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A characterization of compact operators via the non-connectedness of the attractors of a family of IFSs

Alexandru Mihail, Radu Miculescu

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In this paper we present a result which establishes a connection between the theory of compact operators and the theory of iterated function systems. For a Banach space X , S and T bounded linear operators from X to X such that $\|S\| < 1$ and $\|T\| < 1$ and $w \in X$, let us consider the IFS $S_w = (X, f_1, f_2)$, where $f_1, f_2: X \rightarrow X$ are given by $f_1(x) = S(x)$ and $f_2(x) = T(x) + w$, for all $x \in X$. On one hand we prove that if the operator S is compact, then there exists a family $(K_n)_{n \in \mathbb{N}}$ of compact subsets of X such that A_{S_w} is not connected, for all $w \in H \cup K_n$. On the other hand we prove that if H is an infinite dimensional Hilbert space, then a bounded linear operator $S: H \rightarrow H$ having the property that $\|S\| < 1$ is compact provided that for every bounded linear operator $T: H \rightarrow H$ such that $\|T\| < 1$ there exists a sequence $(K_{T,n})_n$ of compact subsets of H such that A_{S_w} is not connected for all $w \in H \cup K_{T,n}$. Consequently, given an infinite dimensional Hilbert space H , there exists a complete characterization of the compactness of an operator $S: H \rightarrow H$ by means of the non-connectedness of the attractors of a family of IFSs related to the given operator.

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