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COMOVEMENTS IN INTERNATIONAL STOCK MARKETS

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Comovements in International Stock Markets

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Abstract

In the paper monthly realized moments for stock market returns for the US, the UK, Germany and Japan are employed to assess the linkages holding across moments and markets over the period 1973-2004. In the light of the theoretical framework proposed in the paper, the results point to a progressive integration of the four stock markets, leading to increasing comovements in prices, returns, volatility and correlation. Evidence of a positive and non spurious linkage between volatility and correlation, and a trend increase in correlation coefficients over time, is also found. All the above mentioned linkages seem to be particularly strong for the US and Europe, while the persistent stagnation of the economy and the weak fundamentals over the 1990s may have been the cause of the more idiosyncratic behavior of the Japanese stock market.

J.E.L. classification: G1, G15, C32.

Keywords: realized volatility, realized correlation, stock markets, financial integration, economic integration.

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1. Introduction

This paper is an empirical assessment of various aspects regarding interconnections among the largest world stock markets, that is the United States, the United Kingdom, Japan and Germany. Many papers have contributed to the debate on the interdependence across international stock markets, considering issues such as volatility spillovers, correlation breakdowns, trends in correlation patterns.

This debate is important under different points of view. Firstly, the gains and motivations of international diversification rely on low correlations across international stock markets. Financial markets integration could have eroded much of the gains from international diversification by making market to comove more closely and enhancing spillovers. Secondly, strong comovements in extreme market realizations may increase the risk of global financial instability, with local market disruptions quickly spreading across countries, independently of fundamental dynamics.

Among the many interesting elements, two deserve particular attention, that is the association between correlation and volatility and the existence of an increasing trend among correlation coefficients. Bennet and Kelleher (1988), Von Furstenberg and Jeon (1989), Bertero and Mayer (1990) and Longin and Solnik (1995), for instance, have found evidence of instability in the correlation patterns characterizing international stock markets, with both volatility and correlation increasing in correspondence of the October 1987 stock market crash, and correlation remaining higher afterwards, also when volatility reverted to pre-crash levels. A positive linkage between correlation and volatility is also documented by King et al., 1994; Karolyi and Stulz, 1996; Solnik et al., 1996; Ramachand and Susmel, 1998; Ang and Bekaert, 1999; Rockinger and Jondeau, 2001; Ball and Touros, 2000; Morana and Beltratti (2002). The results of most of these studies are robust to the upward sample selection bias affecting the computation of the correlation coefficient pointed out by Ronn et al. (2000), Forbes and Rigobon (1999), and Boyer et al. (1999), suggesting a non spurious positive association between stock market volatility and correlation.

Solnik et al. (1996) and Longin and Solnik (1995) have also documented the presence of an increasing trend in the correlation coefficients for international stock markets over the period 1958-1985, which, according to Rockinger and Jondeau (2001), after 1995 would have stabilized at higher levels for European markets, and decreased for pairs including the US and Japanese markets. Similar evidence has also been provided by Bekaert et al. (2005). Hence, albeit international stock markets would have undergone a progressive integration, the evidence would still favour the existence of three regional groups, i.e. the US, Europe and Pacific Basin, rather than a single world market (Engle and Susmel, 1993; Groenen and Franses, 2000).

In this paper the empirical evidence on the linkages across international stock markets has been further assessed following a different approach from the one previously employed in the literature. The original contributions of the paper are as follows. Firstly, monthly realized variance and correlation processes for the four major international stock market indices, namely the US, the UK, Japan and Germany, are employed. The use of realized moments allows a more precise measurement of the features under investigation. Secondly, a unified assessment of the linkages across moments and countries is carried out by means of a common factor model, granting that the linkages uncovered are not spurious. Moreover, not only the existence of common factors across countries and realized moments has been assessed, but also how the importance of these factors has changed over time. The sample (1973 - 2004) allows for such a thorough assessment, encompassing several key episodes of market turmoil, as for instance the oil price shocks of 1973 and 1981, the stock market crashes of 1987 and 2000, the two Gulf Wars, the 1997 East Asian crises, the 1994 Mexican peso collapse, the Russian and LTCM crises. Moreover, by using a larger sample than the one employed in previous studies, up to date evidence concerning trend comovements in prices, returns and volatility, comovements and trend dynamics in correlation processes, and comovements in volatility and correlation processes, is provided, also concerning the last decade of data, which has not been explored so far. Finally, a theoretical framework has been introduced to explain the effects of markets integration on first and higher return moments. It is shown that market integration leads to an increase in return correlations and in the comovement of return correlations, to a positive linkage between volatility and return correlation, and to an increase in the comovement in volatilities.

The main findings of the study are as follows. Evidence of strong linkages across stock markets, involving comovements in prices, returns and volatility, is found. Moreover, evidence of a trend increase in the correlation coefficients over the full time span analyzed, and that the positive linkage between volatility and correlation is robust and non spurious, holding for both bear and bull markets, is also found. All the above mentioned findings are rationalized in terms of the effects of financial and economic integration and fully coherent with the theoretical framework developed in the paper. Linkages across stock markets seem to have in general grown stronger over time, particularly for the US and Europe, while the more idiosyncratic behavior detected for Japan over the 1990s suggests that local factors, i.e. weak fundamentals, may still dominate, over specific periods of time, the trend dynamics generated by globalization and market integration. While the results are still coherent with the evidence of three separate geographic areas for international stock markets, i.e. Europe, the US and the Pacific Basin, the results also show that the heterogeneity between Europe and the US has steadily reduced over time, being the two markets currently strongly integrated.

After this introduction the paper is organized as follows. In section two the data and the construction of the realized processes are discussed. In section three the econometric methodology employed in the paper is introduced. In section four the empirical results are presented. Finally, in section five conclusions are drawn.

2. Theoretical arguments

Assuming away for simplicity the existence of currencies, consider three stock markets (x, y, z) whose (log) returns (r_{j_t}) are measured on the basis of a one factor (F_t) model

$$r_{j_t} = E_{t-1}\left[r_{j_t}\right] + \beta_{j_t} F_t + \varepsilon_{j_t}, \qquad j = x, y, z,$$

where it is assumed that $E\left[\varepsilon_{j_t}\right] = 0$, $E\left[F_t\right] = 0$, $V_{t-1}\left[\varepsilon_{j_t}\right] = \sigma^2_{j_t}$, $V_{t-1}\left[F_t\right] = \sigma^2_{F_t}$, $V\left[\varepsilon_{j_t}\right] = \sigma^2_j$, $C\left[\varepsilon_{i_t}, \varepsilon_{j_t}\right] = 0$, $\beta_{j_t} = C_{t-1}\left[r_{j_t}, F_t\right] / V_{t-1}\left[F_t\right]$, for $j = x, y, z$.[1] By cumulating the ith return process, it is also found that

$$p_{j_t} = p_{j_0} + \sum_{s=1}^{t} r_{j_s} = p_{j_0} + \sum_{s=1}^{t} E_{s-1}\left[r_{j_t}\right] + \sum_{s=1}^{t} \beta_{j_s} F_s + \sum_{s=1}^{t} \varepsilon_{j_i s} \qquad j = x, y, z,$$

where p_{j_t} is the log price index for the generic asset at time period t.

Consider first the case of no integration, that is $\beta_{j_t} = 0, \;\; j = x, y, z, \; \forall t$. It is found

$$r^*_t = E_{t-1}\left[r^*_t\right] + \varepsilon^*_t$$

[1] The conditional mean, variance, covariance, and correlation are denoted as $E_{t-1}[\cdot]$, $V_{t-1}[\cdot]$, $C_{t-1}[\cdot, \cdot]$ and $\rho_{t-1}[\cdot, \cdot]$, respectively. The corresponding unconditional moments are denoted as $E[\cdot]$, $V[\cdot]$, and $C[\cdot, \cdot]$, respectively.

$$p^*_{jt} = p_{j0} + \sum_{s=1}^{t} r^*_{j_s} = p_{j0} + \sum_{s=1}^{t} E_{s-1}\left[r^*_{j_s}\right] + \sum_{s=1}^{t} \varepsilon^*_{j_s} \quad j = x, y, z,$$

where "*" is added to the variables in the equations to denote the case of no integration, in order to better distinguish the two cases in the discussion which follows.

In this case the conditional CAPM, see Merton (1973), suggests that conditional expected returns are proportional to the conditional volatility of the idiosyncratic shocks

$$E_{t-1}\left[r^*_{jt}\right] = \gamma \sigma^*_{jt}, \tag{2.1}$$

for $j = x, y, z$, where the proportionality coefficient is assumed to be equal across markets, due to homogeneous coefficients of risk aversion.

The higher conditional central moments are in this case

$$V_{t-1}\left[r^*_{jt}\right] = \sigma^{*2}_{jt} \tag{2.2}$$

$j = x, y, z$ and all the conditional covariances and correlations between returns are equal to 0, i.e.

$$C_{t-1}\left[r^*_{i_t}, r^*_{jt}\right] = \rho_{t-1}\left[r^*_{i_t}, r^*_{jt}\right] = 0 \tag{2.3}$$

$j = x, y, z$.

Consider now the general case of perfect market integration. In this case the risk premium is proportional to the volatility of the non-diversifiable risk factor

$$E_{t-1}\left[r_{jt}\right] = \gamma \beta_{jt} \sigma_{Ft}, \tag{2.4}$$

$j = x, y, z$, and the higher moments become

$$V_{t-1}\left[r_{jt}\right] = \beta^2_{jt} \sigma^2_{F_t} + \sigma^2_{jt} \tag{2.5}$$

$$C_{t-1}(r_{i_t}, r_{jt}) = \beta_{i_t} \beta_{jt} \sigma^2_{F_t} \tag{2.6}$$

$$\rho_{t-1}\left[r_{i_t}, r_{jt}\right] = \frac{\beta_{i_t} \beta_{jt} \sigma^2_{F_t}}{\sqrt{\beta^2_{i_t} \sigma^2_{F_t} + \sigma^2_{i_t}} \sqrt{\beta^2_{jt} \sigma^2_{F_t} + \sigma^2_{jt}}}, \tag{2.7}$$

$j = x, y, z$.

Hence, even after capital market integration is achieved, the equations clarify that changes in the volatilities of market returns and correlations among returns may be due to (i) shocks to conditional betas, (ii) shocks to volatilities of non-diversifiable factors and of idiosyncratic factors. Yet, a dominant role for global rather than for idiosyncratic shocks may be expected. The key implications of capital market integration are as follows.

Firstly, the conditional expected returns are likely to drop if the volatility of the non-diversifiable risk factor is lower than the volatility of the idiosyncratic risk factors and the betas are not too high. This follows by comparing (2.1) and (2.4). For instance, for market x it is found

$$\frac{E_{t-1}\left[r^*_{x_t}\right]}{E_{t-1}\left[r_{x_t}\right]} = \frac{\sigma^*_{x_t}}{\beta_{x_t}\sigma_{F_t}},$$

and $\frac{E_{t-1}\left[r^*_{x_t}\right]}{E_{t-1}\left[r_{x_t}\right]} > 1$ if $\sigma^*_{x_t} > \beta_{x_t}\sigma_{F_t}$.

Moreover, the common dynamics determined by the returns factor, i.e. $\beta_{j_t}F_t$, should also been detectable in prices, since, in the case of integration, the common component $\sum_{s=1}^{t}\beta_{j_s}F_s$ affects the log price index for the various markets.

Secondly, the proportion of non-diversifiable risk over total risk is likely to increase, while the proportion of idiosyncratic risk is likely to decrease. For instance, assuming that total risk remains unchanged after integration, i.e. $V_{t-1}\left[r_{j_t}\right] = V_{t-1}\left[r^*_{j_t}\right]$, for market j from (2.5) it is found that,

$$\sigma^2_{j_t} + \beta^2_{j_t}\sigma^2_{F_t} = \sigma^{*2}_{j_t},$$

i.e.

$$\sigma^2_{j_t} < \sigma^{*2}_{j_t},$$

since $\beta^2_{j_t}\sigma^2_{F_t} > 0$.

Thirdly, while the total effect on conditional volatility is ambiguous[2], comovement in volatility across markets is likely to increase due to the increased importance of the global volatility factor. In fact, from (2.5) it is found

$$\lim_{\sigma^2_{j_t}\to 0} V_{t-1}\left[r_{j_t}\right] = \beta^2_{j_t}\sigma^2_{F_t}$$

[2]The level of stock market volatility may be fully idependent of the forces driving market integration and explained, for instance, by macroeconomic factors. See Beltratti and Morana (2006).

$j = x, y, z$, i.e., as the contribution of the idiosyncratic components tends to disappear, the global volatility factor becomes the source of volatility fluctuations.

Fourthly, the correlation coefficient also increases if, for each pair, the betas have the same sign. By comparing (2.6) and (2.3), for instance, for asset i and j it is found

$$\rho_t\left[r_{i_t}, r_{j_t}\right] > \rho_t\left[r^*_{i_t}, r^*_{j_t}\right],$$

since

$$\frac{\beta_{i_t}\beta_{j_t}\sigma^2_{F_t}}{\sqrt{\beta^2_{i_t}\sigma^2_{F_t} + \sigma^2_{i_t}}\sqrt{\beta^2_{j_t}\sigma^2_{F_t} + \sigma^2_{j_t}}} > 0.$$

Fifthly, a positive relationship between volatility and correlation can be noted. In fact, by computing the derivative of $\rho_{t-1}\left[r_{i,t}, r_{j,t}\right]$ with respect to $\sigma^2_{F_t}$, it is found

$$\frac{1}{2}\beta_{i_t}\beta_{j_t}\frac{\sigma^2_{F_t}\beta^2_{i_t}\sigma^2_{j_t} + \sigma^2_{F_t}\sigma^2_{i_t}\beta^2_{j_t} + 2\sigma^2_{i_t}\sigma^2_{j_t}}{\left(\sqrt{\left(\sigma^2_{F_t}\beta^2_{i_t} + \sigma^2_{i_t}\right)}\right)^3\left(\sqrt{\left(\sigma^2_{F_t}\beta^2_{j_t} + \sigma^2_{j_t}\right)}\right)^3} > 0$$

i,$j = x, y, z$, if $\beta_{i_t}\beta_{j_t} > 0$.

However, the derivative of $\rho_t\left[r_{i_t}, r_{j_t}\right]$ with respect to $\sigma^2_{i_t}$ and $\sigma^2_{j_t}$ yields

$$\frac{-\frac{1}{2}\sigma^2_{F_t}\beta_{i_t}\beta_{j_t}}{\left(\sqrt{\left(\beta^2_{i_t}\sigma^2_{F_t} + \sigma^2_{i_t}\right)}\right)^3\sqrt{\left(\beta^2_{j_t}\sigma^2_{F_t} + \sigma^2_{j_t}\right)}} < 0$$

$$\frac{-\frac{1}{2}\sigma^2_{F_t}\beta_{i_t}\beta_{j_t}}{\sqrt{\left(\beta^2_{i_t}\sigma^2_{F_t} + \sigma^2_{i_t}\right)}\left(\sqrt{\left(\beta^2_{j_t}\sigma^2_{F_t} + \sigma^2_{j_t}\right)}\right)^3} < 0,$$

i,$j = x, y, z$, respectively, if $\beta_{i_t}\beta_{j_t} > 0$.

Hence, the correlation coefficient would tend to increase with the volatility of the common factor and to decrease with the volatility of the idiosyncratic components.

Finally, comovements in conditional correlations may increase due to a dominant common volatility factor. In fact, from (2.7), after simple algebra it is found

$$\rho_{t-1}\left[r_{i_t}, r_{j_t}\right] = \rho_{t-1}\left[r_{i_t}, r_{k_t}\right]\frac{\beta_{j_t}\sqrt{\beta^2_{k_t}\sigma^2_{F_t} + \sigma^2_{k_t}}}{\beta_{k_t}\sqrt{\beta^2_{j_t}\sigma^2_{F_t} + \sigma^2_{j_t}}}$$

$i,j,k = x,y,z$, and then

$$\lim_{\sigma^2_{j_t},\sigma^2_{k_t}\to 0} \rho_{t-1}\left[r_{i_t}, r_{j_t}\right] = \rho_{t-1}\left[r_{i_t}, r_{k_t}\right].$$

Analogously,

$$\lim_{\sigma^2_{i_t},\sigma^2_{j_t}\to 0} \rho_{t-1}\left[r_{y_t}, r_{z_t}\right] = \rho_{t-1}\left[r_{i_t}, r_{k_t}\right].$$

Moreover,

$$\lim_{\sigma^2_{i_t},\sigma^2_{j_t}\to 0} \rho_{t-1}\left[r_{i_t}, r_{j_t}\right] = 1.$$

While the above discussion has been carried out considering the effects of financial markets integration, the effects of economic integration on stock markets dynamics should be considered as well. As a number of recent paper has provided evidence in favour of the existence of global factors in the world business cycle, with close interactions shown particularly by G-7 countries (see for instance Kose et al., in press; Canova and de Nicolò, 2003; Pesaran et al. 2004; Bagliano and Morana, 2006), it may be expected that common dynamics in international stock markets may also be related to common dynamics in fundamentals. In this light, many channels can be assumed, as for instance the linkage between economic activity and cash flows, and the existence of common dynamics in international interest rates and inflation rates. Moreover, the empirical evidence (see for instance Phylaktis and Ravazzolo, 2002) also suggests that financial and economic integration are closely related, the latter providing a channel for the former as well. Therefore, economic integration provides a further explanation for comovements in international stock markets.

3. The data

Following Andersen et al. (2001) and Barndorff-Nielsen and Shephard (2002), assume that the log $M \times 1$ vector price process, p_t, follows a multivariate continuous-time stochastic volatility diffusion

$$dp_t = \mu_t dt + \Omega_t dW_t,$$

where W_t denotes a standard M-dimensional Brownian motion process, and both the processes for the $M \times M$ positive definite diffusion matrix Ω_t and the M-dimensional instantaneous drift μ_t are strictly stationary and jointly independent of the W_t process.

Then conditional on the sample path realization of Ω_t and μ_t, the distribution of the continuously compounded h-period return

$$r_{t+h,h} = p_{t+h} - p_t$$

is

$$r_{t+h,h} | \sigma \left\{ \mu_{t+\tau}, \Omega_{t+\tau} \right\}_{\tau=0}^{h} \sim N\left(\int_0^h \mu_{t+\tau} d\tau, \int_0^h \Omega_{t+\tau} d\tau \right).$$

The integrated diffusion matrix

$$\int_0^h \Omega_{t+\tau} d\tau$$

can be employed as a measure of multivariate volatility.

By the theory of quadratic variation, under some weak regularity conditions,

$$\hat{\Omega}_{t+h} = \sum_{j=1,\dots,[h/\Delta]} r_{t+j\cdot\Delta,\Delta} r'_{t+j\cdot\Delta,\Delta} \xrightarrow{p} \int_0^h \Omega_{t+\tau} d\tau,$$

i.e. the realized variance covariance matrix estimator is a consistent estimator, in the frequency of sampling ($\Delta \to 0$), of the integrated variance covariance matrix. Moreover, it is also possible to write

$$\hat{\Phi}_{t+h} = \mathbf{A}^{-1/2} \left(\sum_{j=1,\dots,[h/\Delta]} r_{t+j\cdot\Delta,\Delta} r'_{t+j\cdot\Delta,\Delta} \right) \mathbf{A}^{-1/2} \xrightarrow{p} \int_0^h \Phi_{t+\tau} d\tau,$$

where

$$\mathbf{A} = diag \left\{ \sum_{j=1,\dots,[h/\Delta]} r^2_{1_{t+j\cdot\Delta,\Delta}}, \dots, \sum_{j=1,\dots,[h/\Delta]} r^2_{M_{t+j\cdot\Delta,\Delta}} \right\}$$

and

$$\begin{aligned}
\Phi_{t+\tau} &= \mathbf{B}_{t+\tau}^{-1/2}\Omega_{t+\tau}\mathbf{B}_{t+\tau}^{-1/2} \\
\mathbf{B}_{t+\tau} &= diag\left\{\Omega_{(11),t+\tau},...,\Omega_{(MM),t+\tau}\right\}^{-1/2},
\end{aligned}$$

i.e. the realized correlation matrix estimator is a consistent estimator, in the frequency of sampling ($\Delta \to 0$), of the integrated correlation matrix. Moreover, as shown by Barndorff-Nielsen and Shephard (2002), the above estimators also have Gaussian feasible limiting distributions.

On the basis of the above results, monthly realized variance ($\hat{\Omega}_{(ii),t}$), covariances ($\hat{\Omega}_{(im),t}$) and correlations ($\rho_{im,t}$) processes can be computed from daily observations as follows

$$\begin{aligned}
r_{i,t} &= \sum_{j=1}^{H} r_{i,j,t} \\
\sigma_{i,t}^2 &= \hat{\Omega}_{(ii),t} = \sum_{j=1}^{H} r_{i,j,t}^2 \\
\hat{\Omega}_{(im),t} &= \sum_{j=1}^{H} r_{i,j,t} r_{m,j,t} \\
\rho_{i,t}^m &= \hat{\rho}_{im,t} = \hat{\Omega}_{(im),t} / \left(\sqrt{\hat{\Omega}_{(ii),t}}\sqrt{\hat{\Omega}_{(mm),t}}\right),
\end{aligned}$$

where $r_{i,j,t}$ is the return on asset i at daily observation j for month t and M is the number of trading days in the month, ranging from 20 to 23 depending on the month. Only for the computation of the realized correlation process weekly returns have been used, in order to account for non overlapping trading periods. Evidence from Beltratti and Morana (in press), however, suggests that the monthly realized variance estimator computed using daily or weakly returns is not significantly affected by observational noise.

Data Stream stock price indexes have been employed for four countries, namely Germany, Japan, the US, and the UK, yielding a total of 14 processes. The time sample analyzed is from 1/01/1973 through 30/04/2004, for a total of 8175 daily observations and 376 monthly observations.

From the summary statistics for the monthly series analyzed some stylized facts can be noted. Firstly, the mean return is positive for all the countries, being highest for the UK (0.68) and lowest for Japan (0.35). Skewness is negative for all the countries apart from the UK, while excess kurtosis is positive, and particularly strong for the UK. Secondly, coherent with the presence of outliers in returns, realized variances show a large range of variation (0.8 - 692). Differently from returns, realized variances also show strong serial correlation, finding which is coherent with the long memory property generally detected in empirical studies. The strong deviation from normality is expected, being the variance process always positive. Thirdly, the correlation coefficients show a large range of variation (-0.97 - 0.99), with the average correlation coefficient ranging between 0.26 and 0.42. In general, correlations involving Japanese market returns tend to be lower than the ones involving European and US market returns. Interestingly, according to the Ljung-Box test, also the correlation coefficients show strong persistence (in four out of six cases).

4. Econometric methodology

Consider the dependent vector process $\mathbf{y}_t$ of dimension $N \times 1$. Principal component analysis (PCA) decomposes the variance-covariance matrix $\boldsymbol{\Sigma}_y$ as

$$\boldsymbol{\Sigma}_y = \mathbf{A}\boldsymbol{\Lambda}\mathbf{A}'$$

where $\boldsymbol{\Lambda}$ is the diagonal matrix of the eigenvalues of the variance-covariance matrix, and $\mathbf{A}$ is the matrix of the associated orthogonal eigenvectors. The principal components can then be computed as

$$\boldsymbol{\mu}_t^+ = \mathbf{A}'\mathbf{y}_t,$$

and the associated eigenvalues measure the variance of each principal component. The proportion of total variance accounted by the j-th principal component is then $\pi_j = \boldsymbol{\Lambda}_j / \sum_s \boldsymbol{\Lambda}_s$, where $\boldsymbol{\Lambda}_j$ is the j-th element on the main diagonal of the matrix $\boldsymbol{\Lambda}$. On the other hand, the proportion of variance of the i-th variable accounted by the j-th principal component can be computed as $\pi_{i,j} = a_{ij}^2 \boldsymbol{\Lambda}_j / (\sum_j a_{ij}^2 \boldsymbol{\Lambda}_j)$, where a_{ij} is the ijth entry in the $\mathbf{A}$ matrix. By sub sample estimation it is possible to monitor how the proportion of total variance π_j and $\pi_{i,j}$ have changed over time, allowing to assess whether the degree of comovement across processes has increased or decreased over time.

If there are common unobserved components, the variance-covariance matrix $\boldsymbol{\Sigma}_y$ is of reduced rank, and therefore not all the eigenvalues are larger than zero.

The number of common factors is then given by the number of non zero eigenvalues collected in the matrix $\mathbf{\Lambda}$, say k. By denoting the standardized principal components ($\mathbf{\Sigma}_{\mu^{++}} = \mathbf{I}_N$) as

$$\boldsymbol{\mu}_t^{++} = \mathbf{\Lambda}_*^{-1/2}\mathbf{A}_*'\mathbf{y}_t,$$

where $\mathbf{\Lambda}_*$ is the $k \times k$ sub matrix containing the non zero eigenvalues and $\mathbf{A}_*$ the $N \times k$ matrix of the associated eigenvectors, the model may then be rewritten in terms of the common factor representation

$$\mathbf{y}_t = \mathbf{\Theta}\boldsymbol{\mu}_t^{++} + \boldsymbol{\varepsilon}_t, \tag{4.1}$$

where $\mathbf{\Theta} = \mathbf{A}_*\mathbf{\Lambda}_*^{1/2}$ is the $N \times k$ factor loading matrices and $\boldsymbol{\varepsilon}_t$ is an $N \times 1$ vector of idiosyncratic components.

Recent theoretical developments of Bai (2004, 2003), Bai and Ng (2001) have justified the use of the PCA estimator also for dependent processes. In particular, Bai (2003) has considered the generalization of PCA to the case in which the series are weakly dependent processes, establishing consistency and asymptotic normality when both the unobserved factors and idiosyncratic components show limited serial correlation, also allowing for heteroschedasticity in both the time and cross section dimension in the idiosyncratic components. In Bai (2004) consistency and asymptotic normality has been derived for the case of I(1) unobserved factors and I(0) idiosyncratic components, also in the presence of heteroschedasticity in both the time and cross section dimension in the idiosyncratic components. Finally, Bai and Ng (2001) have established consistency also for the case of I(1) idiosyncratic components. As pointed out by Bai and Ng (2001) consistent estimation should also be achieved by PCA in the intermediate case represented by long memory processes, which is the relevant case for some of the processes analyzed in the paper. Monte Carlo evidence that the performance of the principal components approach is indeed not affected by the presence of long memory, being also robust to the presence of moderate noise, is provided in the Appendix.

5. Empirical results

International stock market linkages have been considered under several dimensions. Firstly, the existence of linkages across returns has been investigated. Coherent with the progressive integration of international stock markets and the globalization process, the aim of this former exercise is to assess whether global factors can be identified and their relative importance in determining stock markets dynamics. Secondly, the existence of linkages across volatility processes has been assessed. In a risk based interpretation, comovements in volatility reveal that market risk follows similar dynamics across countries. When one or more persistent common factors can be detected in a set of volatility processes, there exist linear combinations of the processes, and hence portfolios, characterized by lower persistence in volatility, that should be preferred by investors. The effects of the progressive integration of international stock markets should also be reflected

in the dynamics of the conditional correlation process. In fact, market integration should lead to an increase in conditional correlations over time, feature which should be common across correlation processes and that cannot be assessed by analyzing returns directly. Hence, as a third step the analysis of the correlation processes has been carried out. Next the presence of linkages between correlation and volatility has been assessed. Both the dominance of global factors and contagion can explain the presence of such a relationship. It is important to assess the strength and robustness of this linkage, also in the light of the persistence detected in these processes and the implications that this phenomenon may have for forecasting. In all the cases the analysis has been carried out over the period 1973-2004. Moreover, in order to assess whether the strength of comovements has changed over time, the analysis has also been repeated for three sub samples, namely 1973-1982, 1983-1992, 1993-2004.

5.1. Linkages in returns and prices

The system analyzed is composed of the return processes for the four countries considered in the sample. Hence, the common factor model can be written as

$$\mathbf{r}_t = \boldsymbol{\Theta}_r \boldsymbol{\mu}_t^r + \boldsymbol{\varepsilon}_t^r, \tag{5.1}$$

where $\mathbf{r}_t$ is a 4×1 vector of returns, $\boldsymbol{\mu}_t^r$ is a $k \times 1$ vector of common factors, with $k \leq$

4, and $\boldsymbol{\varepsilon}_t^r$ is a 4×1 vector of idiosyncratic components. Time variation in the $4 \times k$ factor loading matrix $\boldsymbol{\Theta}_r$ is assessed by means of the sub sample analysis, while $\boldsymbol{\Theta}_r$ is assumed constant when estimation is performed using the full sample. Since the the common factors are the standardized principal components $\boldsymbol{\mu}_t^r = \boldsymbol{\Lambda}^{-1/2}\mathbf{A}'\mathbf{r}_t$, the relevance of each factor can be measured by the contribution provided by each factor to the explanation of total variance, i.e. by the associated eigenvalue. Moreover, its relevance can also be assessed on the basis of the proportion of the variance of each return process explained by the factor.[3]

The results are reported in Table 1, Panel A. As is shown in the Table, the results are clear-cut, pointing to the presence of a dominant global factor over the period 1973-2004, explaining about 60% of total variance. This latter factor accounts for 61% of variance for Germany, 65% for the US, 74% for the UK and only 38% for Japan. The remaining three factors are idiosyncratic, the second accounting for 60% of variance for Japan, and the third and fourth accounting for 37% and 32% of variance for Germany and the US, respectively.

As pointed out by the sub sample analysis, the lower proportion of variance explained by the first factor for Japan, i.e. the stronger idiosyncratic component, is coherent with the different dynamics of economic fundamentals in the various countries over the 1990s, with stagnation affecting the Japanese economy only.[4] In fact, while for the 1970s the evidence in favour of the presence of a single dominant factor is weak, in the 1980s the evidence of a global factor is clear-cut. The dominant global factor explains 60% of the variance for Germany, 63% for the US, 72% for the UK and 52% for Japan. Over the 1990s this latter proportion falls to 36%, while for the other countries the figures increase to 85% for Germany and 76% for the US and the UK. Hence, the findings are coherent with previous results pointing to separate geographic areas for international stock markets, i.e the US, Europe and the Pacific Basin, albeit the US and European markets appear to have comoved very closely over the last decade. On the other hand, a stronger idiosyncratic behavior can be detected for the Japanese market. As pointed out by the sub sample analysis, the findings show that in the last decade the integration of international stock markets has progressed for the US and European markets, but not for Japan. Hence, country fundamentals for Japan have dominated the trend dynamics generated by globalization over the time period investigated. Since, as shown in the methodological framework, returns dynamics determined by common factors should imply comovements also in stock prices, the analysis has been repeated considering the log price indexes for the four stock markets. Hence, the common factor model can be written as

[3] See the methodological section.

[4] See Morana (2005) for an up to date account of macroeconomic developments in Japan since the 1990s.

$$\mathbf{p}_t = \mathbf{\Theta}_p \boldsymbol{\mu}_t^p + \boldsymbol{\varepsilon}_t^p, \tag{5.2}$$

where $\mathbf{p}_t$ is a 4×1 vector of log stock market index prices, $\boldsymbol{\mu}_t^p$ is a $k \times 1$ vector of common factors, with $k \leq 4$, $\boldsymbol{\varepsilon}_t^p$ is a 4×1 vector of idiosyncratic components, and $\mathbf{\Theta}_p$ is a $4 \times k$ factor loading matrix.

Coherent with the above results, the same pattern detected for returns can also be found for log prices, providing further support to the results of comovement in international stock markets.[5] Also in the light of the evidence in favour of economic integration, we do not think that the observed features may be fully accounted by transitory dynamics, as on the other hand pointed out by Brooks and del Negro (2004).

5.2. Linkages in conditional variance

The system analyzed is composed of the log variance processes for the four countries. The analysis has been carried out on the log variance processes rather than on the variance processes to mitigate the impact of outlying observations (volatility blips) on the estimates, also in the light of the better asymptotic properties of the estimator. The common factor model can then be written as

$$\ln \boldsymbol{\sigma}_t^2 = \mathbf{\Theta}_\sigma \boldsymbol{\mu}_t^\sigma + \boldsymbol{\varepsilon}_t^\sigma, \tag{5.3}$$

where $\ln \boldsymbol{\sigma}_t^2$ is a 4×1 vector of realized log variances, $\boldsymbol{\mu}_t^\sigma$ is a $k \times 1$ vector of common factors, with $k \leq 4$, $\boldsymbol{\varepsilon}_t^\sigma$ is a 4×1 vector of idiosyncratic components, and $\mathbf{\Theta}_\sigma$ is a $4 \times k$ factor loading matrix.

As is shown in Table 1, Panel B, over the period 1973-2004 the bulk of total variance (82%) is explained by the first two factors (60% and 22%, respectively), which according to the variance decomposition can be interpreted as global factors. In fact, the first factor explains 60% of the variance for Japan and the US, 76% of the variance for Germany, and 37% of the variance for the UK. Additional 34%, 19% and 30% of the variance are explained by the second factor for the Japan, the US and the UK, respectively. On the other hand, the second factor does not affect the German stock market. Finally, the last two factors appear to be idiosyncratic, affecting the European market and the US market, respectively. The evidence

[5]For reason of space we do not include detailed results for the log prices, which are available upon request from the authors.

provided by the sub sample analysis points to two factors explaining about 80% of total variance for the former two periods, while for the latter period is the first factor to account alone for 80% of the total variance.

The interpretation of the factors is also clear-cut. In fact, while the first factor bears the interpretation of global factor independently of the sub sample investigated, the second factor captures idiosyncratic dynamics in the various countries, and it is interpretable as Japanese factor for the most recent sub sample. This finding is coherent with the results provided by the analysis carried out on returns, pointing to the protracted stagnation of the Japanese economy over the 1990s as the explanation for the more idiosyncratic behavior shown by the Japanese stock market. As is shown in the Table, the strength of the comovement across European and US markets in the most recent sub period is impressive, with the first factor accounting for about 90% of variance for Germany and the UK and about 80% for the US. On the other hand, the percentage of variance falls to 41% for Japan, for which the second factor accounts for an additional 57% of variance.

5.3. Linkages in conditional correlations

The system analyzed consists of the six correlation processes that can be computed among the four markets in the sample. The common factor model for the correlation processes can then be written as

$$\boldsymbol{\rho}_t = \boldsymbol{\Theta}_\rho \boldsymbol{\mu}_t^\rho + \boldsymbol{\varepsilon}_t^\rho, \tag{5.4}$$

where $\boldsymbol{\rho}_t$ is a 6×1 vector of stock market index prices, $\boldsymbol{\mu}_t^\rho$ is a $k \times 1$ vector of common factors, with $k \leq 6$, $\boldsymbol{\varepsilon}_t^\rho$ is a 6×1 vector of idiosyncratic components, and $\boldsymbol{\Theta}_\rho$ is a $6 \times k$ factor loading matrix. The results of the PCA analysis are reported in Table 2. As is shown in the Table, the results for the period 1973-2004 point to the presence of a dominant factor driving the correlation processes. This factor explains about 42% of total variance. The factor also explains between 45% and 51% of the variance of the correlations involving Germany, and between 33% and 40% of the variance of the correlations excluding Germany. The second factor bears the interpretation of global factor as well, affecting all the correlation processes, albeit to a lower extent relatively to the first factor. In fact, the second factor accounts for about 20% of total variance, with the proportion of explained variance for the various processes falling in the range 10%-29%.

As is shown in Figure 1, coherent with the effects of markets integration, the first factor accounts for the trend increase in correlations over time, with the second factor also contributing to it since the beginning of the 1990s. The process of integration, as measured by the trend increase in the dominant global factor, however does not appear to have been monotonic, given the break in the level, but not in the slope, occurred in 1985. The remaining four factors appear to be idiosyncratic, in none of the cases each one affecting all the series and never explaining more than 12% of total variance. In particular, the third factor accounts for about 34% of the US-UK correlation, also affecting the Germany-US (15%) and Japan-UK (14%) correlations at a lower extent; the fourth factor explains between 11% and 19% of all the correlations apart from the Germany-Japan and Germany-US ones; the fifth factor accounts for 26% of the Germany-Japan correlation and 17% of the Japan-US correlation; finally, the sixth factor accounts for about 21% of the variance of the Germany-Japan correlation and 10% of the UK-Japan correlation.

The sub sample analysis points to some instability in the overall linkages. While the evidence of a global factor is robust across sub samples, the results point to an increasing comovement in the correlation coefficients over time. In fact, in the 1970s the first factor explains only about 32% of total variance and all the remaining factors appear to be idiosyncratic. On the other hand, in the 1980s the first factor accounts for about 38% of total variance, with the second factor affecting five correlations out of six. Finally, in the 1990s the first factor accounts for about 50% of total variance, with the first and second factors jointly accounting between 47% and 87% of the variance of the various processes.

5.4. Linkages between correlation and volatility

The existence of linkages between correlation and volatility has been assessed by means of trivariate models, composed of the log variance processes and the correlation coefficient for each pair of countries. The common factor model can then be written as

$$\boldsymbol{\varphi}_t^{xy} = \boldsymbol{\Theta}_{\varphi_{xy}} \boldsymbol{\mu}_t^{\varphi_{xy}} + \boldsymbol{\varepsilon}_t^{\varphi_{xy}}, \tag{5.5}$$

where $\boldsymbol{\varphi}_t^{xy}$ is a 3×1 vector composed of the realized log variances for the generic assets x and y and of the realized correlation for the two assets, $\boldsymbol{\mu}_t^{\varphi_{xy}}$ is a $k \times 1$ vector of common factors, with $k \leq 3$, $\boldsymbol{\varepsilon}_t^{\varphi_{xy}}$ is a 3×1 vector of idiosyncratic components, and $\boldsymbol{\Theta}_{\varphi_{xy}}$ is a $3 \times k$ factor loading matrix.

The results are reported in Table 3. As is shown in the Table, the findings point to a single dominant factor over the period 1973-2004, explaining between 43% and 60% of total variance. The factor bears the interpretation of volatility factor, explaining the bulk of volatility fluctuations (between 45% and 78%). This latter factor also has a non negligible impact on the correlation coefficient, explaining a proportion of its variance ranging between 26% and 36%. The remaining two factors are idiosyncratic, the former explaining between 27% and 32% of total variance and mostly affecting the correlation coefficient (61% - 72%), the latter explaining between 12% and 21% of total variance and affecting the volatility processes (16% - 46%). The sub sample analysis suggests some instability in the results, pointing to the 1990s as the period in which the impact of volatility on correlation has been in general strongest and to the 1980s as the period in which the impact has been weakest.[6] In Figure 1 the smoothed realized correlation processes are plotted with the corresponding smoothed volatility factors. As is shown in the plot, the linkage between volatility and correlation is positive and particularly strong for the European and US markets. Moreover, the linkage between the two processes seems to hold both when volatility increases or falls. Hence, the findings point to a non spurious and robust relationship relating correlation and volatility.

6. Conclusions

In the paper the existence of linkages across stock markets has been assessed considering several dimensions. Evidence of strong linkages across markets, as measured by comovements in prices and returns and in volatility processes, has been found. A single dominant factor can be found for the stock market prices and returns for Germany, Japan, the US and the UK over the period 1973-2004, while two global factors can be found for the volatility processes. Evidence of a trend increase in the correlation coefficients over time, explained by two dominant factors, and that the positive dependence of correlation on volatility is not spurious and robust, holding for both bear and bull markets, has also been found. All the above mentioned findings are coherent with the theoretical framework, enlightening the effects of market integration on international stock markets. Moreover, the linkages across stock markets seem to have in general grown stronger over

[6] Detailed results for the sub sample analysis are omitted for reasons of space. They are available upon request to the authors.

time, particularly for the US and Europe, while the heterogeneity detected for Japan over the 1990s suggests that, over specific periods of time, local factors, i.e. weak fundamentals, may still dominate the dynamics generated by globalization and integration. While the results are still coherent with the evidence of three separate geographic areas for international stock markets, i.e. Europe, the US and the Pacific Basin, it should be noted that the heterogeneity between Europe and the US has steadily reduced over time and that the two markets are now strongly integrated. In fact, the evidence points to a dominant factor accounting between 76% and 85% of returns variance and between 82% and 90% of volatility variance for the two areas. It may also be expected that, as the Japanese economy will fully accomplish its recovery, comovement with the Western markets will also recover to the previous high levels. Among the implications of the results there is the open question on whether the linkages detected across moments can be exploited for improved forecasting of the variance-covariance matrix and of Value at Risk and whether a structural interpretation of the common factors detected for the various moments can be provided. These issues are left for further research.

7. Appendix: Monte Carlo results

The simulated model is as follows

$$
\begin{aligned}
\begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix} &= I_2 f_t + \begin{bmatrix} v_{1,t} \\ v_{2,t} \end{bmatrix} \\
& \mathbf{v}_t \tilde{\ } n.i.d.(\mathbf{0}, \mathbf{\Sigma}) \\
(1-L)^d f_t &= \varepsilon_t \\
& \varepsilon_t \tilde{\ } n.i.d.(0,1),
\end{aligned}
$$

where $d = \{0, 0.1, 0.2, 0.3, 0.4, 1\}$, $\Sigma = diag(\sigma, \sigma)$, $\sigma = \{2, 1, 0.5, 0.25, 0.125\}$, $t = 1, ..., 400$. The number of replications (n) has been set to 20,000 for each case.

The performance of the estimator has been assessed with reference to the ability of recovering the unobserved factor f_t. The Theil inequality coefficient (IC) and the root mean square forecast error ($RMSFE$) have been employed for the evaluation

$$RMSFE = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(f_t^* - f_t)^2}$$

$$IC = \frac{RMSFE}{\sqrt{\frac{1}{T}\sum_{t=1}^{T} f_t^{*2}} + \sqrt{\frac{1}{T}\sum_{t=1}^{T} f_t^2}},$$

where $f_t^* = \frac{1}{n}\sum_{j=1}^{n}\hat{f}_{j,t}$ and $\hat{f}_{j,t}$ is the estimated unobserved factor at time t for replication j.

The results of the Monte Carlo exercise are reported in Table 4.

Table 4: Monte Carlo results

σ	2	1	0.5	0.25	0.125
UI	0.27	0.10	0.03	0.01	0.00
$RMSFE$	0.43	0.18	0.06	0.01	0.00

The Table reports the Theil inequality coefficient (IC) and the root mean square forecast error ($RMSFE$).

As is shown in the Table, the performance of the estimator is slightly negatively affected by the presence of noise, although the inequality coefficient is very low also for the worst case, which it is not expected to be relevant for realized processes. In addition, the performance of the estimator tends to improve as the noise/signal ratio falls, being extremely satisfactory in terms of inequality coefficient also with moderate noise affecting the observed variables. Finally, the performance of the estimator is not affected by the order of integration of the series, since the results are fully invariant to the order of integration assumed for the unobservable factor.[7] Overall, the findings suggest that the principal components approach may be usefully employed also in the case of long memory processes, being also robust to the presence of moderate noise.

[7] In the Table only the results for $d = 0.4$ are reported, since no changes can be detected for different orders of integration.

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Table 1: Returns and realized log variances

Panel A: Returns

	1973 – 2004					1973 – 1982			
	f_1	f_2	f_3	f_4		f_1	f_2	f_3	f_4
TOT	0.604	0.177	0.138	0.080	TOT	0.663	0.147	0.109	0.081
r_{ge}	0.611	0.003	0.370	0.017	r_{ge}	0.244	0.138	0.334	0.284
r_{ja}	0.377	0.602	0.019	0.001	r_{ja}	0.223	0.263	0.171	0.343
r_{us}	0.653	0.021	0.001	0.325	r_{us}	0.463	0.310	0.220	0.008
r_{uk}	0.743	0.090	0.124	0.044	r_{uk}	0.938	0.062	0.000	0.000
	1983 – 1992					1993 – 2004			
	f_1	f_2	f_3	f_4		f_1	f_2	f_3	f_4
TOT	0.610	0.200	0.140	0.050	TOT	0.681	0.210	0.070	0.040
r_{ge}	0.600	0.100	0.300	0.000	r_{ge}	0.853	0.063	0.081	0.003
r_{ja}	0.520	0.478	0.002	0.000	r_{ja}	0.355	0.644	0.001	0.000
r_{us}	0.626	0.068	0.125	0.181	r_{us}	0.758	0.018	0.168	0.057
r_{uk}	0.723	0.058	0.142	0.077	r_{uk}	0.764	0.032	0.042	0.163

Panel B: realized log variances

	1973 – 2004					1973 – 1982			
	f_1	f_2	f_3	f_4		f_1	f_2	f_3	f_4
TOT	0.598	0.228	0.108	0.065	TOT	0.549	0.213	0.139	0.099
$\ln\sigma^2_{ge}$	0.759	0.001	0.218	0.021	$\ln\sigma^2_{ge}$	0.517	0.045	0.141	0.296
$\ln\sigma^2_{ja}$	0.601	0.340	0.059	0.000	$\ln\sigma^2_{ja}$	0.600	0.377	0.023	0.000
$\ln\sigma^2_{us}$	0.599	0.185	0.001	0.215	$\ln\sigma^2_{us}$	0.543	0.031	0.230	0.197
$\ln\sigma^2_{uk}$	0.367	0.403	0.143	0.087	$\ln\sigma^2_{uk}$	0.513	0.275	0.210	0.002
	1983 – 1992					1993 – 2004			
	f_1	f_2	f_3	f_4		f_1	f_2	f_3	f_4
TOT	0.599	0.245	0.105	0.050	TOT	0.781	0.124	0.066	0.029
$\ln\sigma^2_{ge}$	0.598	0.275	0.126	0.001	$\ln\sigma^2_{ge}$	0.895	0.003	0.056	0.045
$\ln\sigma^2_{ja}$	0.600	0.388	0.011	0.001	$\ln\sigma^2_{ja}$	0.413	0.572	0.015	0.000
$\ln\sigma^2_{us}$	0.594	0.096	0.191	0.118	$\ln\sigma^2_{us}$	0.819	0.053	0.128	0.000
$\ln\sigma^2_{uk}$	0.606	0.013	0.193	0.189	$\ln\sigma^2_{uk}$	0.886	0.004	0.042	0.068

The table reports the percentage of variance accounted by each factor. TOT refers to total variance, r_i refers to the ith return, $\ln\sigma_i^2$ refers to to the ith log variance process. Hence, entry $(1,2)$ in Panel A is the proportion of total variance explained by the first factor, while entry $(2,1)$ is the proportion of variance of the returns for Germany explained by the first factor.

Table 2: Realized correlations

1973 − 2004

	f_1	f_2	f_3	f_4	f_5	f_6
TOT	0.418	0.192	0.117	0.103	0.088	0.081
ρ_{ge}^{ja}	0.508	0.096	0.081	0.010	0.264	0.041
ρ_{ge}^{us}	0.457	0.153	0.148	0.033	0.001	0.209
ρ_{ge}^{uk}	0.445	0.294	0.000	0.138	0.048	0.075
ρ_{ja}^{us}	0.363	0.247	0.028	0.138	0.173	0.051
ρ_{ja}^{uk}	0.395	0.248	0.138	0.111	0.008	0.100
ρ_{us}^{uk}	0.328	0.097	0.335	0.193	0.041	0.005

1973 − 1982

	f_1	f_2	f_3	f_4	f_5	f_6
TOT	0.329	0.172	0.167	0.127	0.113	0.093
ρ_{ge}^{ja}	0.411	0.116	0.002	0.346	0.040	0.084
ρ_{ge}^{us}	0.381	0.171	0.138	0.000	0.085	0.224
ρ_{ge}^{uk}	0.490	0.027	0.027	0.287	0.042	0.126
ρ_{ja}^{us}	0.268	0.108	0.225	0.053	0.343	0.004
ρ_{ja}^{uk}	0.211	0.575	0.009	0.061	0.142	0.003
ρ_{us}^{uk}	0.205	0.023	0.616	0.000	0.036	0.120

1983 − 1992

	f_1	f_2	f_3	f_4	f_5	f_6
TOT	0.387	0.191	0.141	0.117	0.088	0.077
ρ_{ge}^{ja}	0.582	0.001	0.117	0.033	0.265	0.001
ρ_{ge}^{us}	0.303	0.224	0.061	0.240	0.041	0.132
ρ_{ge}^{uk}	0.297	0.479	0.089	0.040	0.012	0.084
ρ_{ja}^{us}	0.411	0.203	0.109	0.053	0.061	0.162
ρ_{ja}^{uk}	0.501	0.144	0.050	0.153	0.069	0.083
ρ_{us}^{uk}	0.185	0.073	0.457	0.213	0.072	0.000

1993 − 2004

	f_1	f_2	f_3	f_4	f_5	f_6
TOT	0.497	0.241	0.104	0.071	0.047	0.040
ρ_{ge}^{ja}	0.763	0.087	0.000	0.003	0.059	0.087
ρ_{ge}^{us}	0.258	0.521	0.019	0.098	0.062	0.041
ρ_{ge}^{uk}	0.105	0.363	0.199	0.312	0.021	0.001
ρ_{ja}^{us}	0.649	0.051	0.203	0.069	0.005	0.023
ρ_{ja}^{uk}	0.689	0.066	0.171	0.025	0.030	0.019
ρ_{us}^{uk}	0.228	0.567	0.017	0.002	0.122	0.063

The table reports the percentage of variance accounted by each factor. TOT refers to total variance, ρ_j^i refers to the jith correlation process. Hence, entry $(1,2)$ in the table is the proportion of total variance explained by the first factor, while entry $(2,1)$ is the proportion of variance of the realized correlation between returns for Germany and Japan explained by the first factor.

Table 3: Realized log variances and correlations

	$GE-JA$				$GE-US$		
	f_1	f_2	f_3		f_1	f_2	f_3
TOT	0.555	0.294	0.151	TOT	0.603	0.265	0.132
$\ln\sigma^2_{ge}$	0.720	0.047	0.234	$\ln\sigma^2_{ge}$	0.721	0.083	0.197
$\ln\sigma^2_{ja}$	0.689	0.093	0.218	$\ln\sigma^2_{us}$	0.724	0.078	0.197
ρ^{ja}_{ge}	0.257	0.741	0.002	ρ^{us}_{ge}	0.364	0.636	0.000
	$GE-UK$				$JA-US$		
	f_1	f_2	f_3		f_1	f_2	f_3
TOT	0.512	0.321	0.167	TOT	0.501	0.292	0.207
$\ln\sigma^2_{ge}$	0.740	0.000	0.260	$\ln\sigma^2_{ja}$	0.527	0.240	0.233
$\ln\sigma^2_{uk}$	0.496	0.343	0.161	$\ln\sigma^2_{us}$	0.639	0.016	0.345
ρ^{uk}_{ge}	0.299	0.620	0.081	ρ^{us}_{ja}	0.338	0.620	0.042
	$JA-UK$				$US-UK$		
	f_1	f_2	f_3		f_1	f_2	f_3
TOT	0.434	0.299	0.268	TOT	0.587	0.289	0.124
$\ln\sigma^2_{ja}$	0.507	0.030	0.464	$\ln\sigma^2_{us}$	0.775	0.029	0.195
$\ln\sigma^2_{uk}$	0.445	0.258	0.297	$\ln\sigma^2_{uk}$	0.707	0.124	0.169
ρ^{uk}_{ja}	0.350	0.608	0.042	ρ^{uk}_{us}	0.279	0.715	0.007

The table reports the percentage of variance accounted by each factor. TOT refers to total variance, $\ln\sigma^2_i$ refers to the ith log variance process and ρ^i_j to the jith correlation process. Hence, entry $(1,2)$ in the table is the proportion of total variance explained by the first factor, while entry $(2,1)$ is the proportion of variance of the realized log variance for Germany explained by the first factor. Results are for the period 1973-2004.

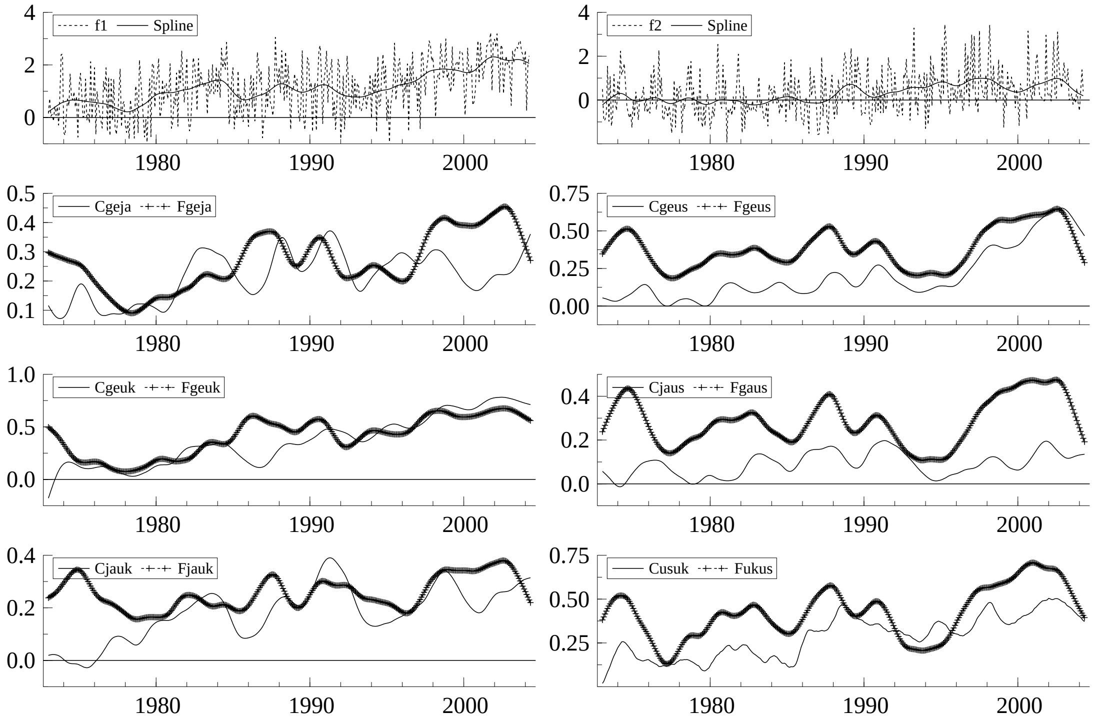

Figure 1: Plots 1-2: realised correlations, first and second factor (f_i) with spline interpolator. Plots 3-8: smoothed realised correlations (C_{ij}) and volatility factors (F_{ij}).