{ij} \geq 0$ represent the vote share of party i in region j. The effective number of parties in the country, $\Omega(\beta)$ can be written as a weighted sum of $\Omega_j(\beta)$, the effective numbers of parties in the regions, as follows:

$$\Omega(\beta) = \frac{1}{H(\beta)} = \sum_{j=1}^{M} \frac{1}{H_j(\beta)} \frac{H_j(\beta)}{H(\beta)} \frac{1}{M} = \sum_{j=1}^{M} \Omega_j(\beta) w_j(\beta) \tag{10}$$

where:

$$\begin{aligned}
H_j(\beta) &= a_j + b_j G_j(\beta) \\
G_j(\beta) &= \frac{1}{\beta + \beta^2} \left(\sum_{i=1}^{N_j} v_{ij}^{1+\beta} - N_j^{-\beta} \right) \\
a_j &= \frac{1}{N_j},\ b_j = \left[\frac{N_j - 1}{N_j^{\beta} - 1} \frac{N_j^{\beta}}{N_j} \right] (\beta + \beta^2) \\
\Omega_j(\beta) &= \frac{1}{H_j(\beta)} \text{ and } w_j = \frac{H_j(\beta)}{H(\beta)} \frac{1}{M}
\end{aligned}$$

The weights w_j in equation (10) may be interpreted as the 'scaled' contribution of inter-party vote-share inequality in a region to overall inter-party vote-share inequality. The scaling factor is the inverse of, M, the number of regions: when $M = 1$, $w_j = 1$ and $\Omega(\beta) = \Omega_j(\beta)$. In the special case, when $\beta = 1$,

$$w_j = \frac{\sum_{i=1}^{N_j} v_{ij}^2}{\sum_{i=1}^{N} v_i^2} \frac{1}{M}$$

which is the ratio of the value of the regional and national Herfindahl indices, scaled by the number of regions.

6 Two Measures of Electoral Disproportionality

Electoral disproportionality measures the degree of discord between the proportion of votes received by the various parties relative to the proportion of parliamentary seats obtained by them. If s_i represents the proportion of seats obtained by party i, then a popular measure of disproportionality, due to Gallagher (1991), is defined as:

$$\rho = \sqrt{\frac{1}{2}\sum(v_i - s_i)^2} = \sqrt{\frac{1}{2}\left(\sum_{i=1}^{N} v_i^2 + \sum_{i=1}^{N} s_i^2 - 2\sum_{i=1}^{N} v_i s_i\right)} \tag{11}$$

which is, essentially, the sum of the values of Herfindahl indices calculated on, respectively, vote and seat shares less the covariance between the seat and vote shares.

Alternatively, one may define the degree of disproprtionality in an electoral system as the ratio of the effective number of parties calculated on *vote* shares (denoted $\Omega_v(\beta)$) to the effective number of parties calculated on *seat* shares (denoted $\Omega_s(\beta)$). On this definition, the degree of disproportionality is:

$$\sigma_\beta = \frac{\Omega_v(\beta)}{\Omega_s(\beta)} \tag{12}$$

such that the system is perfectly proportional when $\sigma = 1$ and 'disproportional' when $\sigma > 1$, with higher values of σ being associated with greater degrees of disproportionality[6] . When $\beta = 1$, the effective number of parties is defined by the L-T measure and the degree of disproportionality is simply the ratio of the Herfindahl indices calculated, respectively, on seat and on vote shares:

$$\sigma_1 = \frac{\Omega_v(1)}{\Omega_s(1)} = \frac{\sum s_i^2}{\sum v_i^2} \tag{13}$$

7 A Numerical Example

Results from the 1997 and 2001 elections in Northern Ireland to the British Parliament (Westminster) were used to put empirical flesh on the above analysis. These elections, which sent a member of parliament from each of 18 constituencies in Northern Ireland to Westminster, were contested on a first-past-the-post basis. Table 1 shows, for the 1997 election, the number of parties that contested the elections from each of the 18 parliamentary constituencies of Northern Ireland (column headed 'N') and also the effective number of parties, $\Omega(\beta)$, when, respectively: $\beta = 1$; $\beta = 0.5$; $\beta = 1.5$. Table 2 does the same for the 2001 parliamentary elections.

[6] When $\sigma < 1$, the effect of the electoral system is to protect smaller parties. This is sometimes used to protect geographical minorities. For example, elections to the US Senate award two seats to every state, regardless of their respective sizes.

The effective number of parties when $\beta = 1$ is the reciprocal of the Herfindahl index calculated on the constituency vote shares and thus corresponds to the L-T measure. The effective number of parties was highest when $\beta = 1.5$ and lowest when $\beta = 0.5$. This was because, given a distribution of vote shares across the parties contesting an election in a constituency, higher values of β result in lower values of the inequality index H. As Figure 1 shows, values of $\beta > 1$ compress the distance between two vote shares, $v_1 < v_2$ so that $\lambda(\beta, v_1, v_2) < v_2 - v_1$; however, these distances are inflated for $\beta < 1$ so that $\lambda(\beta, v_1, v_2) > v_2 - v_1$; lastly, when $\beta = 1$, $\lambda(\beta, v_1, v_2) = v_2 - v_1$. Since the effective number of parties is the reciprocal of the value of the inequality index, H, the result follows.

The penultimate rows of Tables 1 and 2 show, for the 1997 and 2001 elections respectively, the mean of the effective number of parties in the constituencies. These mean values are not the same as the effective number of parties in Northern Ireland. This number - shown in the last row of Tables 1 and Table 2 - is obtained from the effective number of parties in the constituencies as a weighted sum (see equation (10)) not as an arithmetic mean.

Northern Ireland has four major parties. Of these, the Democratic Unionist Party (DUP) and the Ulster Unionist Party (UUP) are the unionist parties and Sinn Fein (SF) and the Social Democratic Labour Party (SDLP) are the nationalist parties. Between them they won all the 18 parliamentary seats in Northern Ireland in 2001 and all, but one, in 1997. Collectively, they received 87% of the vote in 1997 and 92% of the vote in 2001. When $\beta = 1$, the effective number of parties was calculated as 4.6 in 1997 and 4.7 in 2001; with $\beta = 0.5$, the higher value of the inequality index reduced the effective number of parties to 3.7; and with $\beta = 1.5$, the lower value of the inequality index raised the effective number of parties to 5.8. These results illustrate that the the choice of a value for β, the transfer sensitivity parameter, can significantly affect calculations of the effective number of parties in a system.

8 Conclusions

The central message of this paper is that the construction of indices or measures which purport to give scalar representation to vectors of distributive outcomes cannot be wholly based on 'objective' considerations. While there may be unanimity about the desirability of the value of an index falling, consequent upon an egalitarian transfer (the weak principle of transfers), there will inevitably be disagreement about the amount by which it should fall. The strong principle of transfers says that this amount should depend only upon the distance between two distributive positions but, as this paper has shown, the distance between two positions depends critically upon the analyst's preferences about where in the income distribution he/she would most like to see redistribution effected. In short, it depends upon the analyst's 'transfer sensitivity' or, equivalently 'aversion to inequality'.

These remarks apply in full to the measurement of the effective number of parties in a political system. This measure, as has been shown, can be generated by taking the reciprocal of an index drawn from the family of generalized entropy inequality indices. But the family member chosen will determine the value of the inequality index and, hence, influence the calculation of the effective number of parties. Choosing a member from the family reduces to choosing a value of β, the 'transfer sensitivity' parameter. If the choice, as with the L-T measure of the effective number of parties, is $\beta = 1$ then (perhaps, without even being aware of it) the analyst is placing equal weight on transfers at all levels. On the other hand, the analyst who chooses $\beta < 1$ places more weight on transfers between smaller parties while the analyst who chooses $\beta > 1$ places more weight on transfers between larger parties. The point is that the answer to the question "what is the effective number of parties?" depends partly upon the facts of electoral data, which will frame the answer, but it also depends upon what is in the heart of the person to whom the question is addressed.

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Table 1
Effective Number of Parties in Northern Ireland
Westminster Elections, 1997

constituency	β=1	β=0.5	β=1.5	N
East Antrim	4.2	3.5	5.1	9
East Belfast	3.3	2.7	4.1	9
East Londonderry	4.0	3.3	4.8	8
Fermanagh & South Tyrone	2.7	2.4	3.0	5
Foyle	2.6	2.4	2.9	5
Lagan Valley	2.8	2.4	3.2	8
Mid-Ulster	2.9	2.5	3.5	6
Newry & Armagh	2.9	2.6	3.3	5
North Antrim	3.3	2.9	3.8	7
North Belfast	2.8	2.5	3.3	7
North Down	3.7	3.1	4.6	8
South Antrim	2.6	2.5	2.9	6
South Belfast	4.3	3.5	5.4	10
South Down	2.5	2.3	2.8	5
Strangford	3.2	2.8	3.8	7
Upper Bann	3.6	3.0	4.2	8
West Belfast	2.2	1.9	2.5	6
West Tyrone	3.1	2.6	3.7	6
Average over Constituencies	3.2	2.7	3.5	7.1
Northern Ireland	4.6	3.7	5.8	10

N = number of parties contesting election

Table 2
Effective Number of Parties in Northern Ireland
Westminster Elections, 2001

constituency	β=1	β=0.5	β=1.5	N
East Antrim	3.5	3.0	4.1	7
East Belfast	3.7	3.0	4.5	9
East Londonderry	4.0	3.8	4.3	5
Fermanagh & South Tyrone	3.5	3.5	3.6	4
Foyle	2.9	2.6	3.1	5
Lagan Valley	2.7	2.6	2.9	5
Mid-Ulster	2.6	2.4	2.8	4
Newry & Armagh	3.5	3.4	3.6	4
North Antrim	3.0	2.8	3.3	5
North Belfast	3.5	3.0	4.0	6
North Down	2.2	2.0	2.6	6
South Antrim	3.5	3.2	4.0	6
South Belfast	3.2	2.7	3.9	8
South Down	3.3	3.0	3.5	5
Strangford	2.8	2.5	3.3	6
Upper Bann	3.8	3.4	4.1	5
West Belfast	2.1	1.9	2.3	7
West Tyrone	2.9	2.9	2.9	3
Average over Constituencies	3.1	2.9	3.5	5.7
Northern Ireland	4.7	3.7	5.8	9

N = number of parties contesting election

Table 3
Party Vote Shares in Northern Ireland
Westminster Elections, 1997

Constituency	DUP	UUP	SF	SDLP	Others
East Antrim	19.5	38.8	1.6	4.6	35.6
East Belfast	42.6	25.3	2.1	1.6	28.3
East Londonderry	25.6	35.6	9.1	21.7	8.0
Fermanagh & South Tyrone	0.0	51.5	23.1	22.9	2.5
Foyle	21.5	0.0	23.9	52.5	2.0
Lagan Valley	13.6	55.4	2.5	7.8	20.8
Mid-Ulster	36.3	0.0	40.1	22.1	1.5
Newry & Armagh	0.0	33.8	21.1	43.0	2.1
North Antrim	46.5	23.6	6.3	15.9	7.7
North Belfast	0.0	51.8	20.2	20.4	7.6
North Down	0.0	31.1	0.0	4.4	64.5
South Antrim	0.0	57.5	5.5	16.2	20.8
South Belfast	0.0	36.0	5.1	24.3	34.6
South Down	0.0	32.8	10.4	52.9	3.9
Strangford	30.2	44.3	1.2	6.7	17.6
Upper Bann	11.5	43.6	12.1	24.2	8.6
West Belfast	0.0	3.4	55.9	38.7	2.0
West Tyrone	0.0	34.6	30.9	32.1	2.5
Northern Ireland	13.6	32.7	16.1	24.1	13.6

DUP = Democratic Unionist Party
UUP = Ulster Unionist Party
SF = Sinn Fein
SDLP = Social Democratic Labour Party
OTH = Other Parties

Table 4
Party Vote Shares in Northern Ireland
Westminster Elections, 2001

Constituency	DUP	UUP	SF	SDLP	Others
East Antrim	36.0	36.4	2.5	7.3	17.7
East Belfast	42.5	23.2	3.4	2.4	28.5
East Londonderry	32.1	27.4	15.6	20.8	4.1
Fermanagh & South Tyrone	0.0	34.0	34.1	18.7	13.2
Foyle	15.2	6.9	26.6	50.2	1.2
Lagan Valley	13.4	56.5	5.9	7.5	16.6
Mid-Ulster	31.0	0.0	51.2	16.7	1.0
Newry & Armagh	19.4	12.3	30.9	37.4	0.0
North Antrim	49.9	21.0	9.8	16.8	2.6
North Belfast	40.8	12.0	25.2	21.0	0.9
North Down	0.0	56.0	0.8	3.4	39.7
South Antrim	34.8	37.1	9.4	12.1	6.7
South Belfast	0.0	44.8	7.6	30.6	17.0
South Down	15.0	17.6	19.7	46.3	1.3
Strangford	42.8	40.3	2.2	6.1	8.6
Upper Bann	29.5	33.5	21.1	14.9	1.0
West Belfast	6.4	6.2	66.1	18.9	2.3
West Tyrone	0.0	30.4	40.8	28.7	0.0
Northern Ireland	22.5	26.8	21.7	21.0	8.1

DUP = Democratic Unionist Party
UUP = Ulster Unionist Party
SF = Sinn Fein
SDLP = Social Democratic Labour Party
OTH = Other Parties