Spin model with negative absolute temperatures for stock market forecasting

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Abstract

A spin model relating physical to financial variables is presented. Based on this model, an algorithm evaluating negative temperatures was applied to New York Stock Exchange quotations from May 2005 up to the present. Stylized patterns resembling known processes in phenomenological thermodynamics were found, namely, *population inversion* and the *magnetocaloric effect*.

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During recent years there has been increasing interest in the application of statistical mechanics to the study of financial markets and macroeconomic systems in general [1–3]. Thus it is that the neologism "econophysics" has become well-known at the present time. A variety of theoretical models have been proposed, from stochastic to deterministic, trying to explain market dynamics by interaction between agents. In particular, spin models have been the focus of important research works [4–7]. However, a model establishing a nexus between microscopic and macroscopic variables, between pure theory and real finance, is lacking. The principal aim of this work is to interpret the variables of a spin model and their mathematical transcription into financial terms for the application of the model to price trend forecasts in stock markets.

Explicitly, we will consider a stock market as a set of agents or traders similar to spins with a value of ± 1. Since this model is established in order to describe market behavior in the short term, we assume the total number of traders to be constant. The opinion position adopted by the mass media is equivalent to an external field B [4, 8]. We assume an interaction constant J between spins. Thus, the energy of a configuration $\{s_i\}$, $i = 1, \ldots, N$ is specified by

$$E(\{s_i\}) = -B \sum_{i=1}^{N} s_i - J \sum_{i=1}^{N} \sum_{j=i+1}^{N} s_i s_j. \tag{1}$$

This spin model is mathematically equivalent to a lattice gas with N nodes whose occupation variable is defined by $\tilde{s}_i = (s_i - 1)/2$, i.e., $\tilde{s}_i = 1$ if a node is occupied and $\tilde{s}_i = 0$ if it is unoccupied. This consideration is useful when it comes to analyze some issues, such as price time series to be modeled by means of a one-dimensional Brownian motion of a particle suspended in the lattice gas. Assuming such a particle is in approximate thermodynamic equilibrium with the lattice gas, by combining Einstein's equation for mean square displacement [9] $D = \langle x^2 \rangle/(2\tau)$ with the Einstein-Smoluchowski relation [10] $D = \mu k_B T$, we obtain

$$T \propto \langle x^2 \rangle/\tau, \tag{2}$$

where T is the absolute temperature, $\langle x^2 \rangle$ is the mean square displacement, and τ is the mean time between collisions. We then estimate this in financial terms by

$$\tau = 1/n_t, \quad \langle x^2 \rangle_t = \frac{1}{n_t} \sum_{i=1}^{n_t} r_{i,t}^2, \tag{3}$$

$$r_{i,t} = \ln p_{i,t} - \ln p_{i-1,t},$$

where n_t is the number of intraday returns, $r_{i,t}$ the ith return on the tth trading day, and $p_{i,t}$ represents the corresponding prices. The term

$$\sigma_t = \left(\frac{1}{n_t} \sum_{i=1}^{n_t} r_{i,t}^2 \right)^{1/2} \tag{4}$$

is known as (historical) *volatility* in the financial literature [11]. Accordingly, we define the absolute temperature of the spin system on the tth trading day as

$$T_t \propto n_t \langle x^2 \rangle = \sum_{i=1}^{n_t} \left(\ln p_{i,t} - \ln p_{i-1,t} \right)^2 = n_t \sigma_t^2. \tag{5}$$

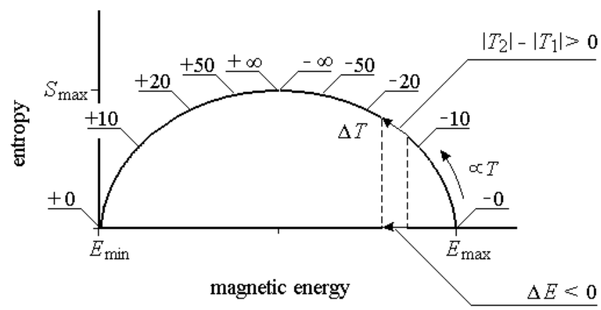

FIG. 1. Entropy vs. energy for a magnetic system. If $\Delta\bar{E} < 0$ and $\mid T_2 \mid - \mid T_1 \mid > 0$, then the temperature sign is negative.

This definition agrees with [2], where the second moments of returns are stated to relate to the system temperature.

Under these conditions, the entropy S and mean energy $\bar{E}$ per spin are bounded from above: $0 \leq S \leq S_{\max}$, and $E_{\min} \leq \bar{E} \leq E_{\max}$. Also the entropy S is a function of energy $\bar{E}$ with an arched shape [12], as shown in Fig. 1.

Next, we define the energy increment $\triangle\bar{E}$ on the tth day to be

$$\Delta\bar{E}_t \propto \frac{\sum_{i=1}^{n_t}(\ln p_{i,t} - \ln p_{i-1,t})v_{i,t}}{\sum_{i=1}^{n_t} v_{i,t}}, \tag{6}$$

where $v_{i,t}$ is the ith lot of shares on the tth day. Such a definition agrees with [1], where money and energy are treated as equivalent concepts. It is also consistent with [6], where logarithmic relative changes of price depend on magnetization increments, and thus on energy per spin.

Negative temperatures are mathematically characterized as follows [13] : $T^{-1} = (\partial S/\partial\bar{E})_X$, i.e., if a positive entropy increment corresponds to a negative energy increment, additional thermodynamic variables X being constant, then the temperature is negative. Thus, it follows that $\Delta\bar{E} < 0$ and $\mid T_t \mid - \mid T_{t-1} \mid > 0$ implies $T < 0$ (as shown in Fig. 1). Furthermore, negative temperatures correspond to high values of energy per spin, such that the system equilibrium is unstable and this tends to result in an increment in absolute values of the temperature $\mid T \mid$ [12]. In this context, it should be mentioned that peaks of "market temperature" are often related to dramatic price movements [14].

At this point, let us digress to consider the Black-Scholes (BS) partial differential equation [2, 3]:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0,$$

where S is a stock price, σ its volatility, V the price of a *derivative*, e.g., a call option [15], and r the risk-free interest rate. As is known, this equation is a particular case of a *diffusion equation*. Indeed, by the following change-of-variable transformation $V = ue^{rt}$, $x = \ln S$, $z = x - (r - \sigma^2/2)t$, and $t' = t_e - t$ [16], the BS equation turns into:

$$\frac{\partial u}{\partial t'} = \frac{1}{2}\sigma^2 \frac{\partial^2 u}{\partial x^2}.$$

This may be interpreted as a *heat transfer* (diffusion) equation [2] where the squared volatility σ^2 is proportional to the *diffusion ratio*, in agreement with [17]. A solution of such an equation is the BS formula [2, 3], which is used for derivatives pricing, e.g., put and call options, and *implied volatility* [18] estimation:

$$V = S\mathcal{N}(d_1) - Ke^{rt'}\mathcal{N}(d_2), \tag{7}$$

where K is the strike price, $\mathcal{N}(d)$ the standard normal cumulative distribution function, $d_1 = [\ln(S/K) + (r + \sigma^2/2)t']/(\sigma\sqrt{t'})$, and $d_2 = d_1 - \sigma\sqrt{t'}$.

Following the above prolegomena, we propose a temperature sign characterizing algorithm as follows: (i) First, we calculate $\Delta\bar{E}_t$ by Eq. (6) on the tth trading day. Then, we calculate temperatures T_t and T_{t-1} by Eq. (5). (ii) Next, we estimate the temperature trend as follows:

(iia) We calculate intraday volatilities σ_{t-1}, σ_t corresponding to the $(t-1)$th and tth trading days by Eq. (4). Next, by entering σ_{t-1} , $S = K = p_{c,t-1}$ ($p_{c,t-1}$ being the closing price on the $[t-1]$th trading day) and an initial value of $t' = 40$ days into the BS formula (7), we get a fictitious price of a fictitious derivative $\hat{p}_t = V$. (iib) We enter $V = \hat{p}_t$ and $S = p_{c,t}$ into the BS formula (7) again, and by a numerical method, e.g., Newton-Raphson, we obtain the implied volatility $\hat{\sigma}_t = \sigma$. Note that the volatility obtained from Eq. (4) is the so-called *historical* volatility, whereas that obtained from the BS formula (7) is referred to as *implied.*

(iii) Finally, having evaluated the following conditions:

$$\Delta\bar{E} < 0 \quad \text{AND} \quad T_t - T_{t-1} > 0, \tag{8}$$

$$\hat{\sigma}_t - \sigma_t > 0, \tag{9}$$

if (8) AND (9) hold, then we will conclude temperature sign to be negative [19]. These conditions may be adjusted by setting nonzero threshold values, which will depend on which particular stock or index is to be analyzed, in order to achieve an optimal predictive result.

Although condition (8) appears to be sufficient, there is a slightly more sophisticated reason for introducing condition (9): As is well-known, a magnetic system is modeled by two subsystems, namely, the *lattice* and the *spins* [12]. These subsystems are in thermal contact and may fluctuate around an equilibrium point. Thus, energy flows between them; e.g., when the spins experience a negative temperature, energy flows *from* the spins *to* the lattice, since negative temperatures are hotter than positive ones [13]. Also note that there is no upper limit for the lattice energy; thus, its Kelvin temperature is always positive. In our own experience, historical volatility relates to lattice temperatures, whereas implied volatility relates to spin temperatures. Thus, historical volatility is a good estimation of spin temperature whenever both subsystems are in mutual equilibrium; however, if they fluctuate, it is necessary to introduce implied volatility for a good estimation of the spin temperature, as for magnitude and sign.

The time series of daily temperatures is a good *leading indicator* for trend changes in markets. By applying the algorithm described above to market data from May 2005 up to the present, we found several temperature patterns, two of which are especially significant since they resemble well-known processes in phenomenological thermodynamics. Indeed, Fig. 2 shows a stylized pattern similar to what appears in so-called *population inversion*: The external field reverses so suddenly that the spins cannot follow such a switch-over. This leaves the spin system in a state of nonequilibrium. During a certain period, the spins reach a new equilibrium state. Through this transition, the temperature increases and decays corresponding to a theoretical passage from $T = -\infty$ to $T = +\infty$ [12]. Note the dramatic change in index trend that occurs on subsequent trading days. By contrast, Fig. 3 shows a stock whose investors are "aligned" in expectation of a generous dividend payout. Such an

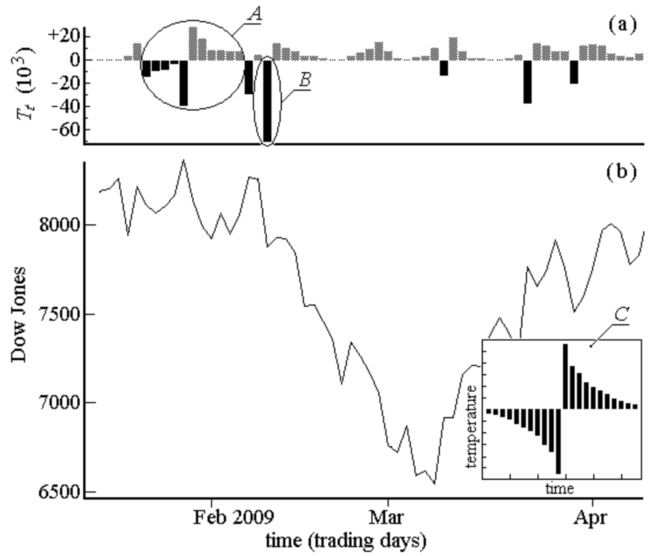

FIG. 2. (a) Time series of temperatures T_t corresponding to (b) the Dow Jones index. The inset C shows how a discrete register of a physical population inversion would be. Compare the inset with detail A.

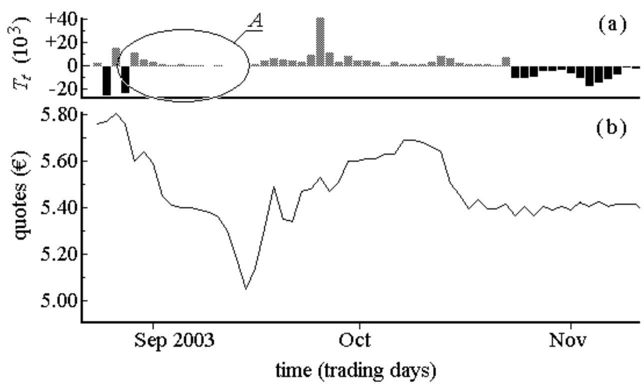

FIG. 3. (a) Time series of temperatures T_t corresponding to (b) quotes of a particular stock. After a dividend payout, this stock suffers an extreme "cooling" near to $T_t = 0$ (detail A).

expectation represents a strong external field that suddenly vanishes on payday. Then, that stock experiences a cooling near to $T = 0$, resembling a *magnetocaloric effect* (detail A). Note the dramatic drop in prices because of investors' trend towards random self-alignment; this may induce an *on-stop orders* collapse depending on "order density."

The magnitude of negative temperature peaks (see detail B in Fig. 2) correlates with subsequent prices or index movements (upward or downward). Figure 4 shows how such magnitudes are correlated for the Dow Jones Industrial Average using a sample consisting of 109 cases of negative temperatures that occurred from May 2005 to January 2012.

An online database daily updated with the latest data from the New York Stock Exchange is freely available to every reader wanting to review graphs like those in Fig. 2 and 3 [20].

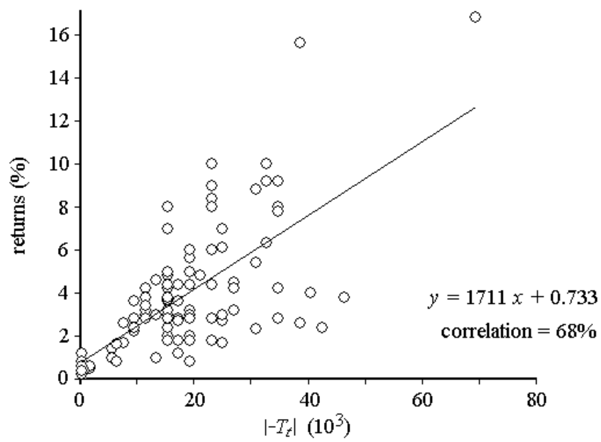

FIG. 4. Correlation and linear regression between magnitude of negative temperature peaks $| -T_t |$ and subsequent movement (upward or downward) of Dow Jones within the 14 following days. The Pearson correlation coefficient equals 68%.

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[18] The volatility may be *historical* or *implied*. See [2, 3].
[19] A feedback loop is also necessary to maximize the correlation coefficient (see Fig. 4) by varying t'.

[20] Http://www.didyf.unizar.es/info/jlsubias/econophysical-trend-indicator.htm.