Mathematics > Logic

Borel's Conjecture in Topological Groups

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(Submitted on 27 Jul 2011 (v1), last revised 5 Jul 2012 (this version, v2))

We introduce a natural generalization of Borel's Conjecture. For each infinite cardinal number κ, let {\sf BC}$_{\kappa}$ denote this generalization. Then ${\sf BC}_{\aleph_0}$ is equivalent to the classical Borel conjecture. Assuming the classical Borel conjecture, $\neg{\sf BC}_{\aleph_1}$ is equivalent to the existence of a Kurepa tree of height $\aleph_1$. Using the connection of ${\sf BC}_{\kappa}$ with a generalization of Kurepa's Hypothesis, we obtain the following consistency results:
(1)If it is consistent that there is a 1-inaccessible cardinal then it is consistent that ${\sf BC}_{\aleph_1}$.
(2)If it is consistent that ${\sf BC}_{\aleph_1}$ holds, then it is consistent that there is an inaccessible cardinal.
(3)If it is consistent that there is a 1-inaccessible cardinal with ω inaccessible cardinals above it, then $\neg{\sf BC}_{\aleph_{\omega}} \, +\, (\forall n<\omega){\sf BC}_{\aleph_n}$ is consistent.
(4)If it is consistent that there is a 2-huge cardinal, then it is consistent that ${\sf BC}_{\aleph_{\omega}}$.
(5)If it is consistent that there is a 3-huge cardinal, then it is consistent that ${\sf BC}_{\kappa}$ holds for a proper class of cardinals κ of countable cofinality.

Comments:	15 pages
Subjects:	**Logic (math.LO)**; General Topology (math.GN); Group Theory (math.GR)
MSC classes:	03E05, 03E35, 03E55, 03E65, 22A99
Cite as:	**arXiv:1107.5383 [math.LO]**
	(or **arXiv:1107.5383v2 [math.LO]** for this version)

Submission history

From: Marion Scheepers [view email]
[v1] Wed, 27 Jul 2011 05:05:29 GMT (17kb)
[v2] Thu, 5 Jul 2012 02:02:06 GMT (17kb)

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