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A Cesàro Average of Hardy-Littlewood numbers

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Let Λ be the von Mangoldt function and $(r_{\text{HL}})(n) = \sum_{\{m_1 + m_2^2 = n\}} \Lambda(m_1)$, be the counting function for the Hardy-Littlewood numbers. Let N be a sufficiently large integer. We prove that $\sum_{\{n \leq N\}} r_{\text{HL}}(n) \frac{\{(1 - n/N)^k\} \Gamma(k + 1)}{\pi^{1/2} \frac{N^{3/2} \Gamma(k + 5/2)}{\Gamma(k + 2)} - \frac{12}{\pi} \frac{N}{\Gamma(k + 2)} - \frac{\pi^{1/2}}{2} \sum_{\rho} \frac{\Gamma(\rho)}{\Gamma(k + 3/2 + \rho)} N^{1/2 + \rho} + \frac{1}{2} \sum_{\rho} \frac{\Gamma(\rho)}{\Gamma(k + 1 + \rho)} N^{\rho} + \frac{N^{3/4 - k/2} \pi^{k+1}}{\sum_{\ell \geq 1} \frac{J_{k+3/2}(2\pi \ell N^{1/2})}{\ell^{k+3/2}} - \frac{N^{1/4 - k/2} \pi^k}{\sum_{\rho} \frac{\Gamma(\rho)}{\Gamma(k + 1/2 + \rho)} (2\pi \ell N^{1/2}) \ell^{k+1/2 + \rho}} + O(\ell^{-k})}$, for $k > 1$, where ρ runs over the non-trivial zeros of the Riemann zeta-function $\zeta(s)$ and $J_{\nu}(u)$ denotes the Bessel function of complex order ν and real argument u .

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