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Weak Tensor Category and Related generalized Hopf algebras

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摘要   There are at least two kinds of generalization of Hopf algebra, i.e. pre-Hopf algebra and weak Hopf algebra. Correspondingly, we have two kinds of generalized bialgebras, almost bialgebra and weak bialgebra. Let ${{\mathscr C}}=({{\mathscr C}},\> \otimes,\>I,\>a,\>l,\>r)$ be a tensor category. By giving up $I,\>l,\>r$ and keeping $\otimes,\>a$ in ${\mathscr{C}}$, the first author got so-called pre-tensor category ${\mathscr{C}}=({\mathscr{C}},\> \otimes,\>a)$ and used it to characterize almost bialgebra and pre-Hopf algebra in {\it Comm. in Algebra}, {\bf 32}(2): 397--441 (2004). Our aim in this paper is to generalize tensor category ${\mathscr{C}}=({\mathscr{C}},\> \otimes,\>I,\>a,\>l,\>r)$ by weakening the natural isomorphisms $l,\>r$, i.e. exchanging the natural isomorphism $ll^{-1}=rr^{-1}=id$ into regular natural transformations $l\overline{l}l=l,\> r\overline{r}r=r$ with some other conditions and get so-called weak tensor category so as to characterize weak bialgebra and weak Hopf algebra. The relations between these generalized (bialgebras) Hopf algebras and two kinds generalized tensor categories will be described by using of diagrams. Moreover, some related concepts and properties about weak tensor category will be discussed.

关键词   Weak tensor category   Weak Hopf algebra   Pre-Hopf algebra   Strictization
分类号

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Abstract There are at least two kinds of generalization of Hopf algebra, i.e. pre-Hopf algebra and weak Hopf algebra. Correspondingly, we have two kinds of generalized bialgebras, almost bialgebra and weak bialgebra. Let ${{\mathscr C}}=({{\mathscr C}},\> \otimes,\>I,\>a,\>l,\>r)$ be a tensor category. By giving up $I,\>l,\>r$ and keeping $\otimes,\>a$ in ${\mathscr{C}}$, the first author got so-called pre-tensor category ${\mathscr{C}}=({\mathscr{C}},\> \otimes,\>a)$ and used it to characterize almost bialgebra and pre-Hopf algebra in {\it Comm. in Algebra}, {\bf 32}(2): 397--441 (2004). Our aim in this paper is to generalize tensor category ${\mathscr{C}}=({\mathscr{C}},\> \otimes,\>I,\>a,\>l,\>r)$ by weakening the natural isomorphisms $l,\>r$, i.e. exchanging the natural isomorphism $ll^{-1}=rr^{-1}=id$ into regular natural transformations $l\overline{l}l=l,\> r\overline{r}r=r$ with some other conditions and get so-called weak tensor category so as to characterize weak bialgebra and weak Hopf algebra. The relations between these generalized (bialgebras) Hopf algebras and two kinds generalized tensor categories will be described by using of diagrams. Moreover, some related concepts and properties about weak tensor category will be discussed.

Key words   Weak tensor category   Weak Hopf algebra   Pre-Hopf algebra   Strictization

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