## Notes and Comment

# Perceiving the centroid of curvilinearly bounded rolling shapes 

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Although its absolute motion is the same, the path of a single moving point is often perceived differently when viewed alone than when viewed as a part of a dynamic configuration of many points. A point's perceived motion path depends on the configuration in which it is embedded, with different configurations producing different perceived paths. The results of our research suggest that the perceptual system selects the motion path of the configural centroid as the common motion for curvilinearly bounded rolling shapes.

To understand better how the perceptual system derives structure from dynamic configurations, we have examined the perception of wheel-generated motions (Proffitt \& Cutting, 1979, 1980; Proffitt, Cutting, \& Stier, 1979; Cutting \& Proffitt, Note 1), a phenomenon with a long history of study from mathematics and philosophy through Gestalt psychology. Like others (Duncker, 1929/1938; Johansson, 1950, 1973; Rubin, 1927; Wallach, 1965), we reduced the stimulus event to configurations of lights on unseen rolling wheels. Figure la shows a single point, P , moving as if attached to a wheel. The curve that describes its path is a cycloid. When a whole wheel is viewed in motion, however, cycloidal motion is not seen. Instead, the perceived motion components are circular rotation and linear translation. These components are also seen if two or more points are placed on an unseen rolling wheel with their configurational centroid at the wheel's center. Figure 1 b represents this perception for two points mounted 180 deg apart.

Our previous research, employing configurations of two, three, and four lights, provided evidence that perceived components of motion are derived by a logically ordered extraction of information. First, motions of the individual lights relative to

[^0]each other are extracted, with all points being seen in circular rotation about the centroid of their configuration. Second, the observer-relative common motion of the whole configuration is seen as the dynamics of the centroid, a motion defined by the residual of the first information extraction. When the centroid of the configuration and center of the unseen wheel coincide, the motions perceived are those of a whole wheel-circular rotation and linear translation as shown in Figure 1b. For those configurations in which these centers do not coincide, one sees circular rotation of smaller radii and translation along cycloidal paths as in Figure 1c. The path of this configuration's centroid is a prolate cycloid.
Heretofore, all studies on wheel-generated motions have used point-light configurations whose centroids could be determined by simple methods of plane geometry. The centroid of a two-light system is halfway between them, the centroid of a three-light system is the center of gravity determined by the method of medians, and the centroid of the particular four-light systems that we used could be determined by the intersection of lines drawn through their vertices. Most shapes, however, require analytical means for the mathematical determination of their centroid. Assuming that the area is defined within any arbitrarily placed coordinate system, the centroid can be determined by the following definite integrals:
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\bar{x}=\frac{\int_{a}^{b} x h(x) d x}{\int_{a}^{b} h(x) d x}, \quad \quad y=\frac{\int_{c}^{d} y l(y) d y}{\int_{c}^{d} l(y) d y},
$$
where $\overline{\mathrm{x}}$ and $\overline{\mathrm{y}}$ are the coordinates for the centroid, $h(x)$ and $l(y)$ are the lengths of the bounded slices of the area taken vertically and horizontally, respectively, and $a$ and $b$ are the extreme values of $x$, and $c$ and $d$ are the extreme values of $y$.

It has been suggested that configural centroids may effect eye movements. Pitts and McCulloch (1947) proposed that the superior colliculus computes the above integrals in determining the center of gravity for the distribution of brightness in the visual field and causes the eyes to move so as to fixate on this centroid. Bruell and Albee (1955) also proposed that spontaneous fixations would be to this centroid of brightness. Richards and Kaufman (1969; Kaufman \& Richards, 1969) found that when observers looked at simple patterns of less than 5 deg of visual angle, their spontaneous fixation tendencies were around the centroid of the patterns. This finding was supported by Virsu (1971); however, Murphy, Haddad,


Figure 1. (a) The cycloidal path traversed by a point, $P$, mounted on an unseen rolling wheel. (b) Two lights, $P$ and $Q$, mounted 180 deg apart are perceived as a system of lights rotating around their centroid, which is also the wheel's center in this configuration, and moving linearly along this point's path. (c) Two lights, $P$ and $Q$, mounted 90 deg apart. This configuration is seen rotating about its centroid and traversing the prolate cycloidal path of this unmarked point. (d) Stimulus $B$ mounted on a wheel in location 5 , two points, $P$ and $Q$, on that shape, and the prolate cycloidal paths that they traverse. (e) One possible perception (not seen) of these points, and all others on the shape, rotating around the center of the motion-generating wheel with that point moving linearly. (f) Another possible perception (actually seen), with these points and all others rotating around the centroid and the whole shape traversing the prolate cycloidal curve traced by this abstract point.
and Steinman (1974) found no configural effects on eye movements for experienced observers instructed to fixate on a specified region. Coren (in press) found that eye movements between fixation points were effected by the configural centroids of the areas within about 5 deg of these points.

Our goal was to determine whether the perceived common motion for rolling shapes would follow the motion path of their centroids. That is, we wished to determine whether the perceptual system would derive the same components of motion for shapes as it does for point-light configurations. Figure if shows this description for one of the moving shapes employed. Figure 1d shows the absolute motion paths of two extreme points on the shape, and Figure le shows an alternative set of motion components that could be derived by extracting the linear motion of the point at the wheel's center and each point's rotation about this center. If the observer sees the shape rotating about its centroid and translating along the path described by this abstract point, as in Figure 1f, then we have evidence that the perceptual system is capable of determining a location within the shape that, in mathematics, requires a calculus procedure. Of course, the perceptual system need not employ
exactly this process, but it must at least perform some calculus-like analog. ${ }^{1}$

Three curvilinearly bounded shapes were fashioned as shown in the left half of each panel in Figure 2. Stimulus A has two axes of symmetry, B has one, and $C$ has none. Each shape was placed on a wheel in six locations. The numbers in Figure 2 on and around each shape mark the location of the wheel's center. Location 1 placed the shape's centroid at the center of the motion-generating wheel. The other locations surrounded the centroid above, below, and to each side. As an example, Figures 1d-1f show Stimulus B mounted in location 5. A metric, $D_{m} / r$, was derived in our previous work that expressed the relative vertical movement perceived in wheel-generated motions. If we call $D_{m}$ the distance of the centroid from the wheel's center and $r$ the distance of the farthest point on the shape from the wheel's center, then the vertical excursion of the centroid is $2 \mathrm{D}_{\mathrm{m}}$ and the vertical excursion of the farthest point is 2 r . The metric $D_{m} / r$ expresses the vertical excursion of the centroid relative to the limit of vertical motion of the shape (Proffitt \& Cutting, 1979). The numbers of the shapes in Figure 2 are ordered such that, as the numbers increase, $D_{m} / r$ increases.


Figure 2. Panels a, b, and $\mathbf{c}$ show Stimuli $\mathbf{A}, \mathrm{B}$, and $\mathbf{C}$, respectively, and the results for each. Stimulus $A$ has two axes of symmetry, Stimulus B has one, and Stimulus $C$ has none. Points numbered 1 through 6 indicate the locations of the wheel's center relative to the shapes. Locations differed for each stimulus to eliminate symmetrical duplications. Location 1 is at the centroid of each stimulus. Note that the centroid for Stimulus $\mathbf{C}$ is outside the bounds of its form. Locations 2 through 6 are at increasing distances from the centroid, yielding increasing values of $D_{m} / r$. These indices are determined by taking the distance between each location and the shape's centroid and dividing it by the distance from the location to the outermost edge of the shape. The plotted points represent the mean judgments of 20 viewers; the vertical lines through them extend one standard error in each direction.

The method of generating the stimuli was similar to that described in our previous work (Proffitt et al., 1979). Each shape was covered with reflectant tape and mounted on a wheel that rolled across a level surface. Bright lights were focused on the wheel as the event was videotaped. When shown on a television monitor, with contrast turned to near maximum and brightness to near minimum, only the rolling shape was visible. Each shape made 2.5 revolutions per trial at $1.75 \mathrm{rev} / \mathrm{sec}$. The maximum visual angle subtended by unmoving shapes was .5 deg ; each dynamic event traversed a visual angle of about 5 deg. There were 18 stimuli ( 3 shapes each at 6 different locations) and a test sequence was constructed
of 4 randomizations of this set for a total of 72 trials. Only the last 3 randomizations were scored; the first served as practice to stabilize use of the judgment scale. Twenty Wesleyan University undergraduates viewed a television monitor on which stimulus trials were presented. They were instructed to rate each event, using a 7 -point scale on "how wheel-like the movement of the event appeared to be," with 7 indicating the most and 1 the least wheel-like movement. Previously, we had found that observers based their judgments on the relative vertical excursion of the configurations' centroid-the less the relative motion, the more events were judged to move like a wheel with linear translation (Proffitt \& Cutting, 1979).

The results are shown for each stimulus in the right halves of each panel in Figure 2. Across all three stimuli, the correlation between observers' judgments and $D_{m} / r$ indices was high ( $\mathrm{r}=-.82, \mathrm{p}<.001$ ), as was interjudge reliability $\left[\mathrm{W}=.53, \chi^{2}(19)=181.2\right.$, $\mathrm{p}<.001]$. Thus, $\mathrm{D}_{\mathrm{m}} / \mathrm{r}$ is a good predictor of judged goodness for wheel-like motions. In other research (Proffitt \& Cutting, 1980), point-light stimuli were directly compared using a paired comparisons design and their motion paths were drawn. The results were entirely consistent with rating scale judgments. Moreover, we had observers rate the similarity of various pairs of point-light stimuli, without suggesting to them the dimension on which they should be judged, and found the metric to be the primary dimension in a multidimensional scaling solution (Cutting \& Proffitt, Note 1). Thus, we take the power of these findings as extending beyond correlational procedures. We infer that observers perceived the translation of these shapes as following the paths of their centroids: The less the vertical hopping of this point, the more wheel-like the event was judged to be,

Each stimulus shape had one exception to an otherwise perfect rank-order correlation for its six locations. Two of these exceptions are amenable to explanations, and a third is not. Stimulus $A$ in location 5 received ratings higher than predicted; however, the center of one of the two lobes in this shape coincided with the wheel's center. Attending to this point would yield the perception of linear translation (Cutting \& Proffitt, Note 1). In addition, Stimulus C in location 6 hugged the rim of the wheel with its curved edge, perhaps causing it to be more suggestive of wheel-like motion than its $D_{m} / r$ metric would predict. The reason Stimulus B in location 4 was given ratings higher than predicted is not clear.

Direct comparison of stimulus shapes can be made only for judgments when the generating center was at location 1. Here, order of symmetry for the shapes appears to contribute to increased judgments (Friedman two-way analysis of variance of ranks: $\chi_{\mathrm{r}}^{2}=5.93, \mathrm{p}=.052$ ). Stimuli $\mathrm{A}, \mathrm{B}$, and C garnered judgments of $5.6,5.4$, and 4.8 , respectively. Sym-
metry considerations fully determine the centroid for Stimulus $A$, determine a line passing through the centroid for Stimulus B, but are irrelevant for determining the centroid of Stimulus C. Therefore, symmetry likely serves an auxiliary function in perceptually locating centroids. This weak symmetry influence was previously found for point-light configurations (Proffitt et al., 1979).

The results suggest that the motion components perceived for shapes undergoing wheel-generated motions are determined in the same manner as are those for point-light configurations. The perceptual system extracts the motion of all points relative to each other, thereby deriving a rotation about the shape's centroid. The common motion of the whole form relative to the observer is seen as the path traversed by this abstract point. That the mathematical determination of a shape's centroid is specified by a calculus procedure implies that the invariants picked up by the perceptual system are a good deal more abstract than has been previously thought. Calculus, as the mathematics of dynamics, may make explicit those procedures that our perceptual systems perform (perhaps analogically) when interpreting events.

## REFERENCE NOTE

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## NOTE

1. We stress the analogical aspect of calculus-like procedure. William James (1890, Vol. 1, p. 196) warned that "The great snare of the psychologist is the confusion of his own standpoint with that of the mental fact about which he is making his report." Our standpoint with regard to the convenience of calculus by no means assures the possession by the perceiver of such a faculty. Kolers and Smythe (1979, p. 166) are particularly cogent in this regard: "One may, for example, model with the differential calculus the behavior of a shortstop fielding a hard grounder, but that is no reason to assume that the shortstop himself uses (or even knows) the differential calculus."
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