On the role of backauditing for tax evasion in an agent-based Econophysics model

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We investigate an inhomogeneous Ising model in the context of tax evasion dynamics where different types of agents are parametrized via local temperatures and magnetic fields. In particular, we analyse the impact of backauditing and endogenously determined penalty rates on tax compliance. Both features contribute to a microfoundation of agent-based econophysics models of tax evasion.

I. INTRODUCTION

One of the first approaches to theoretically account for tax compliance was given by Allingham and Sandmo[1], which incorporates tax rates, potential penalties and audit probabilities as basic parameters in order to evaluate the behavior of expected utility maximizing tax payers. However, it was realized early on that one of the major shortcomings of the Allingham and Sandmo theory is the prediction of much too low levels of tax compliance, as actually observed in industrialized nations. It is believed that these shortcomings are partially due to the insufficient consideration of interaction dynamics among the actors of the tax compliance game, i.e., tax payers, tax advisors, tax authorities, and tax law makers. Another reason might be that many analyses have not incorporated backauditing. Although Allingham and Sandmo already considered backauditing (or lapse of time effects) as a possible extension of their basic theory, the issue of backauditing has been largely neglected in the literature. Noteable exceptions are Alm *et al.*[16], Antunes *et al.*[12] and Hokamp and Pickhardt[15].

It is for these reasons that agent-based models have been set up as a comparatively new tool for analyzing tax compliance issues. In fact, an essential feature of any agent-based model is the direct non-market based interaction among agents, which is combined with some process that allows for changes in individual behavior patterns. Therefore, agent-based tax evasion models may be categorized according to the features of this individual interaction process. In econophysics models this process is commonly described within the Ising model[2] where examples include Zaklan et al.[3,4], Lima and Zaklan[5], and Lima[6].

In contrast, if the interaction process is driven by parameter changes that induce behavioral changes via a utility function and (or) by stochastic processes that do not have physical roots, these models belong to the economics domain. Examples include Mittone and Patelli[7], Davis et al.[8], Bloomquist[9,10], Korobow et al.[11], Antunes et al.[12], Szabó et al.[13], Meder et al.[14], Hokamp and Pickhardt[15], of which some are summarized by Bloomquist[10] and Pickhardt and Seibold[24].

In all agent-based tax evasion models the actual patterns and levels of tax evasion depend on two additional factors: the network structure of society and the tax enforcement mechanism. The network structure is implemented by alternative lattice types and tax enforcement consists of the two economic standard parameters audit probability and penalty rate.

In the present paper we study the effect of alternative backauditing schemes on tax evasion within an multi-agent econophysics model. Moreover, backauditing also enables us to incorporate endogenously determined penalty rates. Both features, backauditing and endogenous penalties, contribute to the microfoundation of econophysics models and, therefore, allow for a more realistic modelling of tax compliance behavior within this modelling frame.

We present our model and formalism in Sec. II. Results are analyzed in Sec. III, where we discuss first the case of homogeneous societies in Sec. III A, which is then compared with the tax evasion of multi agent societies in Sec. III B. We conclude our considerations in Sec. IV.

II. MODEL AND FORMALISM

Our considerations are based on the Ising model, described by the following hamiltonian,

$$H = \sum_{ij} J_{ij} S_i S_j + \sum_i B_i \tag{1}$$

where J_{ij} describes the coupling of Ising variables (spins) $S_i = -1,1$ between lattice sites R_i and R_j. In the present context $S_i = 1$ is interpreted as a compliant tax payer and $S_i = -1$ as a non-compliant one. We implement the model on a two-dimensional 1000×1000 square lattice with nearest-neighbor interactions $J_{ij} \equiv J$ for $|R_i - R_j| = 1$ (lattice constant $a \equiv 1$). We expect that our results are robust with regard to variations of the network structure in analogy to the investigations of Zaklan *et al.* Ref.[3]. Eq. (1) contains also the coupling of the spins to a local magnetic field B_i which together with a local temperature T_i distinguishes the behaviorally different types of agents. Concerning the latter we assume that it is imposed by the coupling of each lattice site i to a heat-bath with temperature T_i[20], as if it were part of a canonical ensemble. We then use the heat-bath algorithm [cf.[17]] in order to evaluate statistical averages of the model. The probability for a spin at lattice site i to take the values $S_i = \pm 1$ is given by

$$p_i(S_i) = \frac{1}{1 + \exp\{-[E(-S_i) - E(S_i)]/T_i\}} \tag{2}$$

and $E(-S_i) - E(S_i)$ is the energy change for a spin-flip at site i. Upon picking a random number $0 \le r \le 1$ the spin takes the value $S_i = 1$ when $r < p_i(S_i = 1)$ and $S_i = -1$ otherwise. One time step then corresponds to a complete sweep through the lattice. We note that a generalization of the model with regard to the incorporation of non-equilibrium dynamics has been recently proposed in Refs.[6,18,19].

Following Ref.[15] we consider societies which are composed of the following four types of agents: (i) *selfish a-type agents*, which take advantage from non-compliance and, thus, are characterized by $B_i/T_i < 0$ and $|B_i| > J$; (ii) *copying b-type agents*, which copy tax behavior of their social environment or neighborhood. This can be modelled by $B_i << J$ and $J/T_i \gtrsim 1$; (iii) *ethical c-type agents*, which are practically always compliant and which are parametrized by $B_i/T_i > 0$ and $|B_i| > J$; (iv) *random d-type agents* which are in principle like c-types, but due to some confusion caused by tax law complexity act by chance within a certain range. We implement this behavior by $B_i << J$ and $J/T_i << 1$. Here and in the following all parameters are measured with respect to $J \equiv 1$. Note that with regard to the previous definitions the analysis in Ref.[4] corresponds to homogeneous societies of b- and d-type agents. Moreover, the model in Ref.[5] studies homogeneous societies of a-type and c-type agents. Our aim instead is the investigation of heterogeneous societies and the influence of different backaudit schemes and endogenously determined penalty rates on the extend of tax evasion.

We consider first the case where the detection of an evading agent in the current period enforces its compliance over the following h time steps or periods. Note that in econophysics models of tax evasion h is regarded as the penalty rate. The aforementioned procedure has been invoked in Refs.[3,4,6,24] and also implemented in a randomized variant in Ref.[5]. In Ref.[5] h has been interpreted as the time over which an agent is ashamed and feels guilty about his behavior after detection. An alternative interpretation would be definite audits by the tax authorities over h time steps after an agent has been detected for the first time, a scheme that is known as conditional future auditing (Alm *et al.*[16]). In addition, we study the situation where an audit may also allow for screening the agent's tax declaration over several years in the past (backaudit). The basic idea is that a backaudit is only performed by the tax authorities upon reasonable initial suspicion, which in our model corresponds to tax evasion in the current period. In the literature this scheme is known as conditional backauditing (see Alm *et al.*[16]). We therefore introduce a probability p_a with which an audit is performed at a given lattice site (agent). If tax evasion is detected in the current period, the backaudit comprises also an inspection of the preceding b_p time steps. Denote with n_e the number of time steps over which the agent was evading during the backaudited periods plus the current period. Then the number of future periods k, over which the agent is forced to be compliant, is set to $k = n_e * h$. Thus, the penalty rate k is now endogenously determined. Note, however, that the above limit of a fixed number of enforced future compliance periods h is recovered in the limit of no backaudit $b_p = 0$.

III. RESULTS

In this section we will first present results for the case of homogeneous societies consisting of either a, b, c- or d-type agents. This allows for extracting their specific behavior patterns under the three different audit schemes which we consider: (i) $b_p = 0$ with penalty rate $h = 4$, (ii) $b_p = 5$ with penalty rate k, (iii) $b_p = 10$ with penalty rate k. It turns out that agent behavior is partially modified in a heterogeneous society due to interaction dynamics between different agent groups.

A. Homogeneous societies

Fig. 1 displays the tax evasion dynamics for a homogeneous system of selfish a-type agents. The endogenous preference with regard to non-compliance is incorporated by a negative field $B = -15$, which is large in magnitude both with regard to the exchange coupling $J \equiv 1$ and the temperature scale $T = 5$.

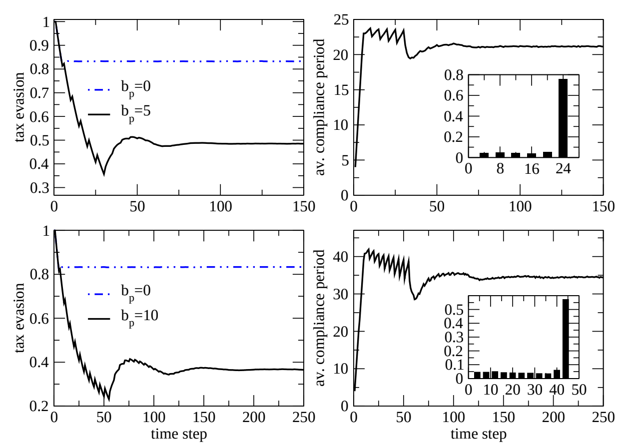

FIG. 1: Tax evasion dynamics for a society of a-type agents (temperature $T = 5$, external field $B = -15$). The upper (lower) panels display tax evasion for backaudit period $b_p = 5$ ($b_p = 10$). In both panels the dashed line corresponds to $b_p = 0$ and $h = 4$. In all cases the audit probability is set to $p_a = 5\%$.

As initial condition we set the value of all agents to -1, i.e. 100% tax evasion[25]. The upper left panel of Fig. 1 compares the time evolution for the extend of tax evasion in case of no backauditing ($b_p = 0$) and a fixed compliance period $h = 4$ (audit probability $p_a = 5\%$) with backauditing ($b_p = 5$). Note that in the latter case the maximum penalty is $k = 6 * 4 = 24$ since the current period is included in the counting of total evasion periods n_e. Up to time step $t_i = 6$ the dynamics of both methods are undistinguishable, whereas for $t_i > 6$ tax evasion without backauditing ($b_p = 0$) rapidly saturates in contrast to the curve with backauditing. In fact, since we start from 100% tax evasion it is unlikely that for the small p_a we are considering an agent which has been forced to compliance, is audited. Therefore, in both methods the extend of tax evasion is initially reduced in each time step by p_a, i.e. decreasing to 95% , 90% , 85% , 80% in time step $t_i = 2, 3, 4$, 5. Agents detected in period $t_i = 1$ can start to cheat again in period $t_i = 6$. In case of no backauditing this fraction of evading agents will be almost compensated by the newly detected evadors which drives the system into the stationary state. On the other hand, backauditing ($b_p = 5$) forces agents which have been detected in $t_i > 1$ to stay compliant over a larger period so that the corresponding curve continues to decrease. The following small upward spikes occur when these agents turn to non-compliance again. The decrease continues until agents can acquire the maximum compliance period of 24 time steps. This is the case after time step $t_i = 30$ where agents which have been convicted in period $t_i = 6$ return to non-compliance. As a consequence the tax evasion probability starts to increase again and slowly approaches a stationary state. The upper right main panel illustrates the time evolution of the average compliance period of detected agents. As anticipated, it rapidly increases within the first 6 time steps and converges to a stationary value of $k \approx 21$ after ~ 80 time steps. Note that the average k is smaller than the maximum value of $k = 24$ due to detected agents which have been compliant within previous $b_p = 5$ time steps. The corresponding distribution is shown in the inset to the upper right panel of Fig. 1. Clearly, most of the detected agents ($\sim 75\%$) are penalized with a compliance period of $k = 24$, whereas the the remaining 25% are forced to compliance over $4, 8, 12, 16, 20$ time steps with almost equal probability of $\sim 5\%$. The lower two panels of Fig. 1 analyze the analogous situation for the larger backaudit period of $b_p = 10$. Clearly this reduces even more the percentage of tax evasion with respect to no backauditing, but on the other hand increases the transient oscillation towards the stationary state.

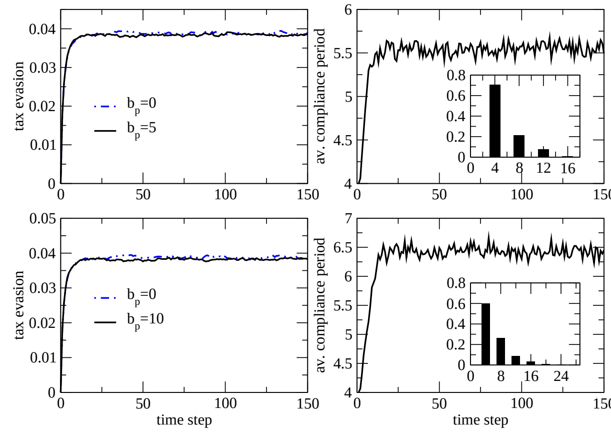

FIG. 2: Same as Fig. 1 but for a society of b-type agents (temperature $T = 2$, external field $B = 0$).

Fig. 2 displays the situation for a homogeneous society of copying b-type agents which are characterized by $B = 0$ and $T = 2$, i.e. the temperature is below the ordering transition of the two-dimensional Ising model $T_c \approx 2.269$. Therefore, b-type agents are guided by a line of action which copies the behavior of its neighbors (i.e. sites which are coupled by $J \equiv 1$). As initial condition all agents are set to full compliance (tax evasion 0%). Due to the small but finite temperature some of the agents occasionally change to non-compliance. However, the small audit probability implies also a small probability that agents are detected twice or more often within b_p time steps. Therefore, for both backaudit periods $b_p = 5, 10$ the majority of agents is sconced with a compliance period of $k = 4$ (cf. insets to the right panels of Fig. 2) so that the average compliance period is only slightly larger than in case of no backaudit $b_p = 0$ (cf. main right panels). As a result, one obtains a small stationary value of tax evasion of $\sim 4\%$ after ~ 15 time steps where the curves with and without backaudit are practically indistinguishable (cf. left panels of Fig. 2). It should be noted that the stationary value is *independent* of the initial condition. Even in case of 100% tax evasion in time step $t_i = 1$ the detected agents have a small probability to return to non-compliance resulting in the same stationary value after a somewhat longer transient oscillation of $\sim 30 - 40$ time steps.

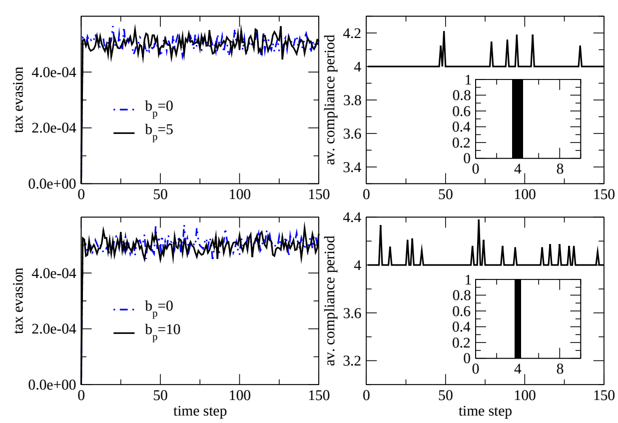

FIG. 3: Same as Fig. 1 but for a society of c-type agents (temperature $T = 5$, external field $B = +15$).

The case of ethical c-type agents is shown in Fig. 3. These are modeled by a strong positive field $B = 15$ (i.e. $B/J > 1$ and $B/T > 1$ with $T = 5$) which enforces compliant behavior. This strong field suppresses the probability for a behavioral change towards non-compliance.

From the right panels of Fig. 3 one can see that the average compliance period is dominantly $k = 4$ (cf. insets), with incidental spikes when some of the agents are detected twice within b_p time steps. Therefore, the number of spikes increases upon increasing the backaudit period from $b_p = 5$ to $b_p = 10$. In any case, backauditing has only a minor effect on c-type agents and the tax evasion rapidly approaches a rather small stationary value of $\sim 5 \cdot 10^{-4}$, which would tend to zero in the limit $B/J, B/T \to \infty$.

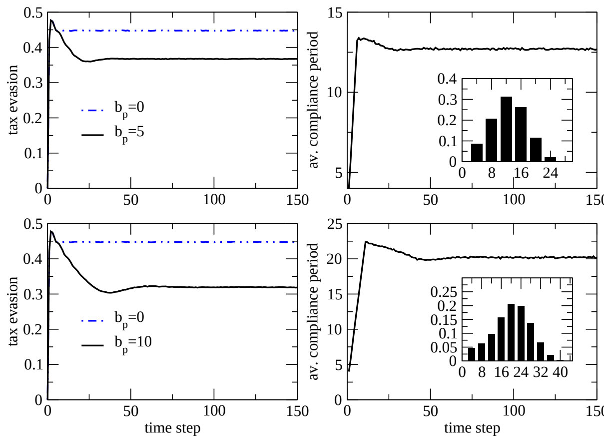

FIG. 4: Same as Fig. 1 but for a society of d-type agents (temperature $T = 20$, external field $B = 0$).

Finally, Fig. 4 displays the case of a homogeneous society of d-type agents which act independently from other agents and, thus, are characterized by a large temperature $T = 20$ and $B = 0$. Setting as initial condition all agents to compliance, the large temperature induces a change to non-compliance for almost half of the society within the first two time steps (cf. left panels of Fig. 4). Then the audit mechanisms drive the system again towards the stationary states, which in case of d-type agents is different for the cases with and without backauditing. In fact, since d-type agents act independently we expect them on average to be half of the time compliant and non-compliant, respectively. The backaudit within $b_p{+}1$ time steps (i.e. including the current period) thus leads to an average enforced compliance period of $k = h \cdot (b_p + 1)/2$. This agrees with the stationary values of the average compliance curves shown in the right panels of Fig. 4 and the actual distribution is shown in the insets. The increased compliance period under backauditing results in a smaller stationary value of tax evasion, as can be seen in the left panels of Fig. 4. Naturally, this value decreases with the length of the backaudit period b_p.

B. Heterogeneous societies

We continue our investigations by considering societies which contain all four behaviorly different types of agents. In order to account for heterogeneity also within the individual agent groups, we allow for parameter variations concerning the characterizing temperature $T_{min} \le T \le T_{max}$ and fields $B_{min} \le B \le B_{max}$. Table I specifies the parameter windows used in the following analysis which correspond to a flat distribution around the same mean values used for the homogeneous societies in the previous subsection.

The results shown in Figs. 5, 6, 7 have been obtained for societies with a fixed share of 35% b-type and 15% d-type agents. We then investigate the influence of different population shares of selfish a-type and ethical c-type agents on the extend of tax evasion and on tax evasion dynamics.

TABLE I: Parameter set for the results shown in Figs. 5 to 7. Note that T and B are measured in units of $J \equiv 1$.

$Agent - type$	T_{min}	T_{max}	B_{min}	B_{max}
a	5	5	-20	-10
b	1	3	0	0
c	5	5	10	20
d	10	30	0	0

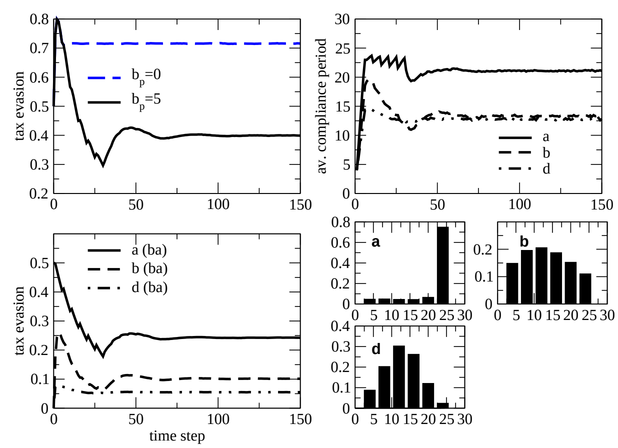

FIG. 5: Tax evasion dynamics for a society consisting of 50% a-type, 35% b-type, and 15% d-type agents. The upper left panel compares the extend of tax evasion and the dynamics of tax evasion for backaudit periods $b_p = 0$ and $b_p = 5$. The lower left panel breaks down the evasion probability to the individual agent types in case of backaudit period $b_p = 5$. The upper right panel displays the average forced compliance period (or penalty) for the individual agent types and the lower right panels show the corresponding distribution. Audit probability is $p_a = 5\%$ in each case.

In Fig. 5 the percentage of a-type agents is set to 50% and the upper left panel compares the extend of tax evasion and the dynamics of tax evasion for backaudit periods $b_p = 0$ and $b_p = 5$. The lower left panel reveals the contributions of the individual agent groups. With respect to the steady state it turns out that the a-type agents contribute with $\sim 25\%$ to the overall tax evasion, which means that $\sim 50\%$ of a-type agents are evading in agreement with the result for the homogeneous a-type society in Fig. 1. Similarly, the d-type agents contribute with $\sim 5\%$ to tax evasion so that approximately 30% of this group is evading, again in agreement with the homogeneous result displayed in Fig. 4. A new feature arises due to the interaction of copying b-type agents with their neighbors. In fact, we observe a corresponding contribution of $\sim 10\%$ to overall tax evasion, which means that $\sim 28\%$ of b-type agents are evading, compared to 4% in the homogeneous society (Fig. 2). This is of course due to the interaction with a-type and to a much smaller extend with d-type agents. As a result the average compliance period of b-types reaches now the value of d-types (upper right panel of Fig. 5), which, together with the a-types, acquire approximately the same value than in the homogeneous case. Also inspection of the forced compliance period (or penalty) distributions (lower right panels of Fig. 5) reveals a much broader spreading as compared to Fig. 2, with the maximum at 12 periods.

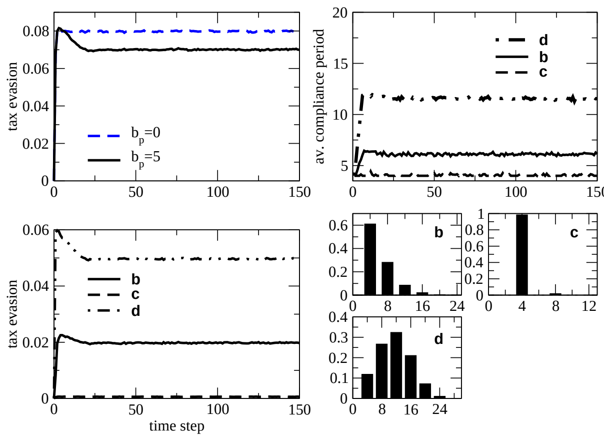

FIG. 6: Same as Fig. 5 but for a society consisting of 35% b-type, 50% c-type, and 15% d-type agents.

The copying feature of b-type agents has much less influence on tax evasion when we replace the 50% a-type by 50% ethical c-type agents, as shown in Fig. 6. In fact, because in the stationary state the large majority of b-types tends to be compliant, the result of Fig. 6 corresponds almost to the result one would obtain by just averaging the results of Figs. 2,3,4, taking into account the parameter distributions.

The case of a society with all four types of agents is shown in Fig. 7. Here, those b-types which are the nearest neighbors of a-type agents are predominantly evading, whereas b-types which are the nearest neighbors of c-types copy the corresponding compliant behavior. In addition, also the fluctuating behavior of d-types is copied by neighboring b-types, similarly to the previous figures 5, 6, 7. As a result, we find that almost $\sim 20\%$ of b-types are evading, a value slightly lower than in Fig. 5, but still larger than in the homogeneous case. Inspection of the

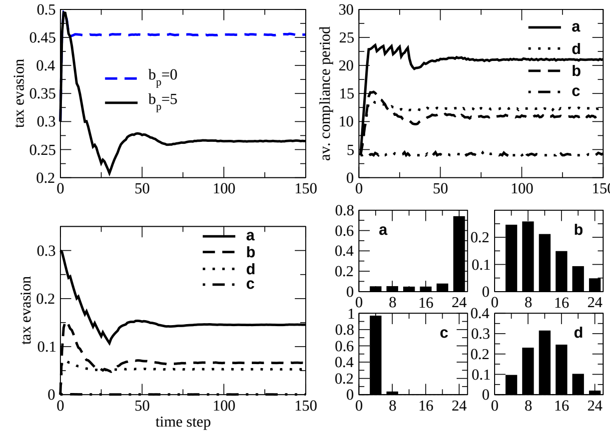

FIG. 7: Same as Fig. 5 but for a society consisting of 30% a-type, 35% b-type, 20% c-type, and 15% d-type agents.

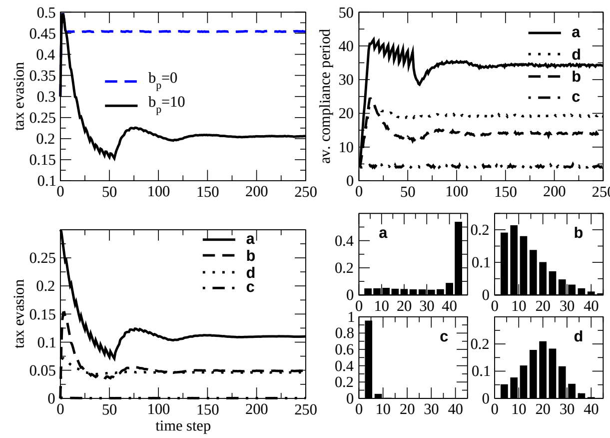

FIG. 8: Same as Fig. 7 but for backaudit period $b_p = 10$.

distribution of the compliance period for b-type agents yields a compromise between the situations shown in Fig. 5 and Fig. 6. In any case, the introduction of backauditing $b_p = 5$ reduces the overall tax evasion from $\sim 45\%$ to $\sim 27\%$.

Finally, we want to briefly address the influence of the penalty parameter h as compared to the backaudit period b_p. In fact, one could argue that for a given society, tax evasion for e.g. $h = 4$ and $b_p = 5$ should be equivalent to $h = 2$ and $b_p = 11$ since in both cases the maximum compliance period of detected evadors is $k_{max} = (b_p + 1) \cdot h = 24$. This would be true for societies of ideal a- or c-type agents which are either always evading or compliant with probability 'one' (i.e. where $B_i \to \pm\infty$). However, in our model we consider the case of finite fields and, moreover, also incorporate b- and d-types with a pronounced distribution of penalty periods (cf. Figs. 2, 4). For $h = 2$ and $h = 4$ this distribution spans the values $k = 2, 4, 6 \ldots 24$ and $k = 4, 8, 12 \ldots 24$, respectively. Therefore a given penalty period k_4 of the

$h = 4$ computation encompasses two penalty periods ($k_2 = k_4$ and $k_2 - 1$) of the $h = 2$ result and, therefore, shifts the average compliance period to slightly larger values. As a result, tax evasion for parameters h and $b_p = 4n + 3$ is always slighltly smaller than for $2h$ and $b_p = 2n + 1$ with n integer.

C. Phase diagrams

In the previous section we have seen that copying b-type agents cause the extend of tax evasion of heterogeneous societies to be different from the corresponding average of homogeneous societies. As noted, this is due to the interaction of b-type agents with behaviorally different types of agents in their neighborhood and the actual interaction dynamics is governed by various parameters, including the agent-type distribution in the population.

Therefore, in Figs. 9a,b,c we show the stationary tax evasion as a function of the percentage of b-type agents for no backauditing ($b_p = 0$) and backauditing ($b_p = 5, 10$), respectively. As we are predominantly interested in the antagonistic influence of a- and c-type agents, the percentage of d-types has been set to zero. The individual curves in Figs. 9a,b,c represent different percentages of c-type agents, so that increasing the value on the horizontal axis $b = 1 - a - c$ corresponds to a decrease of the share of a-type agents.

Consider first the $c = 0$ curve in Figs. 9a,b,c which displays a crossover behavior (visualized through the thin solid lines) as a function of b-types. In the vicinity of $b = 0$ the vast majority of agents is a-type and the speckles of b-types tend to copy the a-type behavior. The reason why tax evasion slightly decreases with the number of b-types is due to the fact that the latter have a larger probability of staying compliant, even in an evading environment (cf. table III in Ref.[24]). On average this probability is $\approx 15\%$ which coincides with the slope of the $c = 0$ curve in Fig. 9 independent of the value of b_p.

In contrast, the behavior around $b = 1$ can be understood from a society of predominantly b-type agents with speckles of a-types. In this limit every a-type contributes to tax evasion proportionally to the corresponding value of the homogeneous society $te_{hom}(a)$ (cf. Fig. 1). In addition, since each a-type agent is surrounded by four b-types, the latter have a larger probability of tax evasion and contribute with $\sim 4a \cdot te_{neighbor}(b)$. The residual fraction of-types $\sim b - 4a$ contributes again with the value of the homogeneous society $te_{hom}(b)$ (cf. Fig. 2). Note that due to the coupling to the a-types one has $te_{neighbor}(b) \gg te_{hom}(b)$ and it is exactly due to these neighbors that the slope in Fig. 9a exceeds the value of 'one'. Since both $te_{hom}(a)$ and $te_{hom}(b)$ increase with the backaudit period b_p we find a decrease of slope for the tangent around $b = 1$ going from Fig. 9a to Fig. 9c.

Finally, consider the horizontal dashed line at 30% tax evasion in Figs. 9a,b,c. This line shows all combinations of different behavioral agent types that are compatible with 30% tax evasion, subject to the underlying parameter specifications. Inspection of Fig. 9 shows that the increase of the backauditing period not only reduces tax evasion for a given agent distribution, but also the possi-

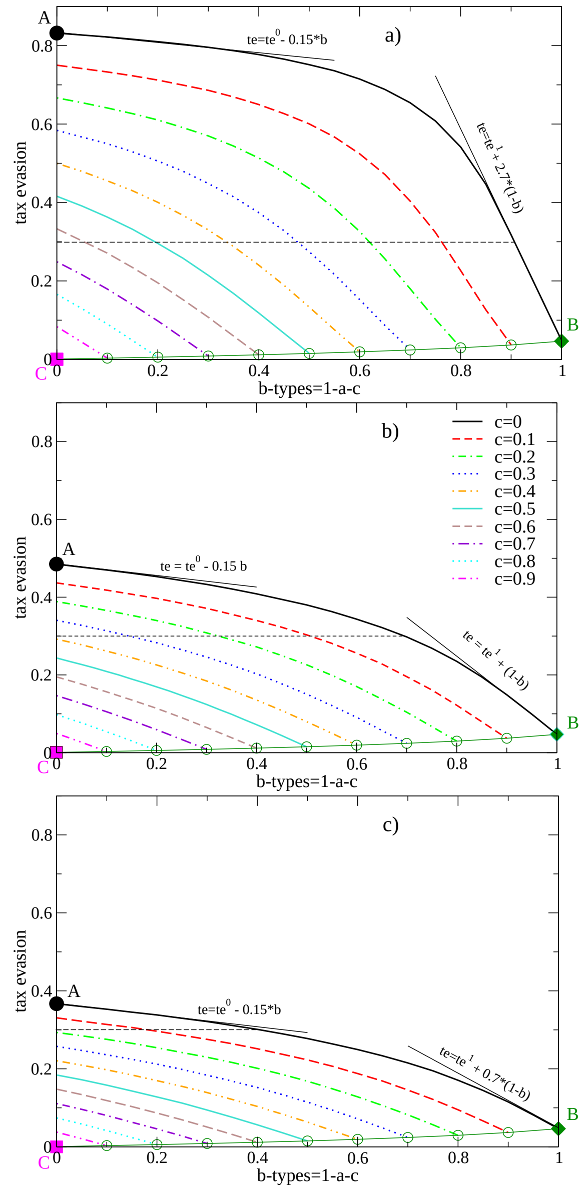

FIG. 9: Stationary values for the extend of tax evasion for audit probability $p_a = 5\%$ as a function of b-types. The individual curves are for different percentages of c-type agents and the fraction of d-types is set to zero. The stationary value is obtained as an average over 30 time steps after the system is thermalized within 270 time steps. We have indicated the tax evasion for 100% a-types (b-types, c-types) by a solid circle denoted A (diamond denoted B, square denoted C). The thin solid lines tangent to the $c = 0$ curve at $b = 0, 1$, respectively, visualize the crossover behavior discussed in the text. Panel a): No backauditing $b_p = 0$, $h = 4$. Panel b): Backauditing $b_p = 5$ and $h = 4$. Panel c): Backauditing $b_p = 10$ and $h = 4$. The horizontal dashed line at 30% tax evasion is also discussed in the text.

ble combinations of agent-types that allow for a specific level of tax evasion (e.g. 30%). Given that these specifications are implemented in a lab experiment with human subjects, it should be possible to derive some information on how closely the agent-based model can predict real human behavior patterns.

IV. CONCLUSION

In this paper we have implemented alternative backauditing schemes, in combination with an endogenously penalty, into a multi-agent econophysics model. Our investigations essentially confirm the results of a few previous analyses on backauditing obtained from a lab experiment with human subjects[16] and from agent-based models of the economics domain[12,15]. In particular, our results show that the compliance rate may increase substantially with the number of backauditing periods, subject to the underlying agent type distribution. To this extent, our findings support the view that introducing long backauditing periods (or lapse of time periods) via tax laws will effectively reduce the level of tax evasion.

Naturally, a-type agents due to their inherent selfish behavior are most affected by backaudits. In addition, we have shown that through backauditing a-types strongly influence the endogeneous compliance of b-types, which can be deduced from the b-type compliance distribution in Figs. 5, 6. However, the higher the share of c-types, i.e. the higher the tax morale in a society, the less effective a backauditing scheme will be. This notwithstanding, a given level of tax evasion may be compatible with different agent type compositions. For example, according to Fig. 9 (dashed horizontal line), a society with observed 30% tax evasion could contain a share of ethical c-types ranging from $c = 0\%$ up to $c \approx 62\%$ (panel a), $c \approx 40\%$ (panel b), and $c \approx 20\%$ (panel c), subject to the underlying parameter setting.

Finally, the incorporation of behaviorally different agent-types, backauditing, and endogenous penalties, adds to the microfoundation of econophysics models and allows for comparing results obtained from such a model with those obtained from human-subject-based lab experiments. As noted, this may permit an empirical determination of the parameters of our model and, in turn, a prediction of the extend of tax evasion under different parameter values. But, of course, this deliniates a future reserach agenda.

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