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Effective results for unit equations over finitely generated domains

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Let A be a commutative domain containing Z which is finitely generated as a Z-algebra, and let a,b,c be non-zero elements of A. It follows from work of Siegel, Mahler, Parry and Lang that the equation (*) ax+by=c has only finitely many solutions in elements x,y of the unit group A* of A, but the proof following from their arguments is ineffective. Using linear forms in logarithms estimates of Baker and Coates, in 1979 Gy\H{o}ry gave an effective proof of this finiteness result, in the special case that A is the ring of S-integers of an algebraic number field. Some years later, Gy\H{o}ry extended this to a restricted class of finitely generated domains A, containing transcendental elements. In the present paper, we give an effective finiteness proof for the number of solutions of (*) for arbitrary domains A finitely generated over Z. In fact, we give an explicit upper bound for the `sizes' of the solutions x,y, in terms of defining parameters for A,a,b,c. In our proof, we use already existing effective finiteness results for two variable S-unit equations over number fields due to Gy\H{o}ry and Yu and over function fields due to Mason, as well as an explicit specialization argument.

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