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Inequalities of Jensen-Pečaric-Svrtan-Fan Type

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Abstract: By using the theory of majorization, the following inequalities of Jensen-Pecaric-Svrtan-Fan type are established: Let I be an interval, $f : I \to \mathbb{R}$ and $t \in I, x, a, b \in I^n$. If

$$a_1 \leq \cdots \leq a_n \leq b_n \leq \cdots \leq b_1, a_1 + b_1 \leq \cdots \leq a_n + b_n; f(t) > 0, f'(t) > 0, f''(t) > 0, f'''(t) < 0$$

then

$$\frac{f(A(a))}{f(A(b))} = \frac{f_{n,n}(a)}{f_{n,n}(b)} \leq \cdots \leq \frac{f_{k+1,n}(a)}{f_{k+1,n}(b)} \leq \frac{f_{k,n}(a)}{f_{k,n}(b)} \leq \cdots \leq \frac{f_{1,n}(a)}{f_{1,n}(b)} = \frac{A(f(a))}{A(f(b))},$$

the inequalities are reversed for $f''(t) < 0, f'''(t) > 0, \forall t \in I$, where $A(\cdot)$ is the arithmetic mean and