

# A subset of $\mathbb{Z}^n$ whose non-computability leads to the existence of a Diophantine equation whose solvability is logically undecidable

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For  $K \subseteq \mathbb{C}$ , let  $B_n(K) = \{(x_1, \dots, x_n) \in K^n : \text{for each } y_1, \dots, y_n \in K \text{ the conjunction } (\forall i \in \{1, \dots, n\} (x_i = 1 \Rightarrow y_i = 1)) \text{ AND } (\forall i, j, k \in \{1, \dots, n\} (x_i + x_j = x_k \Rightarrow y_i + y_j = y_k)) \text{ AND } (\forall i, j, k \in \{1, \dots, n\} (x_i \cdot x_j = x_k \Rightarrow y_i \cdot y_j = y_k)) \text{ implies that } x_1 = y_1\}$ . We claim that there is an algorithm that for every computable function  $f: \mathbb{N} \rightarrow \mathbb{N}$  returns a positive integer  $m(f)$ , for which a second algorithm accepts on the input  $f$  and any integer  $n \geq m(f)$ , and returns a tuple  $(x_1, \dots, x_n) \in B_n(\mathbb{Z})$  with  $x_1 = f(n)$ . We compute an integer tuple  $(x_1, \dots, x_{20})$  for which the statement  $(x_1, \dots, x_{20}) \in B_{20}(\mathbb{Z})$  is equivalent to an open Diophantine problem. We prove that if the set  $B_n(\mathbb{Z})$  ( $B_n(\mathbb{N})$ ,  $B_n(\mathbb{N} \setminus \{0\})$ ) is not computable for some  $n$ , then there exists a Diophantine equation whose solvability in integers (non-negative integers, positive integers) is logically undecidable.

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