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Gaussian Behavior in Generalized Zeckendorf Decompositions

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A beautiful theorem of Zeckendorf states that every integer can be written uniquely as a sum of non-consecutive Fibonacci numbers $\{F_n\}_{n=1}^{\infty}$; Lekkerkerker proved that the average number of summands for integers in $[F_n, F_{n+1})$ is $n/(\phi^2 + 1)$, with ϕ the golden mean. Interestingly, the higher moments seem to have been ignored. We discuss the proof that the distribution of the number of summands converges to a Gaussian as $n \to \infty$, and comment on generalizations to related decompositions. For example, every integer can be written uniquely as a sum of the $\pm F_n$'s, such that every two terms of the same (opposite) sign differ in index by at least 4 (3). The distribution of the numbers of positive and negative summands converges to a bivariate normal with computable, negative correlation, namely $-(21-2\phi)/(29+2\phi) \approx -0.551058$.

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