

# Menu Costs and Phillips Curves

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This paper develops a model of a monetary economy in which individual firms are subject to idiosyncratic productivity shocks as well as general inflation. Sellers can change price only by incurring a real “menu cost.” We calibrate this cost and the variance and autocorrelation of the idiosyncratic shock using a new U.S. data set of individual prices due to Klenow and Kryvtsov. The prediction of the calibrated model for the effects of high inflation on the frequency of price changes accords well with international evidence from various studies. The model is also used to conduct numerical experiments on the economy’s response to various shocks. In none of the simulations we conducted did monetary shocks induce large or persistent real responses.

## I. Introduction

This paper develops a model of a monetary economy in which firms must pay a fixed cost—a “menu cost”—in order to change nominal prices. Menu costs are interesting to macroeconomists because they are often cited as a microeconomic foundation for a form of “price sticki-

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ness” assumed in many New Keynesian models. Without sticky prices these models would not exhibit the real effects of monetary shocks—Phillips curves—that they are designed to analyze.

Under menu costs, any individual price will be constant most of the time and then occasionally jump to a new level. Thus the center of the model will be the firm’s pricing decision to reprice or not to do so. Many New Keynesian models do not examine this decision but instead rely on a simplifying assumption proposed by Calvo (1983) that the waiting time between repricing dates is selected at random from an exponential distribution: Firms choose the size of price changes but not their timing.

As many others are, we are skeptical that the Calvo model provides a serviceable approximation to behavior under menu costs.<sup>1</sup> One reason is that the assumption of a constant repricing rate cannot fit the fact that repricing is more frequent in high-inflation environments. But a second, more important, reason was discovered by Caplin and Spulber (1987), who constructed a theoretical example of an economy with menu costs in which only a small fraction of firms reprice yet changes in money growth are neutral. In their example, there is a stationary distribution of firms’ *relative* prices, and as a monetary expansion proceeds, the firms at the low end of this distribution reprice to the high end. The repricing rate is very low—prices are very “sticky”—but no price stickiness can be seen at the aggregate level. The key to the example is that the firms that change price are not selected at random but are rather those firms whose prices are most out of line.

The Caplin and Spulber example is well designed to exhibit this selection effect, but it is unrealistic in too many respects to be implemented quantitatively. In this paper we capture the selection effect in a new model of menu cost pricing, designed so that it can be realistically calibrated using a new data set on prices, assembled and described by Bils and Klenow (2004) and Klenow and Kryvtsov (2005). This estimation makes use of both cross-section and time-series evidence on the prices of narrowly defined individual goods and summary statistics on the frequency of individual price changes.

The average annual inflation rate in these data is about 2.5 percent and on average 22 percent of prices were changed each month, yet the average price change conditional on a price increase was 9.5 percent. These numbers cannot be understood with a model in which sellers react to aggregate inflation shocks only. We introduce a second, idio-

<sup>1</sup> Another common basis for price stickiness is nominal contracting. Chari, Kehoe, and McGrattan (2000) showed that rational expectations equilibrium models in which firms sign long-term nominal price contracts cannot rationalize the impulse-response functions implied by macroeconomic sticky price models: They do not exhibit nearly enough persistence. Our paper is complementary to theirs.

syncratic shock chosen to rationalize the magnitude of the price changes that do occur at the individual market level. In order to keep the variances of *relative* prices from growing over time, we require this second shock to be mean-reverting. A model with these features is described in detail in Sections II and III, and the calibration is described in Section IV.

Our main finding is that even though monetary shocks have almost no impact on the rate at which firms change prices, the shocks' real effects are dramatically less persistent than in an otherwise comparable economy with time-dependent price adjustment. Simulations of the model's responses to a one-time impulse of inflation show small and transient effects on real output and employment (figs. 4*a* and *b* in Sec. V), in contrast to much larger and more persistent responses of the same model with Calvo pricing. Figure 6 compares before and after distributions of individual prices to illustrate the reason for these different responses. In the menu cost model, a positive aggregate shock induces the lowest-priced firms to increase prices. At the same time, it offsets negative idiosyncratic shocks, and some firms that would otherwise have decreased prices choose to wait. As a result, the lowest-priced firms do most of the adjusting, their adjustments are large and positive, and the economywide price level increases quickly to reflect the aggregate shock. In the Calvo setting, in contrast, firms get the opportunity to reprice randomly, many firms reprice even though they were already close to their desired price, and the average response of prices to the shock is much smaller. It takes longer for the monetary shock to be reflected in prices, and impulse responses become more persistent.

The paper is organized as follows. In Section II we set out the general model. Section III contains the benchmark specification of the model with a constant inflation rate. Section IV describes the data we used and the calibration procedure. We also compare the predictions of the model, as estimated from data from the low-inflation U.S. economy of 1988–97, to international evidence on the frequency of price changes for several countries and time periods, and for the entire Euro area for the period 1995–2000. Although these studies differ in many details and cover a wide range of inflation rates, we found that our model can account extremely well for most of the episodes (see fig. 3 below). Section V then calculates some impulse-response functions. Section VI reintroduces a stochastic shock to the inflation rate and proposes an approximation to the firm's pricing policy. This approximation is then used to examine the behavior of Phillips curves, in the sense of correlations between inflation rates and levels of production and employment. Estimates of the fraction of the variability in these variables that

can be accounted for by monetary shocks in the presence of menu costs are also provided.

The model we describe in the next section builds on the original formulations of the pricing problem of a single firm by Barro (1972) and Sheshinski and Weiss (1977) and on the long literature of other papers that apply  $(S, s)$  type inventory theory to pricing problems.<sup>2</sup> It has proved difficult to situate these pricing models in equilibrium models, but several precedents have been influential and valuable. Lach and Tsiddon (1992) look at individual price distributions in Israel, finding them not to be rectangular and the changes not to be synchronized, even for firms with the same initial price. They suggest that a successful model would need to have idiosyncratic shocks as well as economywide shocks. Bertola and Caballero (1990) and Danziger (1999) also consider models with idiosyncratic as well as aggregate shocks. Dotsey, King, and Wolman (1999) propose a monetary equilibrium model in which the synchronization of price changes is broken by a transient, random shock to the menu cost itself: Firms that draw a high cost have an incentive to postpone repricing. They explore a number of issues numerically using a log-linear approximation. Further developments are described in Willis (2000) and Burstein (2006).

Although several of these earlier papers introduce idiosyncratic as well as aggregate shocks, none does so in a way that quite serves the empirical objectives of this study. In the Dotsey et al. (1999) model and its successors, the idiosyncratic shock affects an individual firm's menu cost and thus influences which firms will reprice at a given time. All firms that do reprice move to the same new price, and that new price is determined entirely by the general inflation shock. To fit the data we use, heterogeneity has to show up in observed prices too. The models of Bertola and Caballero (1990), Danziger (1999), and Gertler and Leahy (2005) are closer to ours and have some of the same qualitative implications. But in these models, all the multiple shocks are random walks, so the variances of relative prices grow linearly over time. Thus these models do not provide theoretical counterparts to the sample moments we use in our calibration.

## II. A Model of Monetary Equilibrium

The theory that we calibrate and simulate in this paper is a Bellman equation for a single price-setting firm that hires labor at a given nominal wage, produces a consumption good with a stochastically varying tech-

<sup>2</sup> Sheshinski and Weiss (1977) analyzed the pricing decision of an individual seller facing a deterministic trend in the desired price level. Versions of this problem, many of them stochastic, have been studied by Frenkel and Jovanovic (1980), Sheshinski and Weiss (1983), Mankiw (1985), Caplin and Leahy (1991), Chang (1999), and Stokey (2002).

nology, and sets product price subject to a menu cost of repricing. We situate our model of a firm in a model of a monetary economy so as to be able to relate its predictions to aggregative evidence. In this economy, there is a continuum of infinitely lived households, each of which consumes a continuum of goods. A Spence-Dixit-Stiglitz utility function is used to aggregate across goods to form current-period utility. Each household also supplies labor on a competitive labor market. Firms hire labor, used to produce the consumption good and to reset nominal prices for the good, and sell goods to consumers. Each firm produces only one of the continuum of consumption goods.

The economy is subject to two kinds of shocks: a monetary shock, which we summarize in the money supply  $m_t$ , and a firm-specific productivity shock  $v_t$ . The log of the money supply is assumed to follow a Brownian motion with drift parameter  $\mu$  and variance  $\sigma_m^2$ ,

$$d\log(m_t) = \mu dt + \sigma_m dZ_m, \quad (1)$$

where  $Z_m$  denotes a standard Brownian motion with zero drift and unit variance. In the absence of the real menu costs associated with changing prices, the evolution of  $m_t$  would have no effect on resource allocation.

There are also firm-specific productivity shocks  $v_t$ , which are assumed to be independent across firms. We assume that  $\log(v_t)$  follows the mean-reverting process:

$$d\log(v_t) = -\eta \log(v_t) dt + \sigma_v dZ_v, \quad \eta > 0, \quad (2)$$

where  $Z_v$  is a standard Brownian motion with zero drift and unit variance, distributed independently of  $Z_m$ .

There is an economywide labor market on which firms hire labor from households at a nominal wage  $w_t$ . The model will be constructed so as to ensure that the log of  $w_t$  also follows the process (1). There is a capital market on which claims to the monetary unit are traded. We adopt the convention that  $E[\int_0^\infty Q_t y_t dt]$  is the value at date 0 of a dollar earnings stream  $\{y_t\}_0^\infty$ , also a stochastic process defined in terms of  $m_t$ .<sup>3</sup>

The *state* of the economy at date  $t$  includes the levels  $m_t$  and  $w_t$  of the money supply and the nominal wage rate. The situation of an individual firm depends also on the price  $p$  that it carries into  $t$  from earlier dates and its idiosyncratic productivity shock  $v_t$ . There is a continuum of firms, so the state of the economy also depends on the joint distribution  $\phi(p, v)$  of these pairs  $(p_t, v_t)$ .

We describe the decision problem of consumers in this environment. At each date  $t$ , each household buys from every seller, and each seller is characterized by a pair  $(p, v)$ , distributed according to a measure

<sup>3</sup> Thus  $Q$  must be multiplied by the appropriate probabilities to obtain the Arrow-Debreu prices.

$\phi_i(p, v)$ . The household chooses a buying strategy  $\{C_i(\cdot)\}$ , where  $C_i(p)$  is the number of units of the consumption good that it buys from a seller who charges price  $p$  at date  $t$ . It also chooses a labor supply strategy  $\{l_i\}$  and a money-holding strategy  $\{\hat{m}_i\}$ , where  $l_i$  is the units of labor supplied and  $\hat{m}_i$  is dollar balances held.

For any buying strategy  $C_i(p)$ , let  $c_i$  be the implied Spence-Dixit-Stiglitz consumption aggregate

$$c_i = \left[ \int C_i(p)^{1-(1/\epsilon)} \phi_i(dp, dv) \right]^{\epsilon/(\epsilon-1)}. \quad (3)$$

Current-period utility depends on  $c_i$  and also on labor supply  $l_i$  and cash holdings  $\hat{m}_i$ , deflated by a price index  $P_i$ . Preferences over time are

$$E \left[ \int_0^\infty e^{-\rho t} \left[ \frac{1}{1-\gamma} c_i^{1-\gamma} - \alpha l_i + \log \left( \frac{\hat{m}_i}{P_i} \right) \right] dt \right]. \quad (4)$$

(It is obvious that the price deflator  $P_i$  will not affect consumer decisions, and it plays no role in the analysis that follows.) The operator  $E(\cdot)$  is defined by the shock processes (1) and (2).<sup>4</sup>

We write the consumer's budget constraint as

$$E \left[ \int_0^\infty Q_i \left[ \int p C_i(p) \phi_i(dp, dv) + R_i \hat{m}_i - W_i l_i - \Pi_i \right] dt \right] \leq m_0, \quad (5)$$

where  $\Pi_i$  is profit income, obtained from the household's holdings of a fully diversified portfolio of claims on the individual firms, plus any lump-sum cash transfers. The term  $R_i \hat{m}_i$ , where  $R_i$  is the nominal interest rate, represents the opportunity cost of holding cash. The household chooses goods demand, labor supply, and money-holding strategies  $\{C_i(\cdot)\}$ ,  $\{l_i\}$ , and  $\{\hat{m}_i\}$  so as to maximize (4), subject to (5), taking  $\{Q_i\}$ ,  $\{R_i\}$ ,  $\{w_i\}$ ,  $\{\Pi_i\}$ ,  $\{\phi_i\}$ , and  $m_0$  as given.

We will use the first-order conditions for consumers to state the prob-

<sup>4</sup> Equilibrium prices and quantities will be modeled as stochastic processes, defined in terms of an initial joint distribution  $\phi_0(p, v)$  of firms by their inherited price  $p$  and productivity level  $v$  and by the evolution of the exogenous processes  $v_t$  and  $m_t$ . Specifically, each firm chooses a pricing strategy that takes the form of a right-continuous step function whose date  $t$  value depends on the histories of its own productivity shocks  $\{v_{i0}^t\}$ , the monetary shocks  $\{m_{i0}^t\}$ , the initial distribution  $\phi_0(p, v)$ , and its inherited initial price  $p_0$ . Consumer strategies depend on the monetary history only. For given firm behavior the initial joint distribution of  $(p_0, v_0)$ , the initial money supply  $m_0$ , and the probabilities implied by (1) and (2) induce a family of probability measures  $\phi_t$  for the prices facing consumers at all dates  $t$ . For any Borel set  $A \subset \mathbf{R}^2$ ,

$$\phi_t(A) = \int \Pr\{(p_t(p_0), v_t)(\omega) \in A | v_0\} \phi_0(dp_0, dv_0).$$

lem solved by firms. These include the first-order condition for money holdings

$$e^{-\rho t} \frac{1}{m_t} = \lambda Q_t R_t, \quad (6)$$

where the equilibrium condition  $\hat{m}_t = m_t$  is imposed. They also include the first-order conditions for consumption choices and labor supply

$$e^{-\rho t} c_t^{-\gamma} c_t^{1/\epsilon} C_t(p)^{-1/\epsilon} = \lambda Q_t p, \quad (7)$$

where the multiplier  $\lambda$  does not depend on time, and

$$e^{-\rho t} \alpha = \lambda Q_t w_t. \quad (8)$$

One can show that there is an equilibrium in which the nominal rate is constant at the level

$$R_t = R = \rho + \mu \quad (9)$$

for all realizations of the two shock processes. In such an equilibrium, (6), (8), and (9) imply

$$w_t = \alpha R m_t, \quad (10)$$

from which it is evident that  $\log(w_t)$  follows a Brownian motion with drift  $\mu$  and variance  $\sigma_m^2$ . We emphasize that the derivation of (10) depends crucially on the assumptions (i) that utility is separable, (ii) that the disutility of labor is linear, and (iii) that the utility of money is logarithmic. Dropping any one of these opens the door to technical complications.

With these facts about equilibrium prices established, we turn to the problem facing an individual firm. At each date, a firm faces consumer demand  $C_t(\cdot)$ , a nominal wage rate  $w_t$ , and a stochastically determined productivity parameter (goods per hour worked)  $v_t$ . The firm enters the period with a price level  $p$  carried over from the past. If it leaves price unchanged, its current profit level is

$$C_t(p) \left( p - \frac{w_t}{v_t} \right).$$

If it chooses any price  $q \neq p$ , its current profit level is

$$C_t(q) \left( q - \frac{w_t}{v_t} \right) - kw_t,$$

where the parameter  $k$  is the hours of labor needed to change price, the real menu cost.

Let  $\varphi(p, v, w, \phi_t)$  denote the present value of a firm that begins at any date  $t$  with the price  $p$  when the productivity and wage shocks take

the values  $v$  and  $w$ , and in which the current, joint distribution of  $(p, v)$  across firms is  $\phi_t$ . This firm chooses a shock-contingent repricing time  $T \geq 0$  and a shock-contingent price  $q$  to be chosen at  $t + T$  so as to solve

$$\begin{aligned} \varphi(p, v, w, \phi_t) = \max_T E_t \left[ \int_t^{t+T} Q_s C_s(p) \left( p - \frac{w_s}{v_s} \right) ds \right. \\ \left. + Q_T \cdot \max_q [\varphi(q, v_{t+T}, w_{t+T}, \phi_{t+T}) - kw_{t+T}] \right]. \quad (11) \end{aligned}$$

Eliminating the multiplier between (7) and (8) and simplifying using (3) yields the demand function facing each firm:

$$C_t(p) = c_t^{1-\epsilon\gamma} \left( \frac{\alpha p}{w_t} \right)^{-\epsilon}. \quad (12)$$

Applying the natural normalization  $Q_0 = 1$  to (8), we obtain

$$Q_{t+s} = e^{-\rho s} \frac{w_t}{w_{t+s}}. \quad (13)$$

Using (12) and (13), we can express the Bellman equation (11) as

$$\begin{aligned} \varphi(p, v, w, \phi_t) = \max_T E_t \left[ \int_t^{t+T} e^{-\rho(s-t)} \frac{w}{w_s} c_s^{1-\epsilon\gamma} \left( \frac{\alpha p}{w_s} \right)^{-\epsilon} \left( p - \frac{w_s}{v_s} \right) ds \right. \\ \left. + e^{-\rho T} \frac{w}{w_T} \cdot \max_q [\varphi(q, v_{t+T}, w_{t+T}, \phi_{t+T}) - kw_{t+T}] \right]. \quad (14) \end{aligned}$$

We call the choices of stopping times  $T$  and prices  $q$  that attain the right side of (14) a firm's *pricing strategy*. We note the simultaneous determination of firms' pricing strategies: For given joint distributions  $\{\phi_t\}$  of prices and productivity levels at current and future dates, each firm's pricing strategy is determined by (14). Conversely, the pricing strategies adopted by all sellers define the distributions  $\{\phi_t\}$  at future dates, given the initial distribution  $\phi_0$ . There is a Nash equilibrium of pricing strategies over a continuum of monopolistically competitive firms.

Finally, a process  $\{\Upsilon_t\}$  with the interpretation that  $\Upsilon_t dt$  is the number of firms that reprice during the time interval  $(t, t + dt)$  is also defined



in equilibrium. The labor market-clearing condition for this economy is then

$$l_i = \int \frac{C_i(p)}{v} \phi_i(dp, dv) + k\Upsilon_i. \quad (15)$$

The equality of goods consumed and goods produced is incorporated in (14).

### III. Special Case: Constant Monetary Growth

Most previous work on menu costs has been simplified by eliminating or avoiding the idiosyncratic shocks,  $\{v_i\}$  in our setup, and focusing on aggregate inflation shocks only. We will initially go in the opposite direction, treating the special case in which the variance  $\sigma_m^2$  of the money growth and wage processes is zero, so that the drift parameter  $\mu$  is simply the constant rate of wage inflation. In this situation, we will seek an invariant joint distribution  $\tilde{\phi}$  for *real* prices  $p/w_i$  and idiosyncratic shocks  $v$ . In this section we formulate, calibrate, and study a Bellman equation for this case of a stationary equilibrium with constant inflation.

The feature of our general equilibrium formulation that makes the firm's Bellman equation (14) hard to analyze is the presence of the distribution  $\phi_i$  as a state variable. Unless we can provide or construct a law of motion for  $\phi_i$ , (14) is just a suggestive formalism. But note that  $\phi_i$  enters (14) only as a determinant of the consumption aggregate  $c_i$ , which acts as a shifter in the demand function facing the individual firm. This feature of the problem can be exploited.

Using (3) and (12), we can express the consumption aggregate in terms of the distributions  $\phi_i$  in general:

$$c_i = \left[ \int \left( \frac{\alpha p}{w_i} \right)^{1-\epsilon} \phi_i(dp, dv) \right]^{1/[\gamma(\epsilon-1)]}. \quad (16)$$

In the case of deterministic money growth, where both the money supply and the nominal wage rate follow a Brownian motion with drift  $\mu$  and variance zero, we can use the change of variable  $x = p/w_i$  and restate (16) as

$$c_i = \left[ \alpha^{1-\epsilon} \int x^{1-\epsilon} \tilde{\phi}_i(dx, dv) \right]^{1/[\gamma(\epsilon-1)]}. \quad (17)$$

In these circumstances, we will conjecture an equilibrium in which the distributions  $\tilde{\phi}_i$  are all equal to an invariant measure  $\tilde{\phi}$ , and the

corresponding consumption aggregate, given by (17), is constant, at some level  $\bar{c}$ . Then we can write (14) as

$$\begin{aligned} \varphi(p, v, w) = \max_T E \left[ \int_0^T e^{-\rho s} \frac{w}{w_s} \bar{c}^{1-\epsilon\gamma} \left( \frac{\alpha p}{w_s} \right)^{-\epsilon} \left( p - \frac{w_s}{v_s} \right) ds \right. \\ \left. + e^{-\rho T} \frac{w}{w_T} \cdot \max_q [\varphi(q, v_T, w_T) - kw_T] \right]. \end{aligned} \quad (18)$$

With the change of variable  $p/w$ , over intervals  $[0, T]$  between repricings,  $\log(x)$  follows a Brownian motion with drift  $-\mu$  and variance zero. Then (18) can be restated, after we cancel and collect terms, as

$$\begin{aligned} \frac{1}{w} \varphi(wx, v, w) = \max_T E \left[ \int_0^T e^{-\rho s} \bar{c}^{1-\epsilon\gamma} (\alpha x_s)^{-\epsilon} \left( x_s - \frac{1}{v_s} \right) ds \right. \\ \left. + e^{-\rho T} \frac{1}{w_T} \cdot \max_{x'} [\varphi(w_T x', v_T, w_T) - kw_T] \right]. \end{aligned} \quad (19)$$

Finally, we seek a solution to (19) of the form

$$\varphi(p, v, w) = w\psi(x, v),$$

where the function  $\psi$  satisfies

$$\begin{aligned} \psi(x, v) = \max_T E \left[ \int_0^T e^{-\rho t} \bar{c}^{1-\epsilon\gamma} (\alpha x_t)^{-\epsilon} \left( x_t - \frac{1}{v_t} \right) dt \right. \\ \left. + e^{-\rho T} \cdot \max_{x'} [\psi(x', v(T)) - k] \right]. \end{aligned} \quad (20)$$

The time-invariant Bellman equation (20) can be studied with familiar methods. The value and policy functions will evidently depend on the parameter  $\bar{c}$ . It is clear that the policy functions will be consistent with an invariant distribution  $\tilde{\phi}$  for  $(x, v)$ , which will also depend on  $\bar{c}$ . Then we find the value of  $\bar{c}$  by solving the fixed-point problem:

$$\bar{c} = \left[ \alpha^{1-\epsilon} \int x^{1-\epsilon} \tilde{\phi}_x(dx, dv; \bar{c}) \right]^{1/[\gamma(\epsilon-1)]}. \quad (21)$$

This completes the construction of the equilibrium.

We studied the problem (20) using a discrete time and state approximation—a Markov chain—following Kushner and Dupuis's (2001) de-

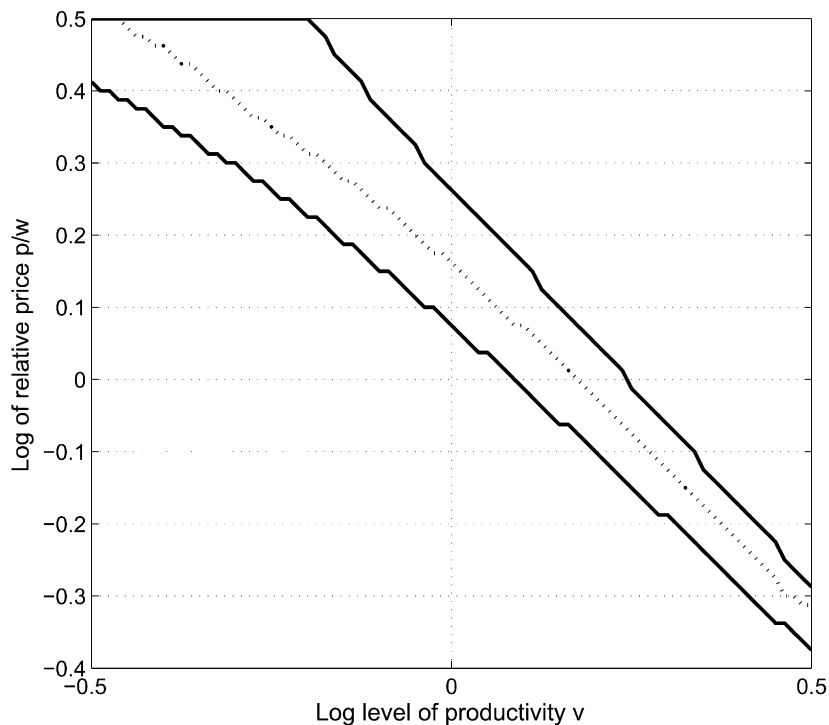


FIG. 1.—Pricing bounds for 0.64 percent quarterly inflation. Solid lines: upper and lower bounds  $U(v)$  and  $L(v)$ . Dotted line:  $g(v)$ .

scription of finite-element methods. That is, we studied the Bellman equation

$$\psi(x, v) = \max \left\{ \Pi(x, v)\Delta t + e^{-r\Delta t} \sum_{x', v'} \pi(x', v'|x, v)\psi(x', v'), \right. \\ \left. \max_{\xi} \left[ \Pi(\xi, v)\Delta t + e^{-r\Delta t} \sum_{x', v'} \pi(x', v'|\xi, v)\psi(x', v') \right] - k \right\}, \quad (22)$$

under the assumption that

$$\Pi(x, v) = c^{1-\epsilon\gamma}(\alpha x)^{-\epsilon} \left( x - \frac{1}{v} \right).$$

The details are given in the Appendix.

Figure 1 illustrates some qualitative features of the optimal pricing policy. It is based on the benchmark parameter values described in the next section, and in particular on the assumption that the aggregate

shock is deterministic:  $\sigma_m^2 = 0$ . To construct the figure, we define the function  $\Omega(v)$  of the productivity shock as

$$\Omega(v) = \max_x [\psi(x, v)],$$

so  $\Omega(v)$  is the value the firm *would* have if it could move costlessly to a new price  $xw$  when the wage is  $w$  and the productivity level is  $v$ . (After this costless move, the menu cost  $k$  is again in force.) The two curves on the figure are the boundaries of the set  $D(v)$  defined by

$$D(v) = \{x > 0 : \psi(x, v) > \Omega(v) - k\},$$

the “region of inaction” on which the firm leaves its price unchanged. Within this region, the firm’s relative price  $x = p/w$  declines at the rate  $\mu$  because of deterministic wage growth, and its productivity level  $v$  moves stochastically as described in (2). When the upper boundary is reached, price is reduced to a point in the interior, indicated as the dotted line on the figure. At the lower boundary, price is raised to the dotted line. Once in the set between the curves, a firm will never leave. The functions defining the boundary of this region are decreasing: high productivity shocks imply price decreases. Note that the inaction intervals  $D(v)$  are wider for low  $v$  values: Getting prices “right” is more important when productivity shocks—and hence quantities sold—are high. Notice too that the firm will occasionally reduce its price, even in an inflationary environment implied by  $\mu > 0$ .

#### IV. Data, Calibration, and a Test

Our basic model lacks many features that a business cycle model needs—it has no capital and no aggregate shocks—but we drew on that literature for the values of the preference parameters  $\rho$ ,  $\gamma$ ,  $\alpha$ , and  $\epsilon$ . We used the annual discount rate  $\rho = .04$ , the risk aversion parameter  $\gamma = 2$ , the elasticity of substitution parameter  $\epsilon = 7$ , and the disutility of labor  $\alpha = 6$ . These  $\rho$  and  $\gamma$  values are conventional. The value of  $\epsilon$  is related to the degree of monopoly power firms have. The elasticity of substitution implies that a firm’s markup—defined as the percentage by which price exceeds marginal cost—is about 16 percent. Estimates of markups typically fall in the 10–20 percent range, implying values of  $\epsilon$  in the 6–10 range.<sup>5</sup> Our results are not sensitive to changes in  $\epsilon$  within that range. We interpreted our linear labor disutility as indivisible labor with lotteries, following Hansen (1985). The value  $\alpha = 6$  implies that 37 percent of the unit time endowment is allocated to work.

<sup>5</sup> See, e.g., Rotemberg and Woodford (1995) and Basu and Fernald (1997). It is not clear to us, we should add, that the estimates reported in these studies are best interpreted as markups in the sense used in this paper.

TABLE 1  
 CALIBRATED PARAMETER VALUES  
 Baseline Values:  $(\eta, \sigma_v^2, k) = (.55, .011, .0025)$

Moment	Data (1)	Model (2)	$\eta = .65$ (3)	$\sigma_v^2 = .015$ (4)	$k = .002$ (5)
Quarterly inflation rate	.0064	.0064	.0064	.0064	.0064
Standard deviation of inflation	.0062	0	0	0	0
Frequency of change	.219	.239	.232	.273	.269
Mean price increase	.095	.097	.094	.104	.092
Standard deviation of new prices	.087	.090	.080	.108	.091

NOTE.—Col. 2 is based on the baseline values. Cols. 3–5 are based on the same values, except for the changes indicated at the head of each column.

For the menu cost parameter  $k$ , the drift parameter  $\mu$ , and the two parameters  $\sigma_v^2$  and  $\eta$  that characterize the idiosyncratic productivity shocks, we used new information on individual prices due to Klenow and Krytsov (2005). This price data set is based on the Bureau of Labor Statistics (BLS) survey and contains about 80,000 time series of individual price quotes in 88 geographical locations. The series are either monthly or bimonthly, depending on the location, for the years 1988–97. The individual price quotes pertain to 123 narrowly defined goods categories. The data set also provides the weights that are used to form the consumer price index from the individual prices. We used the prices and weights for the New York metropolitan area only to calibrate the parameters  $(\mu, \sigma_m^2, \eta, \sigma_v^2)$  and the fixed cost  $k$  of the model described in the previous sections.

For calibrating the model under the assumption of a deterministic trend, we set the variance  $\sigma_m^2$  equal to zero. The actual value, shown in table 1, is .0062. To estimate the inflation rate  $\mu$ , we used the appropriately weighted average (over goods and time) of the observed first differences.

To calibrate the three parameters  $(\eta, \sigma_v^2, k)$ , we calculate three additional sample moments that intuition suggests will convey information. The results are given in the last three rows of the table. The first is the frequency of price change: the average over all months in the data of the fraction of prices that were changed in that month. As shown in the table, this fraction is .219.<sup>6</sup>

Second, we calculated the average log price increase over all prices

<sup>6</sup> There is substantial heterogeneity in the frequency of price changes across different sectors: Airline prices are much more flexible than prices of postage stamps. We considered an alternative version of the model with goods divided into categories with different menu costs and calibrated those costs to the evidence in Bils and Klenow (2004). This multisector model has predictions almost identical to those of the experiments that we report in the paper.

that increased from any date to the next date: .095 in the data. Finally, from among all prices that were increased, we calculated the standard deviation of the new prices. To do that we calculated log deviations from the average,  $z_i(t) = p_i(t) - \bar{p}(t)$ , for each good  $i$  and then computed the standard deviation of  $z_i(t)$  over time for each good  $i$ . Then we averaged over goods  $i$ . This yielded the number .087.

For any values of  $(\mu, 0, \eta, \sigma_v^2, k)$ , we can calculate the corresponding moments predicted by the theory, under the assumption that the probability distribution of  $(x, v)$  is the invariant distribution  $\tilde{\phi}(x, v)$ , say, and that prices are given by the optimal policy function for the dynamic program (18). We then simulated the model of Section III under the parameter values given in the first paragraph of this section plus the “baseline values” indicated in the table to calculate the theoretical moments. These produced the estimates reported in column 2 in the table, headed Model.

This value, .239, shown in the appropriate row of column 2 of the table, is calculated with  $(\mu, \sigma_m^2, \eta, \sigma_v^2, k)$  set equal to (.0064, 0, .55, .011, .0025). Columns 3–5 of the table indicate how the calculated moments change as  $(\eta, \sigma_v^2, k)$  are changed one at a time from these benchmark values. That is, column 3 shows the computed statistics when the parameter vector  $(\mu, \sigma_m^2, \eta, \sigma_v^2, k) = (.0064, 0, .55, .011, .0025)$  is replaced by  $(.0064, 0, .65, .011, .0025)$ . Thus the table shows that the frequency of price changes is insensitive to changes in the rate of mean reversion in the idiosyncratic shock, that it increases with the variance of these shocks, and that it decreases with increases in the menu cost.

There are many studies that try to estimate or calibrate menu costs for particular products. For example, Levy et al. (1997) estimate that the cost of changing prices in supermarkets is about 0.7 percent of firms’ revenue. In our baseline model with  $k = .0025$ , menu costs are about 0.5 percent of revenues. The labor required to adjust prices is equal to 0.5 percent of overall employment.

The treatment of sale pricing is important in microeconomic pricing studies. The BLS flags observations that it regards as sale prices, and the Klenow-Kryvtov data we used had such sale observations removed. Figure 2, taken from Chevalier, Kashyap, and Rossi (2000), shows the time series of actual prices for Triscuits, based on scanner data from a Chicago supermarket chain. On figure 2, temporary sales are evident in the many times the price of Triscuits is reduced for a short time and returned to exactly the former price soon thereafter. Such patterns are of course common to many price series. To obtain a good match between theory and data, then, sales must be either removed from the data or added to the model. As discussed above, we took the first course.<sup>7</sup>

<sup>7</sup> A recent study by Midrigan (2006) uses the Chicago scanner data to calibrate a menu

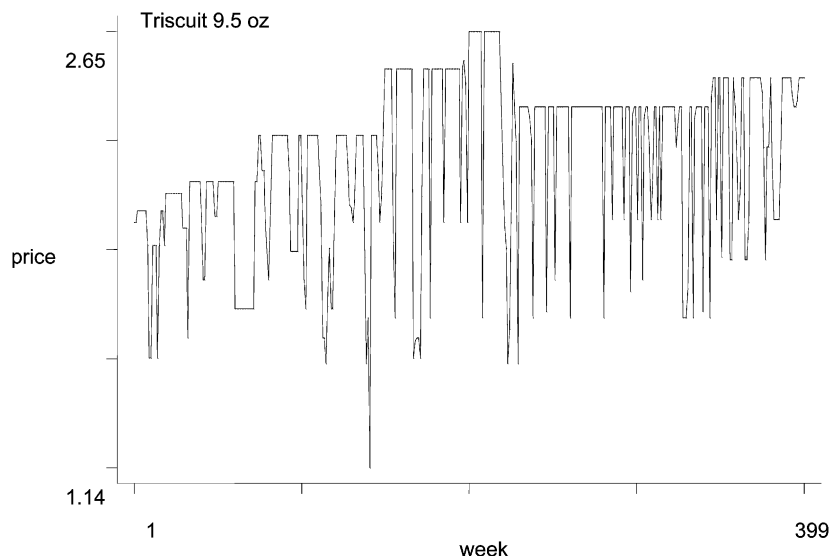


FIG. 2.—Price of Triscuits (9.5 oz.) in Dominick's Finer Foods supermarket in Chicago. Source: Chevalier et al. (2000).

We solved the model, calibrated as just described, for quarterly inflation rates  $\mu$  ranging from zero to 20 percent, calculated the invariant distribution  $\lambda$  in each case, and calculated the fraction of firms that change price each month in this stationary equilibrium. For comparison, we carried out the same calculations for the deterministic Sheshinski-Weiss case in which the variance of the idiosyncratic shocks is set equal to zero. These are the solid and dashed lines shown in figure 3.

The individual points on figure 3 are taken from seven empirical studies of pricing behavior, in addition to the U.S. studies we used in our calibration. These include the studies of Lach and Tsiddon (1992) on Israeli inflations of 1978–79 and 1981–82, Baharad and Eden's (2004) study of the Israeli inflation of 1991–92, Konieczny and Skrzypacz's (2005) analysis of Poland's experience in 1990–96, Gagnon's (2005) study of the frequency of price changes in Mexico during various periods from the late 1990s to 2000, and the Dhyne et al. (2005) study of a variety of countries in the Euro area over the years 1995–2000. The inflation-repricing pair (.64, 21.9) from the Klenow-Kryvtsov data for

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cost model that is similar to ours. He finds too many small price changes to be consistent with our model and argues for a version in which menu costs apply to *groups* of goods. When the menu cost is incurred for a given group, items that are only slightly mispriced are repriced along with the group members that are badly mispriced. Kashyap (1995) also reports a large number of small price changes in a context, catalogue sales, in which prices change infrequently.

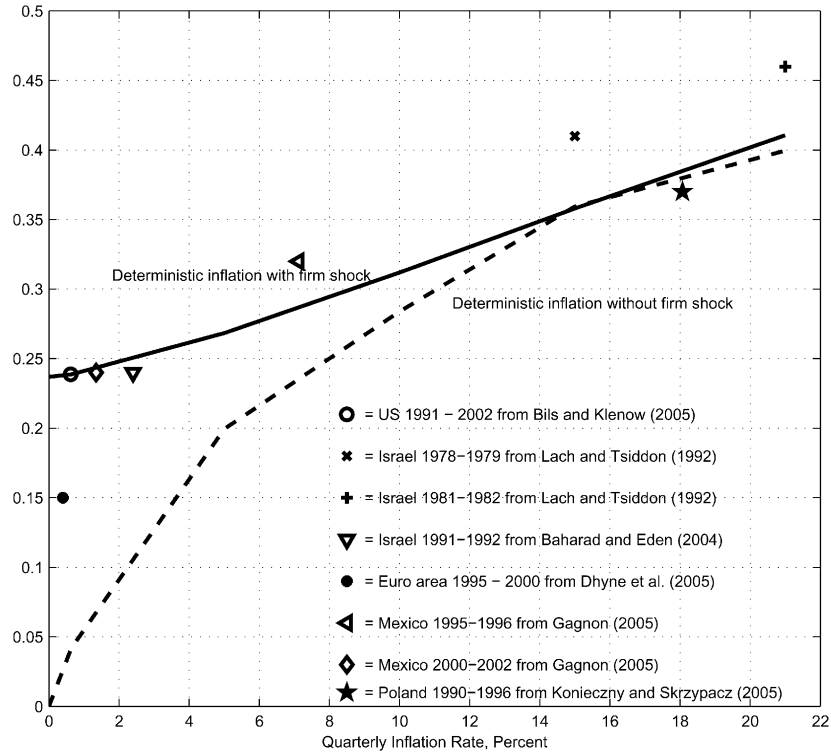


FIG. 3.—Fraction of prices changed each month

the United States is also shown. This pair lies very close to the upper curve, reflecting the fact that we used the Klenow-Kryvtsov data to calibrate our model. The model so calibrated fits the international evidence well, too, in spite of the fact that these studies are based on quite different samples of individual prices and differ in many other details. Our model and the Sheshinski-Weiss model both make the correct, qualitative prediction that the repricing frequency should increase as the rate of inflation increases, but ours gets the magnitude about right at both high and low inflation rates. Since we used only low-inflation data to calibrate the model, this is a genuine out-of-sample test of the theory.

Figure 3 also confirms the necessity of including idiosyncratic shocks if the model is to fit the evidence from low-inflation economies. As inflation rates are reduced, a lot of “price stickiness” remains in the data. Of course, this evidence does not bear on our interpretation of the idiosyncratic shocks as productivity differences, as opposed to shifts in preferences, responses to inventory buildups, or other factors.



## V. Impulse-Response Functions

In this section we consider a thought experiment using the benchmark model with the variance  $\sigma_m^2$  of the inflation rate equal to zero. We subject this economy, assumed to be in the stationary equilibrium corresponding to money growth rate constant at  $\mu$ , to an unanticipated jump from  $m$  to  $(1 + h)m$  in the level of money, after which money growth resumes its original rate  $\mu$ . (By [10] this experiment corresponds to an unanticipated jump in nominal wages from  $w$  to  $[1 + h]w$ .) This experiment will provide intuition for the small effects of monetary policy that we will show in the following section with stochastic inflation.

A monetary disturbance of a one-time shock will take the economy out of the stationary equilibrium we studied in Section III. This fact raises new computational problems, which we deal with as follows. Let  $c(\mu)$  denote the constant value of the consumption aggregate defined in (3) in a stationary equilibrium under the original policy. We construct an equilibrium response in which the original stationary distribution is restored and in which the time path  $\{c_t\}$ ,  $c_0 = c(\mu)$ , and  $c_t \rightarrow c(\mu')$  induced by the shock is perfectly foreseen by firms. Details are provided in the Appendix.

Figures 4a and b plot the impulse-response functions calculated in this way when  $\mu$  equals 1 percent per quarter and  $h = .0125$ . First, note that the initial response in output is less than the size of the monetary shock. Since aggregate output is

$$\begin{aligned} y_t &= \int C_t(p) \phi_t(dp, dv) \\ &= \alpha^{-1/\gamma} \left[ \int \left( \frac{p}{w} \right)^{1-\epsilon} \phi_t(dp, dv) \right]^{(1-\gamma\epsilon)/(\gamma\epsilon-\gamma)} \times \int \left( \frac{p}{w} \right)^{-\epsilon} \phi_t(dp, dv), \end{aligned}$$

the increase of  $w$  to  $(1 + h)w$  can increase total output by at most  $(1 + h)^{(1-\gamma\epsilon)/\gamma} \times (1 + h)^\epsilon = (1 + h)^{1/\gamma}$ .<sup>8</sup>

The increase in  $w$  leads to a temporary increase in the number of the firms changing their prices. This effect is over very quickly, occurring right after the jump in wages, after which the frequency of price changes reverts to its steady-state level. The effect on real output lasts longer, but it also declines to zero by the middle of the first quarter. The decline is fast because many of the firms that do not initially react to the aggregate shock will soon reprice as a result of idiosyncratic shocks. Once a firm decides to reprice for any reason, it will take the higher level of nominal wages into account in choosing the new price.

<sup>8</sup> Also, note that (12) and (3) imply that  $c_t = (w_t/\alpha P_t)^{1/\gamma}$ , where  $P_t$  is the price aggregate defined as  $P_t = [\int p^{1-\epsilon} \phi_t(dp, dv)]^{1/(1-\epsilon)}$ . This relationship shows that the maximum impact of an  $h$  percent shock on the consumption aggregate  $c_t$  is  $(1 + h)^{1/\gamma}$ .

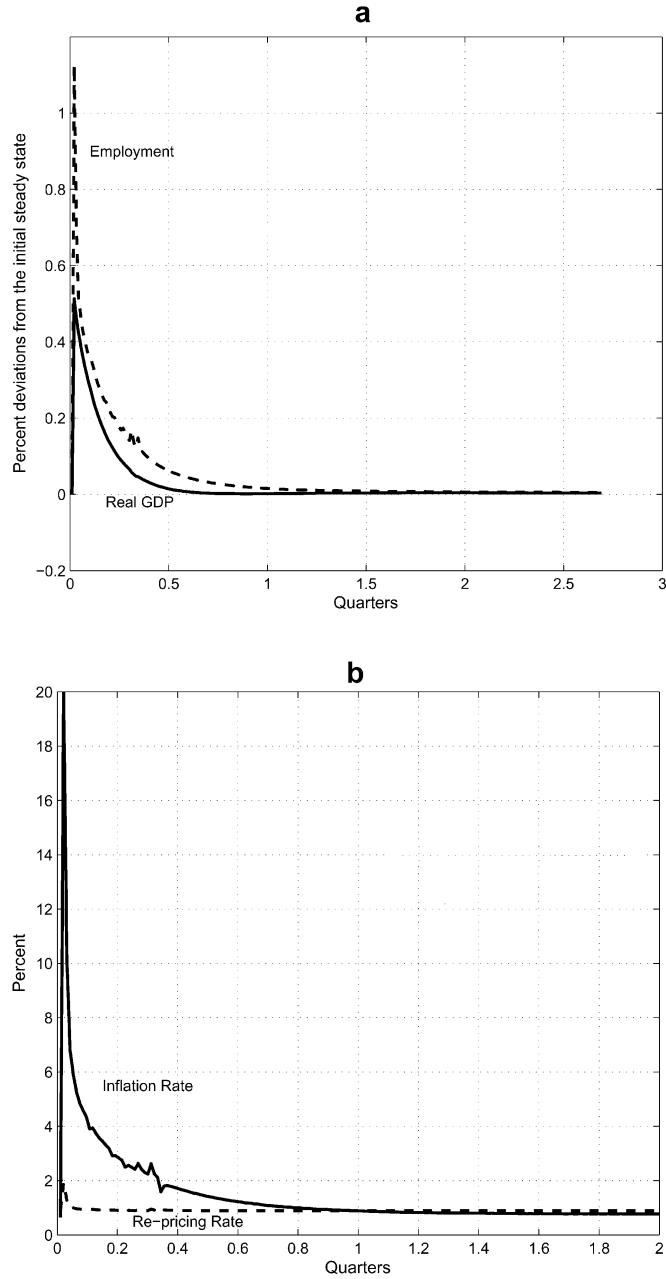


FIG. 4.—Responses to a transient monetary shock. *a*, Responses of employment and output to a one-time increase (impulse) in the level of money of 1.25 percent. The initial level is normalized to one. *b*, Responses of the quarterly inflation rate (percentage) and repricing rate (percentage of firms per day) to a one-time increase (impulse) in the level of money of 1.25 percent.

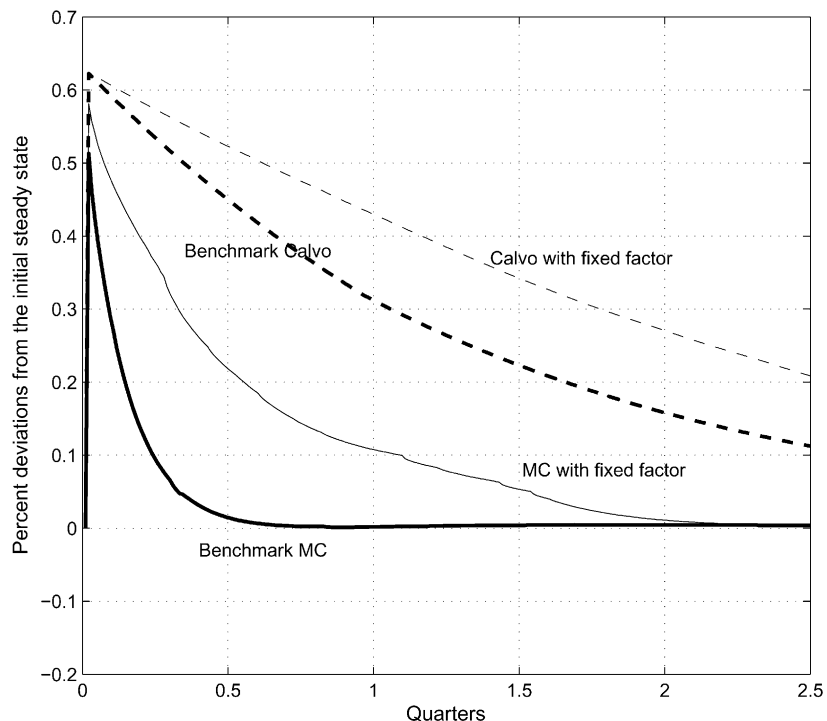


FIG. 5.—Output responses in menu cost and Calvo models

The impulse responses are much more transient than a standard time-dependent model would predict. The two heavy curves in figure 5 compare the output response to the monetary shock described in figure 4a to the output response that would occur in a Calvo (1983) type model, otherwise identical to ours, in which a firm is permitted to reprice in any period with a fixed probability that is independent of its own state and the state of the economy. (The two light, “fixed-factor,” curves are discussed below.) In both simulations we set this fixed repricing probability equal to .23 per month, the frequency predicted by our model. The two curves are very different. The initial response is much larger with “time-dependent” repricing, as compared to our “state-dependent” pricing. Time-dependent pricing also implies a much more persistent effect.

Figure 6 compares before and after distributions of individual prices to illustrate the reason for these different responses. Figure 6a shows repricing behavior in the absence of any aggregate shock. Firms in the menu cost model reprice when idiosyncratic shocks are large enough, and then they reprice to  $p^*$ . The average size of these price adjustments

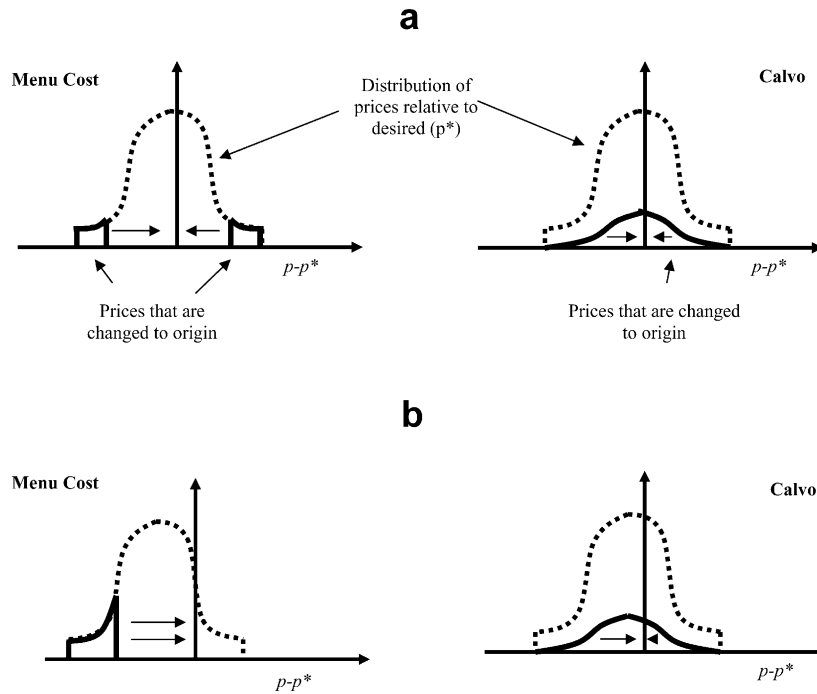


FIG. 6.—Price adjustment in menu cost and Calvo models. *a*, Price adjustment before aggregate shock. *b*, Price adjustment after aggregate shock.

is large. In the Calvo model the firms that adjust prices are chosen randomly, and since many such firms are not far from their desired prices, the average size of the price adjustment is smaller. Increases and decreases of prices in both models are roughly symmetric.

In figure 6*b*, a positive aggregate shock shifts the distribution of the relative prices to the left. In the menu cost environment, this implies that many firms will be outside of the lower bound of their inaction region (see fig. 1) and they increase prices. At the same time, the positive aggregate shock offsets negative idiosyncratic shocks, and firms that would otherwise have decreased prices choose to wait. As a result, the firms in the left-hand tail of the distribution do most of the adjustments, these adjustments are large and positive, and the economywide price level increases quickly to reflect the aggregate shock. In the Calvo setting, in contrast, firms get the opportunity to reprice randomly, the average firm that changes price remains very close to its desired level, and the average response of prices to the shock is much smaller. It takes

longer for the monetary shock to be reflected in prices, and impulse responses become more persistent.<sup>9</sup>

These results can be compared to the previous menu cost literature. In the absence of idiosyncratic shocks, the log-linear approximation of our firms' problem would be equivalent to the setup of Caplin and Spulber (1987). Their result that aggregate shocks are completely neutral would then hold in ours: In a stationary equilibrium the distribution of firms' relative prices would be uniform, and a  $\kappa$  percent increase in  $w$  would cause  $\kappa$  percent of the firms to adjust their prices. The resulting distribution of the relative prices would then be the same as the stationary distribution, and so total output would remain unchanged. The presence of idiosyncratic shocks introduces more complicated distributions of the relative prices, so in our case the shock to nominal wages leads to a real response. However, the main lesson is similar to Caplin and Spulber's: What matters is not so much *how many* prices are changed but *which* prices are changed.

This self-selection effect would lead to a smaller effect of monetary policy relative to time-dependent models in a variety of environments, even though the number of prices that are being changed may appear to be similar. To illustrate this point, we relax the assumptions that labor is fully mobile and enters linearly in both the production and utility functions, and instead introduce a fixed factor so that the production function exhibits decreasing returns. It is known (see, e.g., Chari et al. 2000) that such fixed factors cause monetary shocks to be more persistent in the time-dependent models. We then compare the responses of our benchmark Calvo and menu cost models to a model in which the production function takes the form  $y = (VI)^\theta$  with  $\theta = .8$ . We keep all other parameters the same. These comparisons are shown in figure 5. One can see that both the menu cost and Calvo models do have more persistent impulse responses in the fixed-factor version of the models. However, persistence in the Calvo model (measured by a half-life of a shock) is still five times larger than persistence in a corresponding menu cost model.

## VI. Approximations to a Two-Shock Equilibrium

The analysis so far has been based only on the model with a constant inflation or the same model subjected to a one-time shock. In this section

<sup>9</sup> Klenow and Kryvtsov (2005) provide an empirical decomposition of average inflation into components they label as "time dependent" and "state dependent." They find that in the BLS data the time-dependent component accounts for 88–96 percent of inflation variability. We carried out the same decomposition using simulated series from our menu cost model, in which all price variability is in fact state dependent, and found that the Klenow-Kryvtsov method would attribute 85 percent of inflation variability to the time-dependent component.

we consider the model with stochastic inflation:  $\sigma_m^2 > 0$ . In calculating the impulse-response functions reported in the previous section, we found that the effects of monetary shocks on the consumption aggregate were extremely small. This suggests that there may be little loss in accuracy if we hold  $c_t$  constant at  $\bar{c}$ , say, and simulate a two-shock model with  $\sigma_m^2 > 0$ . The Bellman equation suited for this continues to be (20). The policy function is dependent on this constant  $\bar{c}$ , and again we assume that the implied  $(x, v)$  process has a unique invariant distribution  $\tilde{\phi}(x, v; c)$  (not, of course, the same distribution as when the money shock is deterministic). As before, we obtain the equilibrium (or pseudo-equilibrium) value of  $c$  as the solution to (21). We calculated this solution iteratively. The policy function computed in this way is the policy of a firm that correctly observes the mean level of  $c_t$  but ignores all the fluctuations about this level. We propose this function as an approximation to the true behavior of the firms in a two-shock equilibrium.

To get some idea of the likely accuracy of this approximation, we recalculated the impulse-response functions displayed in figures 4*a* and *b* in Section V (which display a rational expectations equilibrium in which  $c_t$  varies over time) using the constant- $c$  approximation just described. We also increased the size of the initial shock by a factor of four. Figure 7 shows the results for real GDP. Evidently, the approximation works very well for the effects of a one-time shock, even a large one. We take this as an indication that it will also be accurate for stochastic shocks of the same order of magnitude.

We conduct two thought experiments using this approximation. First, we will study the effect of the volatility of inflation on the volatility of the real output by simulating 40 quarters of data.<sup>10</sup> For these simulations we chose  $\sigma_m^2 = (.0062)^2$ , which corresponds to the .0062 standard deviation of quarterly inflation in the Klenow-Kryvtsov data set. The standard deviation of the log level of output is equal to .0006 in our simulation. The standard deviation of actual U.S. quarterly consumption for the same period (1989–98) around linear trend is equal to .015. Thus monetary fluctuations in this model can account for less than 10 percent of the observed fluctuations in output. This estimate is consistent with estimates from other sources (see Lucas's [2003] survey).

In the second experiment we regress the log level of real output on the log difference of the nominal wages, using the simulated series generated by the model:

$$\log(y_t^Q) = \alpha + \beta[\log(w_t^Q) - \log(w_{t-1}^Q)].$$

<sup>10</sup> Since the model is in continuous time (actually, about 40 discrete periods per quarter) and the economic data come in discrete intervals, we aggregate the output of the model into quarterly values by taking the means of the relevant variables over the quarter. Thus the level  $\nu_t^Q$  at quarter  $T$  of any function of time  $\nu(t)$  is defined as  $\nu_t^Q = \int_{T-1}^T \nu(t) dt$ .

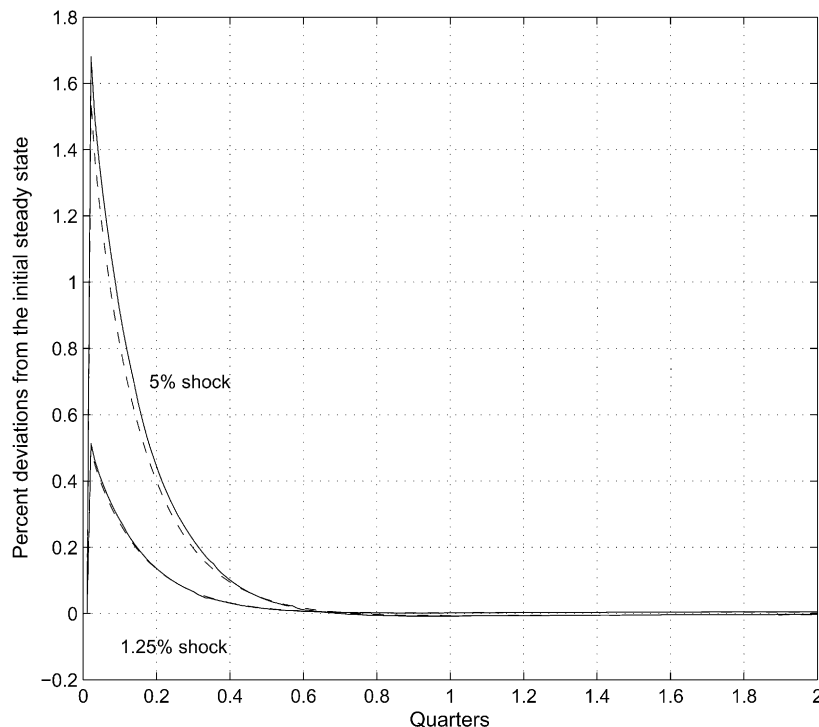


FIG. 7.—Approximate (dashed lines) and exact (solid lines) impulse-response functions: responses of output to a one-time increase (impulse) in the level of money. Initial levels are normalized to one.

In this regression, we obtain the estimate  $\beta = .049$  with the standard error .008. Thus an increase in nominal wage rates leads to an increase in real output, as in standard Phillips curve regressions, but the effect is very small. This conclusion is not sensitive to different specifications of the parameters  $(\mu, \sigma_m)$ .

## VII. Conclusions

We have constructed a model of a monetary economy in which repricing of goods is subject to a menu cost and studied the behavior of this economy numerically. The model is distinguished from its many predecessors by the presence of idiosyncratic shocks in addition to general inflation. We used a data set on individual U.S. prices recently compiled by Klenow and Kryvtsov to calibrate the menu cost and the variance and autocorrelation of the idiosyncratic shocks. We conducted several experiments with the model.

A key prediction of any menu cost model is that the fraction of firms that reprice in a given time interval will increase with increases in the inflation rate. We simulated our model at inflation rates varying from zero to 20 percent per quarter. The results, shown in figure 3, trace out a curve that passes through the inflation rate–repricing rate pair estimated using data from the low-inflation U.S. economy of the 1990s. The same curve also fits very well the low-inflation periods in Mexico and Israel and high-inflation periods in Mexico, Israel, and Poland and reasonably well the low-inflation experience in the Euro area. We note that a model without idiosyncratic shocks could not fit any except the very highest inflations.

We next used the model to calculate the responses of output, employment, and prices to an unanticipated increase in money, equivalent in our setup to an impulse in the nominal wage. The predicted output responses were small and transient, bearing little resemblance to the response characteristics of New Keynesian models based on time-dependent pricing.

These results all refer to a special case in which inflation is deterministic. We also solved an approximation to a more realistic two-shock model. With a realistic inflation variance, this model can account for perhaps one-tenth of the observed variance of U.S. real consumption about trend. A Phillips curve estimated from data generated by the model implies that a one-percentage-point reduction in the inflation rate will depress production by 0.05 percent.

In summary, the model we proposed and calibrated to microeconomic evidence on U.S. pricing behavior does a remarkably good job of accounting for behavior differences between countries with very different inflation rates. It does not appear to be consistent with large real effects of monetary instability. These results seem to us another confirmation of the insight provided by the much simpler example of Caplin and Spulber (1987) that even when most prices remain unchanged from one day to the next, nominal shocks can be nearly neutral: The prices that stay fixed are those for which stickiness matters least, and the prices that are far out of line are the ones that change. Figure 5 substantiates the quantitative importance of this effect.

## Appendix

The construction of approximating Markov chains for the one-shock model of Sections III–V and the two-shock model of Section VI is based on Kushner and Dupuis (2001). This appendix provides the details, based on the two-shock model of Section VI. For the most part, the specialization to the one-shock case is obvious.

In the calculations described below, we fix the grid size  $h$  and define the state space  $S = X \times V$ . To economize on notation, we define  $\tilde{x} = \log(p/w)$  and



$\tilde{v} = \log(v)$ . To find an approximate solution to the two-shock firm's Bellman equation, we fix  $c_i$  at  $\bar{c}$  as described in Section VI, so that the firm's Bellman equation becomes

$$\psi(\tilde{x}, \tilde{v}) = \max_T E \left[ \int_0^T e^{-\rho t} \Pi(\tilde{x}_t, \tilde{v}_t) dt + e^{-\rho T} \max_{\tilde{x}'} [\psi(\tilde{x}', \tilde{v}(T)) - k] \right], \quad (\text{A1})$$

where

$$\Pi(\tilde{x}, \tilde{v}) = \bar{c}^{1-\epsilon} \alpha^{-\epsilon} e^{-\epsilon \tilde{x}} (e^{\tilde{x}} - e^{-\tilde{v}}). \quad (\text{A2})$$

The processes  $(\tilde{x}, \tilde{v})$  are assumed to follow

$$d\tilde{x} = -\mu dt + \sigma_m dZ_m \quad (\text{A3})$$

and

$$d\tilde{v} = -\eta \tilde{v} dt + \sigma_v dZ_v. \quad (\text{A4})$$

Then we approximate the continuous problem (A1) with a discrete problem

$$\begin{aligned} \psi(\tilde{x}, \tilde{v}) = \max \left\{ \Pi(\tilde{x}, \tilde{v}) \Delta t + e^{-\rho \Delta t} \sum_{\tilde{x}', \tilde{v}'} \pi(\tilde{x}', \tilde{v}' | \tilde{x}, \tilde{v}) \psi(\tilde{x}', \tilde{v}'), \right. \\ \left. \max_{\xi} \left[ \Pi(\xi, \tilde{v}) \Delta t + e^{-\rho \Delta t} \sum_{\tilde{x}', \tilde{v}'} \pi(\tilde{x}', \tilde{v}' | \xi, \tilde{v}) \psi(\tilde{x}', \tilde{v}') \right] - k \right\}, \quad (\text{A5}) \end{aligned}$$

where  $\pi$  is a transition function defined on  $S \times S$  that we define in a moment. The time interval  $\Delta t$  is related to the grid size and other parameters by

$$\Delta t = \frac{h^2}{D}, \quad (\text{A6})$$

where

$$D = \sigma_m^2 + \mu h + \sigma_v^2 + \eta \tilde{v} h. \quad (\text{A7})$$

We assume that in a given time interval  $\Delta t$ , at most one of the variables  $\tilde{x}$  and  $\tilde{v}$  changes.<sup>11</sup> Provided that neither  $\tilde{x}$  nor  $\tilde{v}$  is at its upper or lower bound, we assume that if  $\tilde{x}$  changes, it moves either to  $\tilde{x} + h$  or to  $\tilde{x} - h$ ; if  $\tilde{v}$  changes, it moves either to  $\tilde{v} + h$  or to  $\tilde{v} - h$ . The final possibility is that neither of the variables changes and the state remains at  $(\tilde{x}, \tilde{v})$ . The probability of all other transitions is zero. Away from the boundaries of  $S$ , the five nonzero transition probabilities will then be defined by

$$\pi(\tilde{x} + h, \tilde{v}, \tilde{x}, \tilde{v}) = \frac{\sigma_m^2/2}{D}, \quad (\text{A8})$$

$$\pi(\tilde{x} - h, \tilde{v}, \tilde{x}, \tilde{v}) = \frac{(\sigma_m^2/2) + \mu h}{D}, \quad (\text{A9})$$

<sup>11</sup> This means that the Markov chains approximating  $\tilde{v}(t)$  and  $\tilde{x}(t)$  will not be independent for  $h > 0$ , even though the continuous processes are. But independence will hold in the limit as  $h \rightarrow 0$ .

$$\pi(\tilde{x}, \tilde{v} + h, \tilde{x}, \tilde{v}) = \frac{\sigma_v^2/2}{D} \quad (\text{if } \tilde{v} \geq 0), \quad (\text{A10})$$

$$\pi(\tilde{x}, \tilde{v} - h, \tilde{x}, \tilde{v}) = \frac{(\sigma_v^2/2) + \eta\tilde{v}h}{D} \quad (\text{if } \tilde{v} \geq 0), \quad (\text{A11})$$

and

$$\pi(\tilde{x}, \tilde{v}, \tilde{x}, \tilde{v}) = 1 - \frac{\sigma_m^2 + \sigma_v^2 + \eta\tilde{v}h + \mu h}{D} \quad (\text{if } \tilde{v} \geq 0). \quad (\text{A12})$$

(The  $\tilde{v}(t)$  process is symmetric about zero, so the adaptations of [A10] and [A11] for the case  $\tilde{v} < 0$  are obvious.) Transitions at the boundaries are handled by assuming that if, for example,  $\tilde{x}$  hits its upper bound  $\bar{x}$ , then  $\tilde{x}$  goes one step down to  $\bar{x} - h$  with probability  $\pi(\bar{x} - h, \tilde{v}, \bar{x}, \tilde{v})$  and stays at  $\bar{x}$  with probability  $\pi(\bar{x} + h, \tilde{v}, \bar{x}, \tilde{v}) + \pi(\bar{x}, \tilde{v}, \bar{x}, \tilde{v})$  as given by the formulas (A8) and (A12). It is evident that the five probabilities (A8)–(A12) add to one and that the probabilities (A8)–(A11) are positive. That (A12) is nonnegative follows from the fact that  $|\tilde{v}| \leq \bar{v}$ .

The first and second moments of the Markov chain we have just defined, conditional on the current state  $(\tilde{x}, \tilde{v})$  (assumed not be a boundary point of  $S$ ), are readily calculated from (A8)–(A12). They are

$$\frac{E\{\tilde{x}(t + \Delta t) | \tilde{x}(t) = \tilde{x}, \tilde{v}(t) = \tilde{v}\} - \tilde{x}}{\Delta t} = -\mu,$$

$$\frac{E\{\tilde{v}(t + \Delta t) | \tilde{x}, \tilde{v}\} - \tilde{v}}{\Delta t} = -\eta\tilde{v} \quad (\text{if } \tilde{v} \geq 0),$$

$$\frac{\text{Var}\{\tilde{x}(t + \Delta t) | \tilde{x}, \tilde{v}\}}{\Delta t} = \sigma_m^2 + \mu h - \mu^2 \Delta t,$$

$$\frac{\text{Var}\{\tilde{v}(t + \Delta t) | \tilde{x}, \tilde{v}\}}{\Delta t} = \sigma_v^2 + h|\eta\tilde{v}| - (\eta\tilde{v})^2 \Delta t,$$

and

$$\frac{\text{Cov}\{\tilde{x}(t + \Delta t), \tilde{v}(t + \Delta t) | \tilde{x}, \tilde{v}\}}{\Delta t} = -\eta\tilde{v}\mu\Delta t \quad (\text{if } \tilde{v} \geq 0).$$

From (A6) and (A7),

$$\frac{\Delta t}{h} = \frac{h}{\sigma_m^2 + \mu h + \sigma_v^2 + \eta\tilde{v}h} \rightarrow 0 \quad \text{as } h \rightarrow 0.$$

This is the sense in which the conditional, local moments of the approximating chain approximate the conditional, local moments of the continuous-time  $(\tilde{x}(t), \tilde{v}(t))$  process defined by (A3) and (A4). See Kushner and Dupuis (2001, chap. 9) for a proof that this approximation converges in distribution to the continuous-time diffusion process when  $h \rightarrow 0$ .

*Computations of impulse responses.*—It is easiest to describe this construction in

terms of the discrete approximation (22). Initially, we set a limit  $n$  on the number of transition periods and begin with an assumed finite sequence  $\mathbf{c}^n = (c_1, c_2, \dots, c_n)$  of values of the consumption aggregate. Then we define the sequence  $\{\psi_i(x, v, \mathbf{c}^n)\}_{i=1}^n$  of value functions recursively by

$$\psi_n(x, v, \mathbf{c}^n) = \psi(x, v), \tag{A13}$$

where  $\psi(x, v)$  is the solution to (22) at a stationary equilibrium with  $c_i$  constant at  $\bar{c}$ , and

$$\begin{aligned} \psi_i(x, v, \mathbf{c}^n) = \max & \left\{ \Pi(x, v, c_i)\Delta t + e^{-\rho\Delta t} \sum_{v'} \pi(v'|v)\psi_{i+1}(xe^{\mu\Delta t}, v', \mathbf{c}^n), \right. \\ & \left. \max_{x'} \left[ \Pi(x', v, c_i)\Delta t + e^{-\rho\Delta t} \sum_{v'} \pi(v'|v)\psi_{i+1}(x'e^{\mu\Delta t}, v', \mathbf{c}^n) \right] - k \right\} \end{aligned} \tag{A14}$$

for  $i = 1, 2, \dots, n - 1$ . Let  $\{f_i(x, v, \mathbf{c}^n)\}_{i=1}^n$  be the sequence of policy functions corresponding to the value functions  $\{\psi_i(x, v, \mathbf{c}^n)\}_{i=1}^n$  so defined. For given behavior  $\mathbf{c}^n$  of the consumption aggregate, these functions can be calculated by the usual backward induction.

The pricing behavior  $\{f_i(x, v, \mathbf{c}^n)\}_{i=1}^n$  in turn implies a sequence  $\{\tilde{\phi}_i(x, v, \mathbf{c}^n)\}_{i=1}^n$  of joint distributions of real prices and productivity shocks, taking the original invariant distribution  $\phi$  as the initial condition. Individual firm sales are given by (12), and then new values of the consumption aggregate by (3):

$$(\Gamma\mathbf{c})_i = \left\{ \int c_i^{(1-\epsilon\gamma)[1-(1/\epsilon)]} (\alpha x)^{1-\epsilon} \tilde{\phi}_i(dx, dv) \right\}^{\epsilon/(\epsilon-1)}. \tag{A15}$$

The construction described in equations (A13)–(A15) thus defines a function  $\Gamma$  taking an  $n$ -vector  $\mathbf{c}$  into  $\Gamma\mathbf{c}$ .

In our calculations we used the policy functions from the stationary equilibrium with money growth equal to  $\mu$  to generate  $\mathbf{c}(\mu)$  and then iterated using  $\Gamma$  until a fixed point was found. This procedure requires a choice of the length  $n$  of the transition period. We chose  $n$  large enough that the last few terms of the fixed point  $\mathbf{c}^n$  were close to the value  $\mathbf{c}(\mu)$  associated with the new stationary equilibrium. The resulting description of the transition is thus a rational expectations equilibrium in which agents have perfect foresight about the evolution of aggregate variables.

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