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# Counting the number of solutions to the Erdos-Straus equation on unit fractions

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For any positive integer  $n\$ , let  $f(n)\$  denote the number of solutions to the Diophantine equation  $\frac{4}{n} = \frac{1}{x} + \frac{1}{y} + \frac{1}{y} + \frac{1}{z}\$  with  $x,y,z\$  positive integers. The  $\frac{1}{x} + \frac{1}{y} + \frac{1}{y} + \frac{1}{z}\$  with  $\frac{1}{z}\$  positive integers. The  $\frac{1}{y} + \frac{1}{z}\$  solutions conjecture} asserts that  $f(n) > 0\$  for every  $n \ge 2$ . To solve this conjecture, it suffices without loss of generality to consider the case when  $n\$  is a prime  $p\$ . In this paper we consider the question of bounding the sum  $\frac{p}{y}$ . In this paper we consider the question of bounding the sum  $\frac{p}{y}$ . In this paper we consider the question of bounding the sum  $\frac{p}{y}$ . In this paper we consider the asymptotic upper and lower primes. Our main result establishes the asymptotic upper and lower bounds  $\frac{1}{y}$  N  $\log^2 N$   $\frac{1}{y} + \frac{1}{y}$ . To  $\frac{p}{y} = 0(\log^3 p \log \log p)$   $\frac{1}{y}$  for a subset of primes of density arbitrarily close to 1; thus a typical prime has a relatively small number of solutions to the Erd H{o}s-Straus Diophantine equation.

We also establish some related results on f and related quantities, for instance establishing the bound  $f(p) \ln p^{3/5} + O(\frac{1}{\log\log p})$  for all primes p.

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