Interpreting Concatenation and Concatenates

Paul M. Pietroski, University of Maryland

What is the significance of *combining* expressions in a natural human language? A complex expression is not a mere list of words. Combining expressions, as in 'red ball' or 'ball that Pat kicked yesterday' has a semantic effect. But *how* is the meaning of a phrase related to the meanings of its constituents? And how are the meanings of predicates, simple or complex, related to the meanings of sentences and referential devices? Such questions lie at the heart of attempts to understand the kind(s) of compositionality exhibited in human languages.

Elsewhere, I have argued that concatenation signifies conjunction; see Pietroski (2002, 2003, 2005). On this view, phrases like 'red ball' manifest the true character of concatenation: combining 'red' with 'ball' yields a predicate satisfied by things that satisfy 'red' and 'ball'. But examples like (1) seem not to fit this mold.

(1) Pat did not kick every ball yesterday

How can all the constituents of (1) be plausibly analyzed in terms of monadic predicates conjoinable with others? And given such examples, why think concatenation signifies a single operation across diverse constructions, much less the operation of predicate-conjunction? My reply, developed in Pietroski (2005) but presented somewhat differently here, involves a supplementary hypothesis about the role of certain grammatical *relations*.

While concatenation always signifies conjunction, combining a predicate with an argument—as in 'kicked it'—has a grammatical effect that introduces a second semantic factor that is absent in simple cases of combining two predicates. And while a sentence is not a mere conjunction of predicates, the "third" factor may be nothing more than existential closure. Given developments of Davidson's (1967, 1985) work, (2) can be analyzed as in (2a).

- (2) Plum stabbed Green quietly in the hall with a knife
- (2a) ∃e[Agent(e, Plum) & PastStab(e) & Theme(e, Green) & Quiet(e) & In(e, the hall) & With(e, a knife)]

I think such proposals illustrate a more general pattern: concatenation signifies predicateconjunction; certain grammatical relations, akin to prepositions, let us interpret arguments as predicates of "things with participants"; and existential closure, often corresponding to the end of a grammatical "cycle," converts a monadic predicate into something evaluable as true or false.

1. Elementary Cases

It is hardly news that simple cases of predicate-modification correspond to predicate-conjunction, which can be recursive. The phrase 'red ball that Pat kicked' is understood as a conjunction of three predicates, corresponding to the adjective, noun, and relative clause.² Many adverbial modifiers also seem to be predicate-conjoiners; see Davidson (1967), Taylor (1985), Parsons (1990). Sentences like (3-7) exhibit the indicated pattern of validity.

- (3) Plum stabbed Green quickly with a knife
- (4) Plum stabbed Green with a knife quickly
 - $(3) < \longrightarrow (4)$

(5) Plum stabbed Green quickly

.

- (6) Plum stabbed Green with a knife
- $(5) \longrightarrow (7) < (6)$

(7) Plum stabbed Green

But (5) and (6) do *not* jointly imply (3) or (4). Plum may have stabbed Green twice: once with a knife but slowly, and once with a fork quickly. Such facts can be explained by taking seriously paraphrases like (3a), partly formalized in (3b).³

(3a) At least one stabbing of Green by Plum was done quickly and with a knife

(3b) ∃e{PastStabOfGreenByPlum(e) & Quick(e) & ∃x:Knife(x)[With(e, x)]}

Of course, (3b) doesn't reveal semantic structure in the first conjunct. So following

Davidson (1967), one might analyze (3) as in (3c);

 $(3c) \ \exists e \{ PastStabOfBy(Plum, Green, e) \ \& \ Quick(e) \ \& \ \exists x : Knife(x)[With(e, x)] \}$ where 'PastStabOfBy(x, y, e)' means that e was a stab of y by x, and (7) is analyzed as in (7a).

 $(7a) \exists e[PastStabOfBy(Plum, Green, e)]$

But the constituents-as-conjuncts picture can be extended, as suggested in (7b) and (7c).⁴

- (7b) There was a stab such that its Theme was Green, and its Agent was Plum
- $(7c) \exists e[PastStab(e) \& Theme(e, Green) \& Agent(e, Plum)]$

Each word in (7) corresponds to a conjunct in (7c) that may also be associated with a "participation relation." Specifically, one can hypothesize that action verbs and their arguments are understood as conjoinable predicates of events. Then the semantic structure of (3)—the logical form of any proposition expressed with (3)—is as shown in (3d).

(3d) $\exists e \{ Agent(e, Plum) \& PastStab(e) \& Theme(e, Green) \& Quick(e) \& \exists x: Knife(x)[With(e, x)] \}$

The compositional details depend on assumptions about syntax. But suppose the basic constituency structure of (7) is as follows: $[Plum_N [stabbed_V Green_N]]$. Brackets indicate concatenation of expressions. The subscripts reflect a distinction between nouns, including names, and verbs. Let's assume, standardly, that a phrase inherits the label of exactly one constituent, and that a V combined with an N is a V: $[Plum_N [stabbed_V Green_N]_V]_V$. The idea is that for purposes of concatenation, $[stabbed_V Green_N]$ is a V, like the verbs 'stabbed' and 'sang'. Likewise, $[Plum_N [stabbed_V Green_N]_V]$ is a complex V.

A more realistic depiction of (7), $[past_T [Plum_N [stab_V Green_N]_V]_V]_T$, might introduce a tense morpheme that subsequently (transformationally) recombines with the verb. But let's ignore such complications, and indicate any further sentence structure with angled brackets: $\langle [Plum_N [stabbed_V Green_N]_V]_V \rangle$. This distinguishes the sentence from the homophonic phrase $[Plum_N [stabbed_V Green_N]_V]_V$, which can be modified with temporal adverbs. The immediate challenge, though, is to analyze the phrase as a conjunction of predicates.

Let 'Val(e, Σ , \mathbf{A})' mean that e is a value of expression Σ relative to the assignment \mathbf{A} of values to any variables in Σ . Then we can formulate lexical axioms like the following:

Val(e, Green_N, **A**) iff
$$x = Green$$
 Val(e, it_{Ni}, **A**) iff $e = Ai$

$$Val(e, Plum_N, A) iff x = Plum Val(e, stabbed_V, A) iff PastStab(e, A)$$

where in the metalanguage, 'Green' is a logically proper name for a certain gardener, 'Plum' is a label for a certain professor, ' $\mathbf{A}i$ ' stands for whatever \mathbf{A} assigns to the variable with index 'i', and 'PastStab(e, \mathbf{A})' is an appropriate way of relativizing values of the tenseless verb stab_v. We can formulate Conjunctivism in these terms: relative to any assignment \mathbf{A} , an entity e is a value of the phrase formed by concatenating Σ with Σ ' iff e is a value of both concatenates.

Val(e,
$$[\Sigma \Sigma']$$
, **A**) iff Val(e, Σ , **A**) & Val(e, Σ' , **A**)

Alas, it follows from these axioms that (relative to \mathbf{A}): e is a value of [stabbed_v Green_N]_v iff e was both a stab and identical with Green; and e is a value of [Plum_N [stabbed_v Green_N]_v]_v iff e was a stab, identical with Green, and identical with Plum. There is, however, a remedy. And as we'll see, other views face analogous difficulties that call for remedies with worse side-effects.

The fact that [stabbed_V Green_N] is a V can help preserve the idea that concatenation signifies conjunction. While stabbed_V and Green_N are not *themselves* coherently conjoinable,

Conjunctivists can supplement their composition axiom with an auxiliary hypothesis. When a V combines with an N, forming a complex V to be interpreted conjunctively, the N is marked as an argument; and for purposes of interpretation, the concatenates are the V and the N-as-marked. This hypothesis can be encoded by replacing [stabbed_V Green_N]_V with [stabbed_V Green_{N\theta}]_V; where '\theta' is the relevant mark. But the idea, however coded, is that 'stabbed Green' is not a *mere* concatenation of two words. It is a *phrase* of the same grammatical *type* as 'stabbed'. While 'stabbed' and 'Green' are the elements, the phrasal constituents are the verb and its argument.

This makes room for distinguishing the name Green_N, independent of its relation to stabbed_v, from something more complex: Green_N-as-subordinate-sister-of-stabbed_v. Less cumbersomely, we can say that Green_N is "theta marked" by stabbed_v, which is conjoined with Green_N0 as opposed to Green_N. And this formal distinction can be exploited by a system constrained to treat concatenation as a sign of conjunction. If stabbed_v and Green_N cannot have a common semantic value, the event-predicate and entity-label cannot be coherently conjoined, not even if Green_N is construed as a predicate satisfiable by exactly one entity. But the lexical N need not be the *expression* conjoined with verb. So far as interpretation is concerned, the expression conjoined/concatenated with stabbed_v may be Green_{N0}, which is a product of the lexical N *and* its position in the phrase. Even if stabbed_v is concatenated with Green_N "in syntax," such combination may have secondary effects, like Θ -marking.

In which case, $[\text{stabbed}_V \text{ Green}_{N\Theta}]_V$ can be construed as the conjunction of stabbed_V with $\text{Green}_{N\Theta}$. And the expression $\text{Green}_{N\Theta}$ can be interpreted as an event predicate, conjoinable with others, even if Green_N is a label for a certain entity (who may be the Theme of a stabbing). This treats ' Θ ' like the preposition 'of' in 'stabbing of Green': $[\text{stabbing}_V \text{ [of}_P \text{ Green}_N]_P]_V$. Though the

preposition can be viewed as an overt signal for the relevant structural relation. In any case, this leaves room for various interpretations of (the grammatical relation indicated with) ' Θ '.

One might treat ' Θ ' itself as a Theme-marker, as suggested below.

Val(e,
$$\Sigma_{\Theta}$$
, **A**) iff $\exists x[Val(x, \Sigma, \mathbf{A}) \& Theme(e, x)]$; so
Val(e, $Green_{N\Theta}$, **A**) iff $\exists x[Val(x, Green_N, \mathbf{A}) \& Theme(e, x)]$; or simplifying,
Val(e, $Green_{N\Theta}$, **A**) iff Theme(e, Green)

But more useful will be a variant according to which ' Θ ' indicates an abstract relation—being the "internal" participant of—that predicates of certain kinds associate with certain more specific participation relations. Given the axioms below,

Event(e)
$$\rightarrow \forall x[Internal(e, x) \leftarrow Theme(e, x)]$$

$$Val(e, \Sigma_{\Theta}, \mathbf{A}) \text{ iff } \exists x[Val(x, \Sigma, \mathbf{A}) \& Internal(e, x)]$$

$$Val(x, Green_N, \mathbf{A})$$
 iff $Entity(x) & x = Green$

it still follows that internal participants of events are Themes. In particular,

$$Val(e, [stabbed_v Green_{N\Theta}]_v, \mathbf{A}) iff$$

Event(e) & PastStab(e, A) &
$$\exists x[Val(x, Green_N, A) \& Internal(e, x)]$$

So Val(e, $[stabbed_v Green_{N\Theta}]_v$, **A**) iff Event(e) & PastStab(e, **A**) & Theme(e, Green). But there may be other "things" in which entities can "participate" without being Themes.⁸

Perhaps the values of some verbs, as in 'Plum likes Green', are *states*; see Parsons (1990). And especially in light of examples like 'The door was open because Plum opened it', one might want to say that states have Objects (with enduring properties) while events have Themes (that undergo changes). Perhaps some predicates have *ordered pairs* as semantic values,

even if action verbs do not. And in any case, we can say that ordered pairs have internal participants. Consider <stabbed $_{V}$, Green $_{N}>$, identifiable with $\{$ stabbed $_{V}$, $\{$ Green $_{N}\}\}$, whose internal participant is the name Green $_{N}$. The ordered pair <Plum, Green> has Green himself as its internal participant. We can also think about each stabbing of Green as an event with Green as its internal participant. And we can say that Green is the internal participant of his death, even if a death has no external participant. This fits with independent reasons for treating 'Green died' as a transformation of [died $_{V}$ Green $_{N\Theta}]_{V}$, in which the verb takes Green $_{N}$ as an internal argument. 9

In short, we can regard events, states, and ordered pairs as species of a broader genus: things in which entities participate. We can invent a predicate satisfied by ordered pairs of the form $\langle x, Green \rangle$. Likewise, there can be a predicate satisfied by anything that has Green as its internal participant. And we can hypothesize that $Green_{N\Theta}$ is such a predicate. In which case, if stabbed_v is a predicate of events whose internal participants are Themes, Conjunctivism implies that [stabbed_v $Green_{N\Theta}$] is a predicate satisfied by e iff e was a stab whose Theme was Green.

Unlike died_V , $\operatorname{stabbed}_V$ easily combines with two grammatical arguments. But the obvious proposal is that when $[\operatorname{stabbed}_V \operatorname{Green}_{N\Theta}]_V$ combines with Plum_N , forming a V-phrase to be interpreted conjunctively, Plum_N is marked as the argument of this phrase; where for interpretive purposes, the concatenates are $[\operatorname{stabbed}_V \operatorname{Green}_{N\Theta}]_V$ and Plum_N -as-marked. The resulting expression is $[\operatorname{Plum}_{N\Theta} [\operatorname{stabbed}_V \operatorname{Green}_{N\Theta}]_V]_V$, with ' Θ ' as the mark of a lexical V, and ' Θ ' as the mark of a complex V with a Θ -constituent. Of Given appropriate axioms about events and ' Θ ',

Event(e)
$$\rightarrow \forall x [External(e, x) \leftarrow Agent(e, x)]$$

$$Val(e, \Sigma_{\Theta}, \mathbf{A}) \text{ iff } \exists x[Val(x, \Sigma, \mathbf{A}) \& External(e, x)]$$

it follows that $Val(e, [Plum_{N\underline{\Theta}} [stabbed_v Green_{N\Theta}]_v]_v, \mathbf{A})$ iff $Agent(e, Plum) \& PastStab(e, \mathbf{A}) \& Theme(e, Green).$

Adverbial modifers can be added at any point. So given the biconditionals below,

Val(e, [with a knife]_A, **A**) iff $\exists x$:Knife(x)[With(e, x)]

we get the desired result.

Val(e,
$$[[Plum_{N\Theta}]_{V}]$$
 [[stabbed_V Green_{NO}]_V [with a knife]_A]_V, **A**) iff Agent(e, Plum) & PastStab(e, **A**) & Theme(e, Green) & $\exists x: Knife(x)[With(e, x)] & Quick(e)$

Combining a complex V with an adjunct creates another V, not a full *sentence*. But eventually, such modification comes to an end. Let's assume that at this point, perhaps associated with tense, a phrase is marked as complete: nothing more can be added to it; though it may undergo transformations, or serve as a sentential constituent. Using angled brackets to indicate this culminating aspect of sentence-construction, (7) has the grammatical form shown in (7G).

(7) Plum stabbed Green

(7G)
$$\langle [Plum_{N\Theta} [stabbed_v Green_{N\Theta}]_v]_v \rangle$$

This invites an obvious thought about the significance of marking a phrase as complete. Let ' τ ' and ' $_{\perp}$ ' stand for the potential semantic values of sentences. Given the following axioms,

$$Val(\top, \langle \Sigma \rangle, \mathbf{A})$$
 iff $\exists e[Val(e, \Sigma, \mathbf{A})]$

$$Val(\bot, \langle \Sigma \rangle, \mathbf{A}) \text{ iff } \neg Val(\top, \langle \Sigma \rangle, \mathbf{A})$$

the *sentence* (7) has the value \top iff \exists e[Agent(e, Plum) & PastStab(e, **A**) & Theme(e, Green)]. Given exactly two sentential values, as in classic truth-conditional semantics, Conjunctivists can go on to analyze sentential negation as a monadic predicate satisfied by and only by \bot .

Suppose the grammatical structure of (8) is as shown in (8G),

(8) Plum didn't stab Green

$$(8G) \langle [NEG \langle [Plum_{N\Theta} [stabbed_v Green_{N\Theta}]_v]_v \rangle] \rangle$$

with NEG as a functional element that combines with a sentence to form an expression that can be marked as complete and subsequently interpreted as another sentence. (This treats NEG, in effect, as a sentential adjunct.) And consider the following hypothesis: Val(e, NEG, \mathbf{A}) iff $\mathbf{e} = \bot$.

If concatenation signfies conjunction, then attaching NEG creates a conjunctive predicate.

$$\begin{split} & Val(e, [\text{NEG} \ \langle [\text{Plum}_{\text{N}\underline{\Theta}} \ [\text{stabbed}_{\text{V}} \ Green_{\text{N}\Theta}]_{\text{V}}]_{\text{V}} \rangle], \, \textbf{A}) \ iff} \\ & e = \bot \ \& \ Val(e, \ \langle [\text{Plum}_{\text{N}\Theta} \ [\text{stabbed}_{\text{V}} \ Green_{\text{N}\Theta}]_{\text{V}}]_{\text{V}} \rangle, \, \textbf{A}). \end{split}$$

Marking this complex predicate as a complete sentence indicates existential closure.

$$\begin{aligned} &Val(\top, \left<[\mathsf{NEG} \left<[\mathsf{Plum}_{\mathsf{N}\underline{\Theta}} \left[\mathsf{stabbed}_{\mathsf{V}} \ \mathsf{Green}_{\mathsf{N}\Theta}\right]_{\mathsf{V}}\right]\right>, \mathbf{A}) \ \mathsf{iff} \\ &\exists \mathsf{e}[\mathsf{e} = \bot \ \& \ \mathsf{Val}(\mathsf{e}, \left<[\mathsf{Plum}_{\mathsf{N}\Theta} \left[\mathsf{stabbed}_{\mathsf{V}} \ \mathsf{Green}_{\mathsf{N}\Theta}\right]_{\mathsf{V}}\right]_{\mathsf{V}}\right>, \mathbf{A})] \end{aligned}$$

So as desired, (8) has the value \top iff (7) has the value \bot . This shows that NEG and stabbed can each be predicates, coherently conjoinable with others, without being coherently conjoinable with each other. The intervening existential closure lets a predicate of truth values and a predicate of events appear in the same (matrix) sentence, even though concatenation signifies conjunction.

Conjunctivists can diagnose many apparent counterexamples this way: a grammatical argument, marked as such, is interpreted as a predicate of "things" in which semantic values of the argument "participate;" and marking a phrase as complete, with the requisite number of arguments for the relevant predicate, corresponds to existential closure. Suppose that (9G) reflects the grammatical structure of (9), suppressing embedded structure for simplicity.¹¹

(9) Plum stabbed Green, or Peacocke shot Mustard

(9G)
$$\langle [\langle Plum \ stabbed \ Green \rangle_{\Theta} \ [or \ \langle Peacocke \ shot \ Mustard \rangle_{\Theta}]] \rangle$$

The idea is that OR takes two sentential arguments. And suppose the values of OR are ordered

pairs of truth values— $\langle \tau, \tau \rangle$, $\langle \tau, \pm \rangle$, and $\langle \pm, \tau \rangle$. Then relative to any assignment **A**, (9G) has the value τ iff at least one thing satisfies the following three conditions: its external participant is the value of $\langle \text{Plum stabbed Green} \rangle$; it is a value of OR; and its internal participant is the value of $\langle \text{Peacocke shot Mustard} \rangle$. If any value of OR satisfies the first and third condition, then either Plum stabbed Green or Peacocke shot Mustard.

2. A Different Picture (for Comparison)

Davidson (1967) would have analyzed (5) along the lines of (5a).

- (5) Plum stabbed Green quickly
- (5a) ∃e[PastStabbingOfBy(Plum, Green, e) & Quick(e)]

There are reasons, noted below, for spelling out the first conjunct conjunctively as in (5b);

(5b) ∃e[Agent(e, Plum) & PastStab(e) & Theme(e, Green) & Quick(e)] see Pietroski (2005) for a review. But one can go this far, as Davidson (1985) did, without saying that the ampersands reflect *concatenation* of a semantically *monadic* predicate with two arguments. Perhaps (5b) reflects a thematically structured *lexical* meaning of a semantically *ternary* verb combined with two arguments. This is, however, one way of formulating the issue.

Do the ampersands directly reflect the significance of concatenation, as opposed to an interaction of lexical meaning and nonconjunctive concatenation? We can also ask why there is no verb 'quabbed' such that 'Plum quabbed Green' is true iff $\exists e[Agent(e, Plum) \ v \ PastStab(e) \ v$ Theme(e, Green)], and no adverb 'glickly' such that 'Plum stabbed Green glickly' is true iff $\exists e[PastStabOfBy(Plum, Green, e) \ v \ Quick(e)]$. Is this because supralexical concatenation signfies conjunction, or because only certain kinds of lexical meanings can enter into semantic composition—or both, or neither? An increasingly common view is that with regard to simple cases of adjunction, combining one predicate with another does indeed signify predicate-

conjunction; see Heim and Kratzer (1998). The disagreements tend to be about cases of predicate-argument combination. But it is worth considering the pure "Functionist" hypothesis that concatenation in a human language always signifies function-application—as in a Fregean *Begriffsschrift*, and as suggested by various developments of Montague (1970, 1973).

Functionism is often illustrated by supposing that a verb like 'stabbed' indicates a binary function, $\lambda y.\lambda x.$ Stabbed(x, y), from entities to functions from entities to truth values. Then [stabbed_v Green_N]_v indicates the function $\lambda x.$ Stabbed(x, Green); and [Plum_N [stabbed_v Green_N]_v] indicates truth iff Stabbed(Plum, Green). But event variables can be added. Suppose 'stabbed' indicates the function $\lambda y.\lambda x.\lambda e.$ PastStabbingOfBy(e, x, y), which maps entities to functions from entities to functions from events to truth values. Then [Plum_N [stabbed_v Green_N]_v]_v indicates $\lambda e.$ PastStabOfBy(e, Plum, Green), which maps events to truth values. This distinguishes sentences, which involve existential closure, from semantically monadic V-phrases. And this makes room for a Functionist account of adverbs. But ancillary hypotheses are needed.

Suppose that 'quickly' indicates $\lambda e.Quick(e)$, which maps events to truth values. Neither $\lambda e.Quick(e)$ nor $\lambda e.PastStab(e, Plum, Green)$ maps the other to truth. So at least initially, [[Plum_N [stabbed_V Green_N]_V]_V quickly_A]_V presents a difficulty for Functionism, much as [Plum_N [stabbed_V Green_N]_V]_V presents a difficulty for Conjunctivism. Likewise, if the adjective 'red' and noun 'ball' indicate functions from entities to truth values, $\lambda x.Red(x)$ and $\lambda x.Ball(x)$, neither maps the other to truth. So the phrase [red_A ball_N]_N presents a difficulty for Functionism. But there is a familiar proposed remedy: when one predicate is adjoined to another, one predicate indicates a "higher-order" function than it does when appearing by itself as a main predicate.

This suggestion can be encoded in many ways. But to make explicit the parallel with Conjunctivist treatments of arguments, let's mark the subordinate status of 'red' in 'red ball' as

follows: $[red_{A^{\uparrow}} \ ball_N]_N$. The idea is to distinguish the lexical adjective red_A from the adjunct $red_{A^{\uparrow}}$, which will indicate a higher-order function. Let ' $\|\Sigma\|$ ' stand for the Functionist value of the expression Σ , ignoring assignment variability for simplicity. And consider the axioms below,

$$\|red_A\| = \lambda x.Red(x) \qquad \qquad \|ball_N\| = \lambda x.Ball(x)$$

$$\|\Sigma_{\scriptscriptstyle \uparrow}\| = \lambda F.\lambda x.\|\Sigma\|(x) \& F(x) \qquad \qquad \|quickly_A\| = \lambda e.Quick(e)$$

which have consequences for 'red' and 'quickly' as adjuncts.

$$\begin{aligned} \|\mathrm{red}_{\mathrm{A}^{\uparrow}}\| &= \lambda F.\lambda x. \|\mathrm{red}_{\mathrm{A}}\|(x) \ \& \ F(x) \\ &= \lambda F.\lambda x. \mathrm{Red}(x) \ \& \ F(x) \end{aligned}$$

$$\|\mathrm{quickly}_{\mathrm{A}^{\uparrow}}\| &= \lambda F.\lambda e. \|\mathrm{quickly}_{\mathrm{A}}\|(x) \ \& \ F(x) \\ &= \lambda F.\lambda e. \mathrm{Quick}(e) \ \& \ F(e) \end{aligned}$$

This gives Functionists their desired results: $\|\text{red}_{A^{\uparrow}}\|(\|\text{ball}_{N}\|) = \lambda x.\text{Red}(x)$ & Ball(x); and $\|\text{quickly}_{A^{\uparrow}}\|(\|\text{Plum}_{N}\|\text{stabbed}_{V}\text{Green}_{N}\|_{V}\|) = \lambda e.$ Quick(e) & PastStabOfBy(e, Plum, Green). So one can maintain that concatenation signifies function-application, even in cases of adjunction, by hypothesizing a certain "type-shifting" significance for the grammatical relation between a modifying expression and the predicate it modifies.

One can similarly maintain that concatenation signifies conjunction, even in cases of predicate-argument combination, by hypothesizing a certain "prepositional" significance for the grammatical relation between an argument and the predicate it saturates. Neither view is *ad hoc* or intrinsically simpler than the other. Though for several reasons, I find adjunct-adjustment less plausible overall than argument-adjustment; see Pietroski (2005), drawing on many authors. Adjuncts are recursive in ways that arguments are not. We have independent reasons for saying that thematic roles are associated with predicate-argument relations. Functionists have additional work to do, in explaining why human languages do not exhibit certain lexical meanings: $\lambda y.\lambda x.\lambda e.Agent(e, x) v PastStab(e) v Theme(e, y); \lambda z.\lambda y.\lambda x.StabbedWith(x, y, z); etc. And the idea that every predicate corresponds to a function, a$ *set*of some sort, creates difficulties (related

to vagueness and Russell's paradox) that Conjunctivists can avoid. But in any case, one can *supplement* a hypothesis about the significance of concatenation with a hypothesis about the significance of certain grammatical relations.

Since adjuncts differ semantically from arguments, some such supplementation is required, unless we simply encode the difference with distinct composition principles. Many theorists do just this; see Higginbotham (1985), Larson and Segal (1995), Heim and Kratzer (1998). For example, given a basically Functionist idiom, one can adopt the following axioms.

$$\|[\boldsymbol{\Sigma}_{\text{pred}} \, \boldsymbol{\Sigma}_{\text{arg}}]\| = \|\boldsymbol{\Sigma}_{\text{pred}}\|(\|\boldsymbol{\Sigma}_{\text{arg}}\|) \qquad \qquad \|[\boldsymbol{\Sigma}_{\text{pred}} \, \boldsymbol{\Sigma}_{\text{ad}}]\| = \lambda x. \|\boldsymbol{\Sigma}_{\text{pred}}\|(x) \; \& \; \|\boldsymbol{\Sigma}_{\text{ad}}\|(x)$$

But if one such composition principle has a conjunctive character, and there is empirical pressure to replace 'PastStabOfBy(x, y, e)' with 'PastStab(e) & Agent(e, x) & Theme(e, y)', simplicity suggests that we explore the possibility of making do with a Conjunctivist composition principle.

3. Plural Variables

Earlier, I said that $Val(x, it_{N1}, \mathbf{A})$ iff $x = \mathbf{A}1$. Relative to any assignment \mathbf{A} of values to variables, with 'it' as the first variable, x is a value of 'it' iff \mathbf{A} assigns x to the first index. Given this, Conjunctivists can say that $Val(e, [stabbed_v it_{N1\Theta}]_v, \mathbf{A})$ iff $PastStab(e, \mathbf{A})$ & Theme(e, $\mathbf{A}1$). But what about 'them', as in (10)?

(10) Green stabbed them

Moreover, speakers can use (11) to report that Green and Plum together stabbed six turnips,

(11) They stabbed six turnips

without implying that either stabber stabbed six.

If each assignment assigns exactly one value to each variable, then the value of a plural variable is presumably a plural entity—an entity with other entities as elements; see, e.g., Link (1983, 1998), Schwarzschild (1996). But we can reject this singularist conception of variables, and

let an assignment assign values to a plural variable; see Boolos (1984). Instead of associating 'them' with a set of turnips, and 'they' with the set {Plum, Green}, we can associate 'them' with each of the demonstrated turnips and 'they' with each of the people demonstrated. This conception of value-assignments is less familiar, but important for the account of quantification that follows.

Consider a domain with exactly five "basic" entities: a, b, c, d, and e. The possibilities for "things demonstrated" are shown below; the blank reflects cases of demonstrating nothing.

cba	cb	ca	c	ba	b	a	_
dcba	dcb	dca	dc	dba	db	da	d
ecba	ecb	eca	ec	eba	eb	ea	e
edcba	edcb	edca	edc	edba	edb	eda	ed

We can interpret this diagram in terms of thirty-one (non-null) entities: five singletons, and twenty-six plural entities; where each plural entity, with two or more singletons as elements, can be *the* value of a plural variable relative to an assignment. But other interpretations are possible. Consider the twelfth cell, indicated with 'dba'. Instead of thinking about the set {d, b, a} as the value of a variable, we can think about three entities—d, b, and a—as the values of that variable.

To highlight this contrast, imagine binary numerals, with our five entities numbered as follows: a, 1; b, 10; c, 100; d, 1000; and e, 10000. Then the twelfth cell would be indicated with '01011', which designates the sum of three entity correlates: 01011 = 1000 + 10 + 1. One can hypothesize that this arithmetic relation reflects a metaphysical relation that each value of a plural variable bears to potential values of singular variables. From this perspective, '01011' stands for an entity x_{pl} such that y is an element of x_{pl} iff y is identical with d or b or a. Alternatively, we can read '01011' as five answers to yes/no questions about whether a certain

entity, perhaps with others, is assigned to a given variable: (e, \bot) , (d, \top) , (c, \bot) , (b, \top) , (a, \top) .

From this perspective, lattice structures indicate possibilities for assigning *one or more* values to a variable. And there is nothing puzzling about assigning more than one value to a variable. Assigning exactly one entity to a singular variable, like 'it', is akin to an act of demonstrating that entity alone. Likewise, an act of demonstrating several things is akin to assigning more than one entity to a plural variable. Given a tendentious semantic theory, one might insist that what we call an act of demonstrating several things is really an act of demonstrating a plural thing (with elements). But prima facie, this is the fancy idea in need of theoretical support. And there is much to be said in favor of the hypothesis that human languages employ plural variables, each of which can have many values relative to an assignment; see Boolos (1998), Schein (1993, forthcoming), Higginbotham (1998), Pietroski (2003, 2005).

To formulate such hypotheses, we need appropriate notation. Let 'X', unlike 'x', be a metalanguage variable that can be assigned one or more values. Then ' \exists X[...X...]' means that there are one or more things, the Xs, such that *they* satisfy the condition [...X...]; where the plural condition may or may not be such that *they* satisfy it iff *each of them* satisfies a corresponding singular condition. (Foreshadowing: some things are turnips iff each of them is a turnip, since 'turnip' is a distributive predicate; though some turnips can form a circle, or form circles, even if no one of them forms any circle.) Correlatively, let 'Xx' mean that x is one of the Xs. Intuitively, 'Xx' says that it_x is one of them_x; where this does not mean that it_x is an element of it_x, with 'it_x' having exactly one collectionish value relative to each assignment.

On this interpretation, ' $\exists X [\forall x (Xx < -> Fx)]$ ' means that one or more things are such that each thing is one of them iff it is an F; where this does not mean that there is a set such that each thing is an element of that set iff that thing is an F. The difference is vivid with examples like

' $\exists X[\forall x(Xx < -> x \notin x)]$ '. Given ZF set theory, no *set* is such that every thing is an element of that set iff that thing is nonselfelemental. But there are some things such that each thing is one of them iff it is nonselfelemental. Correspondingly, $\exists X[\forall x(Xx < -> Fx)]$ iff $\exists xFx$.

In this sense, introducing a variable that can have values relative to an assignment introduces nothing new. As Boolos (1998, p.72) says, "We need not construe second-order quantifiers as ranging over anything other than the objects over which our first-order quantifiers range...a second-order quantifier needn't be taken to be a kind of first-order quantifier in disguise, having items of a special kind, collections in its range." This matters. For in providing semantic theories of natural human languages, we can employ plural variables, each of whose values is among the things we quantify over when we employ singular quantification—instead of employing plural entities, whose elements are somehow more basic, as the only potential values for plural variables that still take only one value per assignment. This permits quantification over collections: one can posit sets without taking them to be the only values of plural variables. The issue here concerns semantic typology, not ontology. But plural variables make a difference.

In particular, they make room for essentially plural predicates. Some things can *together* satisfy an essentially plural predicate even if no one thing can satisfy the predicate. Boolos (1984) offers, among others, the example 'rained down'; some rocks can rain down even if no thing can. Schein (1993) offers 'clustered'; some elms can be clustered in the middle of the forest even if no single thing can be clustered anywhere. And importantly, given some things, they are sure to be plural in way that no thing can be. Unsurprisingly, 'plural' is a plural predicate *par excellence*. So we can introduce a pair of restricted quantifiers, ' $\exists X$:Plural(X)' and ' $\exists X$:¬Plural(X)'; where the latter is equivalent to ' $\exists x$ ', and $\exists X$:Plural(X)[$\forall x$:Xx(Fx)] iff $\exists x \exists y$ [Fx & Fy & $x \ne y$]. By contrast, $\exists X$:¬Plural(X)[$\forall x$:Xx(Fx)] iff one or more things *such they are not more than one* are such that

each of them is an F. So if $\exists X: Plural(X)[\forall x: Xx(Fx)], \exists X: \neg Plural(X)[\forall x: Xx(Fx)].$

This makes room for theories according to which: $Val(X, them_{N_1}, \mathbf{A})$ iff $\forall x \{Xx < ->$ Assigns($\mathbf{A}, x, 1$)} & Plural(X); where 'Assigns($\mathbf{A}, x, 1$)' means that \mathbf{A} assigns x, perhaps along with one or more other things, to the first variable. The idea is that some things are (together) values of them_{N1} relative to \mathbf{A} iff they are the things that \mathbf{A} assigns to the first index.

4. Plural Arguments

It is also plausible that $Val(X, turnip_N, \mathbf{A})$ iff $\forall x: Xx[Turnip(x)]$. This biconditional is compatible with an axiom like ' $\forall x[Val(x, turnip_N, \mathbf{A})$ iff Turnip(x)]', according to which: given anything, it, is a value of 'turnip' iff it, is a turnip. But we can equally well describe the meaning of 'turnip' as follows: given any one or more things, they, are values of 'turnip' iff each, of them, is a turnip. Conjunctivism is easily recast in these terms. One or more things are values of the phrase $[\Sigma \Sigma']$ iff those things are values of each concatenate.

$$Val(E, [\Sigma \Sigma'], \mathbf{A}) \text{ iff } Val(E, \Sigma, \mathbf{A}) \& Val(E, \Sigma', \mathbf{A})$$

So we can handle 'six' and 'six turnips' as follows, bearing in mind that the 's' in 'turnips' may mark agreement (as in 'zero/1.5/no turnips'), as opposed to intuitive plurality.

$$Val(X, six_{\Delta}, \mathbf{A}) iff Six(X)$$

$$Val(X, [six_A turnips_N]_N \mathbf{A}) iff Six(X) & \forall x: Xx[Val(x, turnip_N, \mathbf{A})]$$

Relative to any assignment, some things are values of 'six turnips' iff they are six *and* each of them is a turnip. We can represent the nondistributive character of six_A as above, taking the absence of distribution on the right of 'iff' to be significant. Or we can mark the essentially plural character of the predicate, as in 'Val(X, six_A , **A**) iff SIX(X)'. No fewer than six things can be six in this sense. No one thing, not even a six-membered thing, can be a value of 'six turnips'.

We can now return to 'stabbed them'. If stabbed_v is a distributive event predicate, unlike

rained_v, we can say that some things are values of stabbed_v iff each of them was a stab.

$$Val(E, stabbed_v, \mathbf{A}) iff \forall e: Ee[Event(e) \& PastStab(e, \mathbf{A})]$$

Conjunctivism tells us what to say next.

$$Val(E, [stabbed_v them_{Nl\Theta}]_v, \mathbf{A}) iff$$

$$\forall$$
e:Ee[Event(e) & PastStab(e, **A**)] & Val(E, them_{N10}, **A**)

One or more things are values of the V-phrase relative to \mathbf{A} iff: each of them was a stab relative to \mathbf{A} ; and they satisfy the condition imposed by them_{N1\theta} relative to \mathbf{A} . At this point, we must tweak the earlier (singularist) characterization of how ' Θ ' influences interpretation. But instead of saying that *the* value of them_{N1\theta} relative to \mathbf{A} is *the* internal participant of *an* event, we can say that the values of them_{N1\theta} relative to \mathbf{A} *are* the internal participants of one or more events. And we want to say this, not just to preserve Conjunctivism, but because it is independently plausible.

Prima facie, (10) does not require any one event be a stabbing of all the demonstranda.

(10) Green stabbed them

Green may have stabbed one turnip in the kitchen at dawn, another in hall at noon, and a third in the library at dusk. A theorist, bent on maintaining a singularist conception of variables, might insist that the truth of (12) *does* require a single plural-event with at least one element per thing stabbed. But again, this is the fancy idea in need of support, given a less ontologically loaded option. Let 'Internal(E, X)' mean that the Xs are the internal participants of the Es. This can be spelled out in terms of 'Internal(e, x)' and first-order quantifiers.

Internal(E, X) iff $\forall e: Ee\{\exists x: Xx[Internal(e, x)]\} \& \forall x: Xx\{\exists e: Ee[Internal(e, x)]\}$

That is, the Xs are the internal participants of the Es iff: each E has an X as its internal participant, and each X is the internal participant of an E; or equivalently, no E has an internal participant that is not an X, and no X fails to be the internal participant of an E. Given this

generalization of the singular 'Internal(e, x)', to allow for plural variables, we can generalize the axiom specifying the semantic role of ' Θ '. In place of a singular axiom,

$$Val(e, \Sigma_{\Theta}, \mathbf{A}) \text{ iff } \exists x[Val(x, \Sigma, \mathbf{A}) \& Internal(e, x)]$$

we can offer a potentially plural variant by capitalizing.

$$Val(E, \Sigma_{\Theta}, \mathbf{A}) \text{ iff } \exists X[Val(X, \Sigma, \mathbf{A}) \& Internal(E, X)]$$

If Σ has exactly one value relative to A, this is a purely formal distinction. But if

$$Val(X, them_{N1}, \mathbf{A}) iff \forall x \{Xx \leftarrow Assigns(\mathbf{A}, x, 1)\} \& Plural(X)$$

then given that $Val(E, them_{N1\Theta}, \mathbf{A})$ iff $\exists X[Val(X, them_{N1}, \mathbf{A}) \& Internal(E, X)]$, we get the consequence noted below.

Val(E, [stabbed_V them<sub>N1
$$\Theta$$</sub>]_V, **A**) iff \forall e:Ee[Event(e) & PastStab(e, **A**)] & \exists X[\forall x{Xx <-> Assigns(**A**, x, 1)} & Plural(X) & Internal(E, X)]

One or more events are values of 'stabbed them' relative to \mathbf{A} iff those events are such that: each of them was a stab (relative to \mathbf{A}), and their internal participants were the things assigned by \mathbf{A} to the plural variable. The condition imposed by stabbed_v is distributive, while the condition imposed by them_{N1\theta} is not. But this is compatible with Conjunctivism, which imposes no conditions apart from conjoinability on the conditions imposed by each concatenate. If it aids comprehension, ' $\exists X[\forall x \{Xx < -> Assigns(\mathbf{A}, x, 1)]\}$ & Plural(X) & Internal(E, X)]' can be replaced with ' ιX :Assigns(\mathbf{A} , X, 1)[Plural(X) & Internal(E, X)]', using a potentially plural descriptor. But the idea, however encoded, is that one or more Es are values of the plural variable them_{N1\theta} iff the things assigned to the variable are the internal participants of those Es.¹⁴

Similarly, given that

$$Val(E, \Sigma_{\Theta}, \mathbf{A})$$
 iff $\exists X[Val(X, \Sigma, \mathbf{A}) \& External(E, X)]$

we get the desired result for plural demonstrative subjects.

 $Val(E, [They_{N2\Theta} [stabbed_v them_{N1\Theta}]_v]_v, \mathbf{A}) iff$

ιX:Assigns(A, X, 2)[Plural(X) & External(E, X)] &

∀e:Ee[Event(e) & PastStab(e, **A**)] &

tX:Assigns(A, X, 1)[Plural(X) & Internal(E, X)]

This does not require that each value of the V-phrase be a composite thing, with events as parts, that has a plural-entity as its sole Agent and a plural-entity as its sole Theme. It says that one or more events, which may have occurred at disparate times and places, are values of the V-phrase relative to assignment **A** iff those events satisfy three conditions: their External participants (Agents) are the things that **A** assigns to the second variable; each of them was a stab; and their Internal participants (Themes) are the things that **A** assigns to the first variable.

Note that "collective" readings do not imply cooperation; see Gillon (1987), Davies (1989), Higginbotham and Schein (1989), Schein (1993). If five professors wrote six papers, it *may* be that the five worked together. But each professor may have acted alone. Or there may have been partial cooperation: perhaps McKay and McBee coauthored three papers, and their rivals wrote three; perhaps Brown, Jones, and Smith wrote one paper, Jones, Smith and McKay wrote another, and so on. There are many ways for 'Five Xs wrote six Ys' to be nondistributively true. Conjunctivist theories with plural variables can capture this indifference to cooperation.

On any such view, ' \exists E[...E...]' means that *one or more* things are such that *they* satisfy the (perhaps complex) condition imposed. This should come as no surprise. As Ramsey (1927) noted, a sentence like 'Plum stabbed Green' implies *at least one* stabbing of Green by Plum, with no further commitment concerning the number of stabbings. And [stabbed_V [six_A turnips_N]_{NΘ}]_V poses no special difficulties, assuming that Val(E, Σ_{Θ} , **A**) iff \exists X[Val(X, Σ , **A**) & Internal(E, X)].

 $Val(E, [stabbed_V [six_A turnips_N]_{N\Theta}]_V, \mathbf{A}) iff$

 \forall e:Ee[Event(e) & PastStab(e, **A**)] & Val(E, [six_A turnips_N]_{N Θ}, **A**)

 $Val(E, [six_A turnips_N]_{N\Theta}, \mathbf{A}) iff \exists X \{SIX(X) \& \forall x : Xx[Turnip(x)] \& Internal(E, X)\}$

Relative to any assignment, some things are values of 'stabbed six turnips' iff: each of those things was a stab, and six turnips were the internal participants (Themes) of those things.

This captures the collective reading of (11).

(11) They stabbed six turnips

The demonstranda were the Agents of some events, each a stabbing, whose Themes were six turnips. The eventish entailments and nonentailments of (12) can also be captured,

(12) They stabbed six turnips with three knives on Monday without saying that (12) implies *an* event such that: its Agent was the collection of the demonstrated individuals; it was composed of some stabs; its Theme was a collection of six turnips; it was done with a collection of three knives; and it occurred (scattered) on the relevant day. We also get, without hard work, the result that some things are values of 'stabbed turnips' iff: each of those things was a stab, and (some) turnips were the Themes of those things.¹⁵

But what about distributive readings of (11) and (12), according to which each of the demonstrated individuals stabbed six turnips? And what about the singular (13)?

(13) Plum stabbed every turnip

This might seem to halt the Conjunctivist train. But we already have the apparatus needed to analyze 'every' and 'every turnip' as monadic predicates conjoinable with others.

5. Frege-Pairs as Values of Quantifiers

Suppose the grammatical structure of (13) is as shown in (13G), ignoring for a moment the internal structure of the quantificational phrase.

(13G)
$$\langle [[every turnip]_1 \langle [Plum_{N\Theta} [stabbed_v t_{1\Theta}]_v]_v \rangle] \rangle$$

Let me stress three aspects of this now common hypothesis, which posits a transformation not audibly signalled in English: 'every turnip' is displaced from its original (direct object) position, leaving a trace; it recombines with the "open" sentence created by the displacement; and the resulting combination is marked as a sentence. Now let me elaborate (13G), indicating that 'every' is a determiner that takes a (singular) noun as its internal argument, and an open sentence as its external argument. But this time, ignore the internal structure of the embedded sentence.

$$(13G+) \langle [[every_D turnip_{N\Delta}]_{D1} \langle Plum stabbed t_1 \rangle_{\Delta}]_D \rangle$$

I use ' Δ ' and ' $\underline{\Delta}$ ', instead of ' $\underline{\Theta}$ ' and ' $\underline{\Theta}$ ', so that we can remain agnostic for now about whether being an argument of a determiner differs semantically from being an argument of a verb.

Conjunctivism implies that relative to any assignment A, (13G+) gets the value τ iff one or more things are such that they are values of both major constituents.

$$\exists E\{Val(E, [every_D turnip_{N\Delta}]_{D1}, \mathbf{A}) \& Val(E, \langle Plum stabbed t_1 \rangle_{\Delta}, \mathbf{A})\}$$

Despite initial appearances, this specification of what (13) means has a perfectly coherent gloss that turns out to be theoretically attractive. The variable 'E' can range over things of the form $<\tau$, x> and $<\bot$, x>; where x is a potential value of a (singular) variable like ' t_1 '. Given the entity Green, we have the ordered pairs $<\tau$, Green> and $<\bot$, Green>; and given any potential value A1 of the variable ' t_1 ', we have $<\tau$, A1> and $<\bot$, A1>. Call these abstracta, each of which has an entity as its internal participant and a sentential value as its external participant, *Frege-Pairs*.

We already appealed to Frege-Pairs, in effect, by construing 'dba'/'01011' as a way of answering questions about whether or not a given assignment assigns a certain entity, perhaps along with others, to a given variable. My suggestion now is that determiners like 'every' are (plural) predicates satisfiable by Frege-Pairs. Initially, this might seem strange. But (13) is true iff there are some Frege-Pairs $_{\rm E}$ such that: each of them $_{\rm E}$ has $_{\rm T}$ as its external participant; the turnips

are their_E internal participants; and each of them_E has τ as its external participant iff Plum stabbed its internal participant. We can encode this biconditional fact more formally,

$$\begin{split} Val(\top, \langle [[every_D \ turnip_{N\Delta}]_{D1} \ \langle Plum \ stabbed \ t_1 \rangle_{\underline{\Delta}}]_D \rangle, \ \pmb{A}) \ iff \\ \\ \exists E \{ \forall e: Ee[External(e, \, \top)] \ \& \ \exists X: \forall x [Xx <-> Turnip(x)] \{ Internal(E, \, X) \} \ \& \\ \\ \forall e: Ee \{ External(e, \, \top) <-> \exists x: Internal(e, \, x) [Plum \ stabbed \ x] \} \} \end{split}$$

assuming at least one turnip, for simplicity. We can also adopt the following axiom for 'every'.

$$Val(E, every_D, \mathbf{A}) \text{ iff } \forall e: Ee[Frege-Pair(e) \& External(e, \top)]$$

On this view, one or more things are values of 'every' iff each of them is of the form $\langle \top, x \rangle$.

So if the two biconditionals below are consequences of plausible semantic principles,

$$Val(E, turnip_{N\Delta}, \mathbf{A}) \ iff \ \exists X : \forall x [Xx <-> Turnip(x)] \{Internal(E, X)\}$$

 $Val(E, \langle Plum \ stabbed \ t_1 \rangle_{\Delta}, \mathbf{A}) \ iff$

$$\forall$$
e:Ee{External(e, \top) <-> \exists x:Internal(e, x)[Plum stabbed x]}

Conjunctivists can handle (13). Determiners and their arguments can be conjoinable predicates that impose (plural) conditions on Frege-Pairs. In Pietroski (2005), I argue that such biconditionals do follow from independently plausible principles. Here, I present the gist.

In $\langle [Plum_{N\underline{\Theta}} [stabbed_v it_{N1\underline{\Theta}}]_v \rangle$, neither noun is marked as plural. We can specify values for singular arguments in various ways. But for present purposes, consider the following.

$$Val(X, Plum_N, \mathbf{A}) \ iff \ \forall x (Xx <-> x = Plum)$$

$$Val(X, it_{N1}, \mathbf{A}) \ iff \ \forall x \{Xx <-> Assigns(\mathbf{A}, x, 1)\} \ \& \ \neg Plural(X)$$

The result

$$\begin{aligned} & Val(\top, \langle [Plum_{N\underline{\Theta}} \ [stabbed_V \ it_{1\underline{\Theta}}]_V \rangle, \textbf{A}) \ iff \\ & \exists E \{\exists X [\forall x (Xx <-> x = Plum) \ \& \ External(E, X)] \ \& \ Val(E, stabbed_V, \textbf{A}) \ \& \\ & \exists X [\forall x (Xx <-> Assigns(\textbf{A}, x, 1) \ \& \ \neg Plural(X) \ \& \ Internal(E, X)] \} \end{aligned}$$

can be simplified, by letting 'A1' signify the thing that A assigns to the first variable.

$$Val(\tau, \langle Plum \ stabbed \ it_1 \rangle, \mathbf{A}) \ iff \ Plum \ stabbed \mathbf{A}1.$$

Assuming that traces of displaced D-phrases are relevantly like singular demonstratives,

$$Val(X, t_1, \mathbf{A}) \text{ iff } \forall x \{Xx \leftarrow Sasigns(\mathbf{A}, x, 1)\} \& \neg Plural(X)$$

we get a similar result: Val $(\top, \langle \text{Plum stabbed } \mathbf{t}_1 \rangle, \mathbf{A})$ iff Plum stabbed A1.

As expected, the open *sentence* has a *sentential* value (τ or \bot) relative to any assignment of a value to the variable. Given any assignment **A**, Tarski (1933) showed us how to think about a variant assignment **A'**—just like **A** except perhaps with regard to what **A** assigns to a certain index, say '1'—and the value of the open sentence relative to **A'**. A familiar move, at this point, is to introduce some trick for *reconstruing* the open sentence: as a predicate whose values are entities, like people and turnips; or as an abstract predicate whose values are sets. ¹⁷ But Functionist reconstrual of the open sentence turns out to be unneeded and unwanted.

In $[[every_D \ turnip_{N\Delta}]_{D1} \langle Plum \ stabbed \ t_1 \rangle_{\underline{\Delta}}]_D$, the open sentence is the external argument, while the noun is the internal argument of the determiner. Correspondingly, the external participant of a Frege-Pair is \top or \bot , while the internal participant is an entity. With this in mind, consider again the proposed biconditionals.

$$Val(E, every_D, \mathbf{A}) \ iff \ \forall e: Ee[Frege-Pair(e) \ \& \ External(e, \ \top)]$$

$$Val(E, turnip_{N\Delta}, \mathbf{A}) \ iff \ \exists X: \forall x[Xx <-> Turnip(x)] \{Internal(E, X)\}$$

$$Val(E, \langle Plum \ stabbed \ t_1 \rangle_{\underline{\Delta}}, \mathbf{A}) \ iff$$

$$\forall e: Ee \{External(e, \ \top) <-> \exists x: Internal(e, \ x)[Plum \ stabbed \ x]\}$$

Behind this formalism is a simple idea. Start with the turnips, and pair each with \top or \bot . There will be many ways of doing this, since each turnip can be associated with either sentential value. (In the case at hand, each turnip might or might not have been stabbed by Plum.) Given

any Frege-Pairs_E that associate all and only the turnips with sentential values, they_E are (nondistributively) values of turnip_{N Δ}. Suppose there are exactly five turnips: a, b, c, d, e. Then three ways of associating the turnips with \top or \bot are indicated below.

The first five Frege-Pairs are (together) values of $turnip_{N\Delta}$, as are the next five, and the next five. But of these, only the last five Frege-Pairs are values of $every_D$.

There are many other ways of satisfying the condition imposed by every_D.

$$<\tau, a> <\tau, c> <\tau, e>$$
 $<\tau, f>$ $<\tau, b> <\tau, d> <\tau, f> <\tau, g>$

But none of these are choices of Frege-Pairs that are also values of turnip_{N Δ}. One or more things are values of turnip_{N Δ} iff they pair *all and only* the turnips with τ or \bot . While turnip_{N Δ} doesn't care about which value a given turnip is paired with, turnip_{N Δ} does require that no turnip be omitted, and that no nonturnip be included. (This is what one expects the restrictor in a restricted quantifier to do.) By contrast, every_D doesn't care about which entities are paired with values; it simply imposes the condition that each entity be paired with τ . The phrase [every_D turnip_{N Δ}]_D cares about *both* dimensions of Frege-Pairs. In our example, one or more things are values of this conjunctive predicate iff they are the following: $\langle \tau, a \rangle, \langle \tau, b \rangle, \langle \tau, c \rangle, \langle \tau, d \rangle$, and $\langle \tau, e \rangle$.

With regard to $\langle \text{Plum stabbed } t_1 \rangle_{\underline{\Delta}}$, the idea is that this open-sentence-as-marked-by-a-D doesn't care about either dimension of Frege-Pairs *per se*. Rather, it imposes a condition on how entities can be paired with sentential values. More specifically, each value of $\langle \text{Plum stabbed } t_1 \rangle_{\Delta}$

conforms to the condition imposed by the open sentence: \top iff Plum stabbed the entity in question; where for each Frege-Pair, the entity in question is its internal participant. (What else?) Many choices of turnipless Frege-Pairs are sure to be choices of Frege-Pairs that meet this requirement. Plum stabbed Green, or he didn't. So either $<\top$, Green> or $<\bot$, Green> is, all by itself, a value of \langle Plum stabbed $t_1\rangle_{\underline{\Delta}}$. And if Plum stabbed Green, but not Scarlet or White, then $<\top$, Green> and $<\bot$, Scarlet> and $<\bot$, White> are together values of \langle Plum stabbed $t_1\rangle_{\underline{\Delta}}$. But the values of [every_D turnip_{N Δ}]_D are also values of \langle Plum stabbed $t_1\rangle_{\underline{\Delta}}$ iff Plum stabbed each turnip.

So we want to preserve the content, if not the form, of the biconditional below.

Val(E,
$$\langle \text{Plum stabbed } \mathbf{t}_1 \rangle_{\Delta}, \mathbf{A})$$
 iff

$$\forall$$
e:Ee{External(e, \top) <-> \exists x:Internal(e, x)[Plum stabbed x]}

Given a variable and any assignment **A**, each Frege-Pair can be viewed as a recipe for creating a Tarski-variant **A'**: given $< \top$, x > or $< \bot$, x >, replace whatever **A** assigns to the variable with x; see Pietroski (2005). So we can rewrite ' $\exists x$:Internal(e, x)[Plum stabbed x]' as follows.

$$\exists A':A' \approx_1 A[Internal(e, A'1) \& Plum stabbed A'1]$$

And by replacing ' $\underline{\Delta}$ ' with ' Δi ', where i is the index of the relevant determiner phrase, we can formulate a general principle; cf. Heim and Kratzer (1998).

for any index *i*, Val(E,
$$\langle ... \rangle_{\Delta i}$$
, **A**) iff

$$\forall e: \text{Ee}\{\text{External}(\mathbf{e}, \ \top) < -> \exists \mathbf{A'}: \mathbf{A'} \approx_i \mathbf{A}[\text{Internal}(\mathbf{e}, \ \mathbf{A'}i) \ \& \ \text{Val}(\ \top, \ \langle \ \dots \ \rangle, \ \mathbf{A'})]\}$$

Given that the Es are Frege-Pairs, each of which has an internal element that is the thing assigned to the ith variable by some i-variant of \mathbf{A} , we can rewrite the condition above;

for any index *i*, Val(E,
$$\langle ... \rangle_{\Delta i}$$
, **A**) iff

$$\forall$$
e:Ee{External(e, \top) <-> \exists **A'**:**A'** $\approx_{i:e}$ **A**[Val(\top , $\langle ... \rangle$, **A'**)]}

where $\mathbf{A'} \approx_{i:e} \mathbf{A}$ iff $\mathbf{A'}$ is just like \mathbf{A} , except that $\mathbf{A'}i$ is the entity (associated with \top or \bot) in e.

Indeed, this may be a Conjunctivist justification for appeal to Tarski-variants in *natural* language semantics. And given Tarski-variants, along with existential closure, Conjunctivists can handle examples with multiple quantifiers and relative clauses, as in (14); see Pietroski (2005).

(14) Every professor who found Green stabbed every turnip

One would like a still simpler general principle for external arguments of determiners.¹⁸ And we can rewrite once more, since each Frege-Pair has τ or \bot as its external participant.

$$Val(E, \langle ... \rangle_{\Delta i}, \mathbf{A}) \text{ iff } \forall e: Ee\{External(e, \exists \mathbf{A'}: \mathbf{A'} \approx_{i:e} \mathbf{A}[Val(\tau, \langle ... \rangle, \mathbf{A'})])\}$$

This says that one or more things_E are values of $\langle ... \rangle_{\Delta i}$ relative to **A** iff each_e of them_E is such that its_e external participant is $(\top \text{ iff } \top \text{ is})$ the value of $\langle ... \rangle$ relative to the variant of **A** that replaces **A***i* with its_e entity. This hypothesis about ' Δ_i '—or more precisely, about the significance of being an indexed argument of a determiner—is no more complex or *ad hoc* than available alternatives.

The biconditional for turnip_{N Δ}

$$Val(E, turnip_{N\Delta}, \mathbf{A}) \text{ iff } \exists X \{ \forall x [Xx <-> Turnip(x)] \& Internal(E, X) \}$$

suggests a hypothesis about ' Δ ', the mark of a determiner's internal argument.

$$Val(E, \Sigma_{\Delta}, \mathbf{A}) \text{ iff } \exists X \{ \forall x [Xx <-> Val(x, \Sigma, \mathbf{A})] \& Internal(E, X) \}$$

Relative to any assignment A, one or more things_E are values of Σ as Δ -marked iff (all and only) the values of Σ are (together) the internal participants of those_E things. The earlier axiom for ' Θ '

$$Val(E, \Sigma_{\Theta}, \mathbf{A})$$
 iff $\exists X[Val(X, \Sigma, \mathbf{A}) \& Internal(E, X)]$

was a little different. But this matters only for internal arguments like the bare plural 'turnips', with no independent element (like the index on them $_{\Theta 1}$) requiring that *all* values of the unmarked expression be internal participants of the relevant Es. So if the internal argument is somehow indexed, or otherwise forces "maximization," the Δ/Θ distinction is eliminable. In any case, the Conjunctivist hypothesis is natural enough: *the* turnips, and not merely some turnips, are the

relevant internal participants when 'turnip' is the internal argument of a determiner.¹⁹

The proposal can be extended to determiners other than 'every'.

$$Val(E, no_D, \textbf{A}) \ iff \ \forall e: Ee[Frege-Pair(e)] \ \& \ \neg \exists e: Ee[Internal(e, \ \top)]$$

$$Val(E, most_D, \mathbf{A}) iff \forall e: Ee[Frege-Pair(e)] \& \exists Y \exists N \{Outnumber(Y, N) \& \}$$

$$\forall e[Ye \leftarrow Ee \& External(e, \top)] \& \forall x[Nx \leftarrow Ee \& External(e, \bot)]$$

If words like 'six' can be displaced determiners, as in $\langle [[six_D \ turnips_{N\Delta}]_{D1} \ \langle Plum \ stabbed \ t_1 \rangle_{\underline{\Delta}}]_D \rangle$, Conjunctivists can capture at least many distributional readings with axioms like the following.

$$Val(E, six_D, \mathbf{A})$$
 iff

$$\forall e$$
:Ee[Frege-Pair(e)] & $\exists Y \{ Six(Y) \& \forall e [Ye <-> Ee \& External(e, $\tau)] \}$$

Work remains. But Conjunctivism is compatible with a wide range of predicate-argument combinations, even given that determiner-predicate combinations are second-order examples. By ascribing limited significance to grammatical relations, we can represent lexical meanings in a way that lets us view concatenation as a way of *conjoining* predicates.

Notes

- ¹ See Castañeda (1967), Carlson (1984), Taylor (1985), Higginbotham (1985), Parsons (1990), and for more recent developments, Higginbotham, Pianesi, and Varzi (2000).
- ² Comparative adjectives like 'big' introduce familiar complications that I cannot address here. See Pietroski (2005, forthcoming) for development of the idea—related to the account of plurality below, and to Higginbotham (1985)—that Adam is a big ant iff: the ants are such that Adam is a big one (of them); i.e., Adam is an ant *and* a big one.
- ³ I use 'x' for mnemonic convenience, without type-restrictions on metalanguage variables. But human language variables may differ, in ways we can note with ' \exists e:Event(e)' and ' \exists x:Entity(x)'.
- ⁴ Davidson spelled the ternary predicate 'PastStabOfBy' differently, using 'Stabbed', but this is irrelevant to his theory. Like Parsons (1990), if not for his reasons, I countenance the possibility of stabs (a.k.a. stabbings) without stabbers and stabs without stabbees; see also Borer (2004).
- ⁵ Either by verb-raising or tense-lowering. See Pollack (1989) and Cinque (1999) for discussion of functional elements posited above the basic "V-shell."
- ⁶ See Higginbotham (1985), Larson and Segal (1995); cp. Tarski (1933). This simple picture will be modified to account for plurality. But let's not worry here about whether each name has a unique bearer: perhaps $Val(x, Green_{Ni}, \mathbf{A})$ iff x is both a bearer of $Green_N$ and associated with index i; see Burge (1973), Katz (1994). Baker (2003) argues that all nouns are indexed.
- ⁷ See Gruber (1965), Fillmore (1968), Jackendoff (1972), Chomsky (1981), Higginbotham (1985). See Hornstein (2001) for a view that takes the idea of theta-*marking* quite seriously.
- ⁸ See Dowty (1979, 1991) for related discussion, with different emphases; see also Baker (1997).
- ⁹ See Burzio (1986), Belletti (1988). Whatever the grammatical structure of intransitive examples like 'I sang/dreamt/counted', there is a sense in which any singing/dreaming/counting is *of*

something (a tune, a dream, some numbers). By contrast, a death (as opposed to a murder) need not be *by* something. See Hale and Keyser (1993), Tenny (1994). One shouldn't read too much into 'Theme': Themes are internal participants of potential values of event predicates. But in paradigmatic cases, the Theme of a event lets us "measure" the event in Tenny's sense: a stabbing of Green has occurred when Green is impacted in a certain way.

¹⁰ Put another way, Green_N and Plum_N are the arguments of stabbed_V and [stabbed_V Green_{N Θ}]_V. Or perhaps Plum_N is the argument of a covert verb that combines with the intrinsically intransitive stabbed_V. See Williams (1981), Marantz (1984), Hale and Keyser (1993), Chomsky (1995), Kratzer (1996), Baker (2003). We could recode, as in Pietroski (2005), using 'ext-' and 'int-' to reflect external and internal arguments of the verb: [ext-Plum_N [stabbed_V int-Green_N]].

¹¹ See Larson and Segal (1995), whose treatment of sentential connectives and transitive verbs is almost Conjunctivist.

 12 If the modification comes earlier, as in [[stabbed_v Green_N]_v quickly_A]_v, this is a further complication; see Kratzer (1996), Chung and Ladusaw (2003). Cases like 'There was a stabbing in the kitchen' and 'Plum kicked Green the ball' pose further difficulties; see Borer (2004).

¹³ I take no stand here on the utility of a Boolos-style interpretation for other projects. But one should not confuse empirical hypotheses, about natural languages and children who acquire them, with claims about how logicians should interpret all their second-order variables.

¹⁴ See Schein (1993), which I draw on. In Pietroski (2005), I used a slightly different principle.

$$Val(E, \Sigma_{\Theta}, \mathbf{A}) \ iff \ \exists X [\forall x \{Xx < -> Val(x, \Sigma, \mathbf{A})\} \ \& \ Internal(E, X)]$$

¹⁵ Perhaps the "bare" plural object really combines with a covert element like 'some'. But this is not required; see Chierchia (1998). My thanks to Ivano Caponegro and Veneeta Dayal for discussion that helped me see a difficulty here for Pietroski (2005); cf. note 14.

¹⁶ See May (1985), Higginbotham and May (1981), and for discussion relevant to the labelling, Hornstein and Uriagereka (1999). Pietroski (2003) includes some Whiggish history.

¹⁷ Frege (1892) spoke of Concepts and their Value-Ranges. See Pietroski (2005) for discussion in the context of questions about what distinguishes a sentence of *natural* language from a list.

¹⁸ We can't say that Val(E, $\langle ... \rangle_{\Delta i}$, **A**) iff $\exists X[Val(X, \langle ... \rangle, A)$ & External(E, X)]. Relative to each assignment **A**, each value of $\langle Plum \text{ stabbed } t_1 \rangle_{\Delta 1}$ would have the same external participant (τ or \bot) depending on whether or not Plum stabbed **A**1. But the $\Delta i/\underline{\Theta}$ distinction is due to indices, interpreted in terms of assignment-variants, which are indispensible.

¹⁹ One can describe 'most' as a monadic predicate of Frege-Pairs, but specify this predicate in terms of 'Outnumber(Y, N)', which can be cashed out in terms of one-to-one correspondence. While space constraints forbid discussion, this rewrite of "generalized quantifier theory" implies (trivially) that determiners are conservative in Barwise and Cooper's (1981) sense; see also Higginbotham and May (1981). Every bottle fell iff every bottle is a bottle that fell, since: whenever the bottles are the internals of some Frege-Pairs_F, each of them_F is such that its external is τ *iff its internal fell* iff its external is τ *iff its internal is a bottle that fell*. See Pietroski (2005).

References

Baker, M. (1997): Thematic Roles and Grammatical Categories. In L. Haegeman (ed.), *Elements of Grammar*, Dordrecht: Kluwer, pp. 73-137.

—(2003): Verbs, Nouns, and Adjectives. Cambridge: Cambridge University Press

Barwise, J. and Cooper, R. (1981): Generalized Quantifiers and Natural Language. *Linguistics and Philosophy* 4:159-219.

Belletti, A. (1988): The Case of Unaccusatives, Linguistic Inquiry 19:1-34.

Boolos, G. (1984): To be is to be the Value of a Variable (or the Values of Some Variables). *Journal of Philosophy* 81: 430-50.

—(1998): Logic, Logic, and Logic. Cambridge, MA: Harvard University Press.

Borer, H. (2004). Structuring Sense. Oxford: Oxford University Press.

Burge, T. (1973): Reference and Proper Names. Journal of Philosophy 70:425-39.

Burzio, L. (1986): Italian Syntax. Dordrecht: Reidel.

Carlson, G. (1984): Thematic Roles and their Role in Semantic Interpretation. Linguistics 22: 259-79.

Castañeda, H. (1967): Comments. In Rescher (1967).

Chierchia, G. (1998): Reference to Kinds Across Languages. *Natural Language Semantics* 6: 339-405.

Chomsky, N. (1981): Lectures on Government and Binding. Dordrecht: Foris.

—(1995): The Minimalist Program. Cambridge, MA: MIT Press.

Chung, S. and Ladusaw, W. (2003): Restriction and Saturation. Cambridge, MA: MIT Press.

Cinque, G. (1999): *Adverbs and Functional Heads—A Cross-Linguistic Perspective*. Oxford: Oxford University Press.

Davidson, D. (1967): The Logical Form of Action Sentences. In Rescher (1967).

—(1985): Adverbs of Action. In Vermazen, B. and Hintikka, M. (eds.), *Essays on Davidson: Actions and Events* (Oxford: Clarendon Press).

Davies, M. (1989): Two Examiners Marked Six Scripts. Linguistics and Philosophy 12: 293-323.

Dowty, D. (1979): Word Meaning and Montague Grammar. Boston: Reidel.

—(1991): Thematic Proto-roles and Argument Selection. *Language* 67:547-619.

Fillmore, C. (1968): The Case for Case. In E. Bach and R. Harms (ed.), *Universal in Linguistic Theory* (New York: Rinehart and Wilson).

Frege, G. (1892): Function and Concept. Geach and Black (1980).

Gillon. B. (1987): The Readings of Plural Noun Phrases in English. *Linguistics and Philosophy* 102:199-219.

Gruber, J, 1965: Studies in Lexical Relations. Doctoral dissertation, MIT.

Hale, K. and Keyser, S. (1993): On Argument Structure and the Lexical Expression of Syntactic

Relations. In Hale, K. and Keyser, S.J., eds., The View From Building 20. Cambridge, MA: MIT Press.

Heim, I. and Kratzer, A. (1998): Semantics in Generative Grammar. Oxford: Blackwell.

Herburger, E. (2001): What Counts. Cambridge, MA: MIT Press.

Higginbotham, J. (1985): On Semantics. Linguistic Inquiry 16: 547-93.

—(1998): On Higher-Order Logic and Natural Language. Proc. of the British Academy 95: 1-27.

Higginbotham, J. and May, R. (1981): Questions, Quantifiers, and Crossing. Linguistic Review 1: 47-79.

Higginbotham, J. and Schein, B. (1989): Plurals. NELS XIX, Proceedings GSLA (UMass: Amherst).

Higginbotham, J., Pianesi, F. and Varzi, A. (eds.) (2000): *Speaking of Events*. Oxford: Oxford University Press.

Hornstein, N. (1995): Logical Form: From GB to Minimalism. Oxford: Blackwell.

Hornstein, N. and Uriagereka, J. (1999): Labels and Reprojection: A Note on the Syntax of Quantifiers.

University of Maryland Working Papers in Linguistics, vol. 8.

Jackendoff, R. (1972): Semantic Interpretation and Generative Grammar. Cambridge, MA: MIT Press.

Katz, J. (1994): Names Without Bearers. *The Philosophical Review* 103:1-39.

Kratzer, A. (1996): Severing the External Argument from its Verb. In J. Rooryck and L. Zaring (eds.), *Phrase Structure and the Lexicon*. Dordrecht (Kluwer Academic Publishers).

Larson, R. and Segal, G. (1995): Knowledge of Meaning. Cambridge, MA: MIT Press.

Link, G. (1998): Algebraic Semantics in Language and Philosophy. Stanford: CSLI.

Marantz, A. (1984): On the Nature of Grammatical Relations. Cambridge, MA: MIT Press.

May, R. (1985): Logical Form: Its Structure and Derivation. Cambridge, MA: MIT Press.

Montague, R. (1974): Formal Philosophy. New Haven: Yale University Press.

Parsons, T. (1990): Events in the Semantics of English. Cambridge, MA: MIT Press.

Pietroski, P. (2002): Function and Concatenation. In Preyer and Peters (2002).

—(2003): Quantification and Second-Order Monadicity. *Philosophical Perspectives* 17: 259-298.

—(2005): Events and Semantic Archiecture. Oxford: Oxford University Press.

Pollack, J. (1989): Verb Movement, Universal Grammar, and the Structure of IP. *Linguistic Inquiry* 20: 365-424.

Ramsey, F. (1927): Facts and Propositions. Aristotelian Society Supplementary Volume VII:153-70.

Rescher, N. (ed) (1967): The Logic of Decision and Action. Pittsburgh: University of Pittsburgh Press.

Schwarzschild, R. (1996): Pluralities. Dordrecht: Kluwer.

—The Grammar of Measurement (2002): in Brendan Jackson (ed), Proceedings of Semantics and

Linguistic Theory XII. Ithaca: CLC Publications, Department of Linguistics, Cornell University.

Schein, B. (1993): *Plurals*. Cambridge, MA: MIT Press.

—forthcoming. Conjunction Reduction Redux. Cambridge, MA: MIT Press.

Tarski, A. (1933): "The Concept of Truth in Formalized Languages," in Logic, Semantics,

Metamathematics (trans. J.H. Woodger, 2nd edition, ed. J Corcoran), Indianapolis: Hackett (1983).

—(1944): The Semantic Conception of Truth. *Philosophy and Phenomenological Research* 4:341-75.

Taylor, B. (1985): Modes of Occurrence. Oxford: Blackwell.

Tenny, C. (1994): Aspectual Roles and and the Syntax-Semantics Interface. Dordrecht: Kluwer.

Williams, E. (1981): Argument Structure and Morphology. The Linguistic Review 1: 81-114.