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A Generalization of Ankeny and Rivlin's Result on the Maximum Modulus of Polynomials not Vanishing in the Interior of the Unit Circle

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Abstract: For an arbitrary entire function f(z), let $M(f,r) = \max_{|z|=r}|f(z)|$. For a polynomial p(z) of degree n, it is known that $M(p,R) \leq R^n M(p,1)$, $R > 1$. By considering the polynomial p(z) with no zeros in $|z| < 1$, Ankeny and Rivlin obtained the refinement $M(p,R) \leq \{(R^n+1)/2\}M(p,1)$, $R > 1$. By considering the polynomial p(z) with no zeros in $|z| < k$, $(k \geq 1)$ and simultaneously thinking of $s^{\rm th}$ derivative $(0 \leq s < n)$ of the polynomial, we have obtained the generalization \begin{displaymath} $M(p^{(s)},R) \leq$ \left\{\begin{array}{l} $(1/2)\{\frac{d^s}{dR^s}(R^n + k^n)\}(2/(1+k))^n M(p,1)$, $R \geq k$,\ $(1/(R^s+k^s))[\{\frac{d^s}{dx^s}(1+x^n)\}_{x=1}]((R+k)/(1+k))^n M(p,1)$, $1 \leq R \leq k$,\end{array}\right. of Ankeny and Rivlin's result.

Key Words: Polynomial, maximum modulus principle, not vanishing in the interior of unit circle, generalization, s^{th} derivative

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